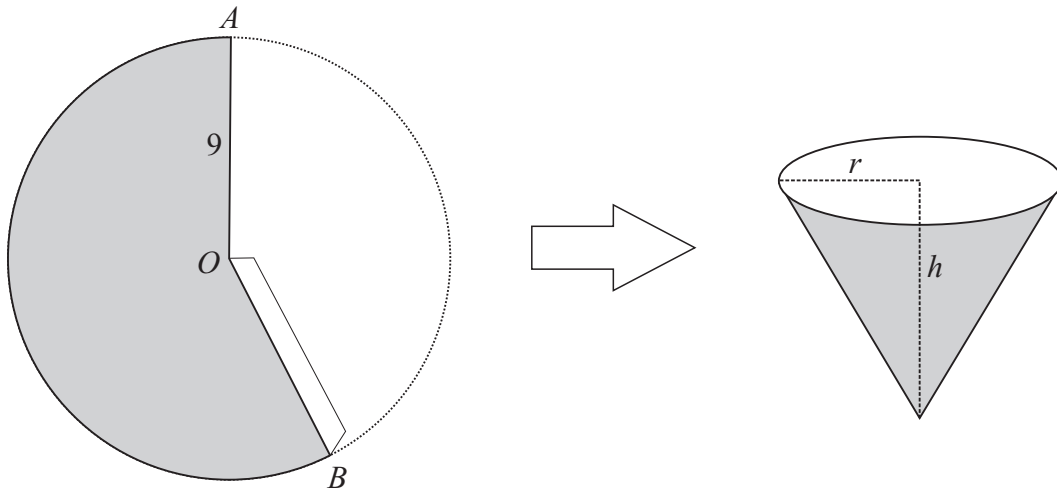


LC 2012 (SET E): PAPER 1

QUESTION 8 (50 MARKS)

Question 8 (a)



The radius in the circle will form the slant height l in the cup.

$$l^2 = h^2 + r^2$$

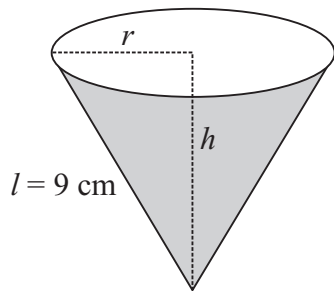
$$81 = h^2 + r^2$$

$$\therefore r^2 = 81 - h^2$$

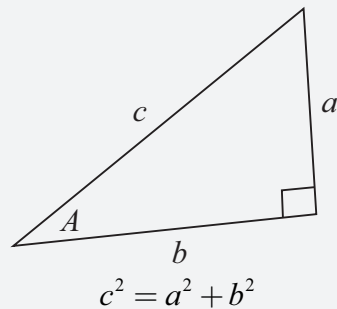
$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (81 - h^2) h$$

$$= \frac{\pi}{3} h(81 - h^2)$$



FORMULAE AND TABLES BOOK
Trigonometry (page 16)
 Pythagoras' theorem

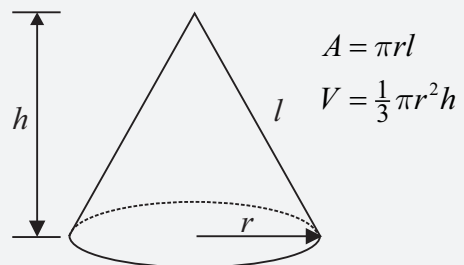


MARKING SCHEME NOTES

Question 8 (a) [Scale 10B (0, 7, 10)]

- 7: • $r^2 = 81 - h^2$ and stops
 • Any reasonable first step

FORMULAE AND TABLES BOOK
Surface area and volume:
Cone [page 10]



Question 8 (b)

$$\frac{\pi}{3}h(81-h^2) = \frac{154\pi}{3} \leftarrow \text{Put the volume } V \text{ equal to } \frac{154\pi}{3}$$

$$h(81-h^2) = 154$$

$$81h - h^3 - 154 = 0$$

$$h^3 - 81h + 154 = 0 \leftarrow \text{Solve this cubic equation using the factor theorem.}$$

FACTOR THEOREM

If k is a root of a polynomial equation $P(x) = 0$, then $(x - k)$ is a factor of $P(x)$ and vice versa.
or

For a polynomial $P(x)$, $P(k) = 0 \Rightarrow P(x) = (x - k)Q(x)$, where $Q(x)$ is a polynomial of degree one less than $P(x)$.

$$h = 1: (1)^3 - 81(1) + 154 = 154 \neq 0$$

$$h = 2: (2)^3 - 81(2) + 154 = 8 - 162 + 154 = 0 \Rightarrow (h - 2) \text{ is a factor}$$

