

## LC 2012 (SET E): PAPER 1

### QUESTION 6 (25 MARKS)

#### Question 6 (a)

$$f(x) = e^{-\frac{1}{2}x^2}$$

$$f'(x) = e^{-\frac{1}{2}x^2} \times \left(-\frac{1}{2}(2x)\right) = -xe^{-\frac{1}{2}x^2}$$

$$\begin{aligned} f''(x) &= -x(-xe^{-\frac{1}{2}x^2}) + e^{-\frac{1}{2}x^2}(-1) \\ &= x^2e^{-\frac{1}{2}x^2} - e^{-\frac{1}{2}x^2} \leftarrow \text{Factorise} \\ &= e^{-\frac{1}{2}x^2}(x^2 - 1) \end{aligned}$$

**FORMULAE AND TABLES BOOK**  
Calculus: Derivatives [page 25]

$$y = e^x \Rightarrow \frac{dy}{dx} = e^x$$

$$y = e^{f(x)} \Rightarrow \frac{dy}{dx} = e^{f(x)} \times f'(x)$$

#### MARKING SCHEME NOTES

##### Question 6 (a)

**First derivative: [Scale 10B (0, 7, 10)]**

7: • Any reasonable first step

**Second derivative: [Scale 10B (0, 7, 10)]**

7: • Any reasonable first step

#### Question 6 (b)

$$\frac{d^2y}{dx^2} = 0$$

POINT OF INFLECTION: Put  $\frac{d^2y}{dx^2} = 0$  and solve for  $x$ .

$$e^{-\frac{1}{2}x^2}(x^2 - 1) = 0$$

$x^2 - 1 = 0 \leftarrow$  Factorise the quadratic.

$$(x+1)(x-1) = 0$$

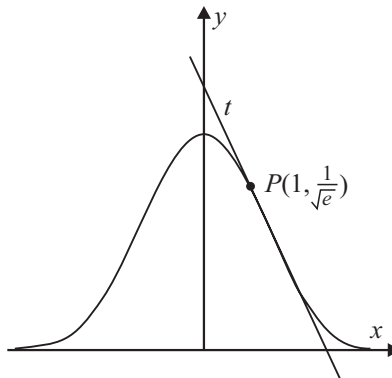
$\therefore x = -1, 1 \leftarrow$  The point  $P$  is in the first quadrant so ignore  $x = -1$ .

$$x = 1: y = e^{-\frac{1}{2}(1)^2} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \Rightarrow P\left(1, \frac{1}{\sqrt{e}}\right)$$

Finding the slope  $m$  of the tangent  $t$  at  $P$  by differentiating the function.

$$\frac{dy}{dx} = -xe^{-\frac{1}{2}x^2}$$

$$\left(\frac{dy}{dx}\right)_{x=1} = -(1)e^{-\frac{1}{2}(1)^2} = -\frac{1}{\sqrt{e}}$$



Equation of tangent  $t$ :  $P\left(1, \frac{1}{\sqrt{e}}\right), m = -\frac{1}{\sqrt{e}}$

$$y - \frac{1}{\sqrt{e}} = -\frac{1}{\sqrt{e}}(x - 1)$$

$$\sqrt{e}y - 1 = -1(x - 1)$$

$$\sqrt{e}y - 1 = -x + 1$$

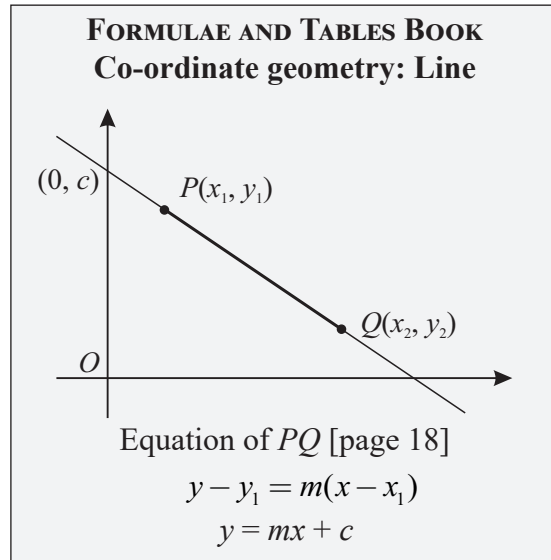
$$x + \sqrt{e}y - 2 = 0$$

Cuts the  $x$ -axis: Put  $y = 0$

$$y = 0: x + \sqrt{e}(0) - 2 = 0$$

$$\therefore x = 2$$

$$\therefore (2, 0)$$



**MARKING SCHEME NOTES**

**Question 6 (b) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Any reasonable first step
  - $y - 0 = m(x - 2)$
  - States  $f''(x) = 0$  and stops
- 4:**
- Full solution with one or two errors
  - Correct tangent and stops