There are 2 methods to find the quadratic factor

METHOD 1: Lining up (A cubic is a quadratic multiplied by a linear factor)

$$h^3 - 81h + 154 = (h - 2)(h^2 + kh - 77)$$

$$h^3 + 0h^2 - 81h + 154 = h^3 + (k - 2)h^2 + (-2k - 77)h + 154$$

$$\therefore 0 = a - 2 \Rightarrow k = 2$$

$$\therefore h^3 - 81h + 154 = (h - 2)(h^2 + 2h - 77) = 0$$

Solve the quadratic equation:

$$h^2 + 2h - 77 = 0$$

$$a = 1, b = 2, c = -77$$

$$h = \frac{-2 \pm \sqrt{2^2 - 4(1)(-77)}}{2}$$

$$= \frac{-2 \pm \sqrt{312}}{2} = -1 \pm \sqrt{78}$$

$$\therefore h = -9.83, 7.83$$

ANSWERS: $h = 2$ cm, 7.83 cm

METHOD 2: DIVISION

$$\begin{array}{r}
 h^2 + 2h - 77 \\
 h - 2 \overline{) h^3 + 0h^2 - 81h + 154} \\
 \underline{\mp h^3 \pm 2h^2} \\
 2h^2 - 81h + 154 \\
 \underline{\mp 2h^2 \pm 4h} \\
 -77h + 154 \\
 \underline{\pm 77h \mp 154} \\
 0
 \end{array}$$

FORMULAE AND TABLES BOOK
Algebra: Roots of the quadratic equation

$$ax^2 + bx + c = 0 \text{ [page 20]}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

MARKING SCHEME NOTES
Question 8 (b) [Scale 20D* (0, 6, 12, 18, 20)]

6: • Any reasonable first step

12: • Any step beyond $\frac{154\pi}{3} = \frac{\pi}{3}h(81-h^2)$

18: • Substantial correct progress made (e.g. 1 root found + some further correct progress)
• Solution by correct method, but with some errors

Question 8 (c)

$$V = \frac{1}{3}\pi h(81 - h^2)$$

$$= 27\pi h - \frac{1}{3}\pi h^3$$

$$\frac{dV}{dh} = 27\pi - \pi h^2 = 0$$

$$\therefore 27 - h^2 = 0$$

$$27 = h^2$$

$$\therefore h = \sqrt{27} = 27^{\frac{1}{2}}$$

FIND TURNING POINTS
(LOCAL MAXIMUM/MINIMUM)

Put $\frac{dy}{dx} = 0$ and solve for x

$$V = 27\pi h - \frac{1}{3}\pi h^3$$

$$V_{\text{Max.}} = 27\pi(27^{\frac{1}{2}}) - \frac{1}{3}\pi(27^{\frac{1}{2}})^3$$

$$= 27^{\frac{3}{2}}\pi - \frac{1}{3}(27^{\frac{3}{2}})\pi$$

$$= \frac{2}{3}(27^{\frac{3}{2}})\pi$$

$$\approx 294 \text{ cm}^3$$

MARKING SCHEME NOTES

Question 8 (c) [Scale 5C* (0, 3, 4, 5)]

3: • Any reasonable first step

4: • Finds h correctly and stops

• Finds $\frac{dV}{dh}$ correctly and sets $\frac{dV}{dh} = 0$

• Error(s) in derivative, and finishes correctly or nearly correctly

Question 8 (d)

	cups in part (b)		cup in part (c)
radius (r)	8.77 cm	4.43 cm	7.35 cm
height (h)	2 cm	7.83 cm	5.2 cm
capacity (V)	$\frac{154\pi}{3} \approx 161 \text{ cm}^3$	$\frac{154\pi}{3} \approx 161 \text{ cm}^3$	294 cm ³

Question 8 (e)

The diameters of the first and third cups are too large compared to their heights. The middle cup has the most reasonable shape ($r = 4.43 \text{ cm}$, $h = 7.83 \text{ cm}$)

MARKING SCHEME NOTES

Question 8 (d) [Scale 5C* (0, 3, 4, 5)]

3: • Any one correct entry

• Any correct relevant work

4: • Five or six correct entries (based on previous work)

Question 8 (e) [Scale 5B (0, 3, 5)]

0: • Incorrect answer, with or without explanation

3: • Correct answer, without explanation

• Cup with maximum volume

Question 8 (f)

$$r = 4.43 \text{ cm}$$

$$h = 7.83 \text{ cm}$$

$$V = 161 \text{ cm}^3$$

The circumference of the rim of the cup is equal to the arc length of the sector that forms it.

$$\text{Circumference of the rim of the cup} = 2\pi(4.43) = 27.83 \text{ cm}$$

Arc length of circle:

$$s = r\theta$$

$$27.83 = 9\theta$$

$$\therefore \theta = \frac{27.83}{9} = 3.09 \text{ rads} = 3.09 \times \frac{180^\circ}{\pi} \approx 177^\circ$$

MARKING SCHEME NOTES**Question 8 (f) [Scale 5C* (0, 3, 4, 5)]**

- 3:**
- Reasonable work with any one of $A = \pi r^2$, $C = 2\pi r$, $A = \pi r l$, $A = \frac{1}{2} r^2 \theta$
 - Any reasonable first step
- 4:**
- Correct answer in radians