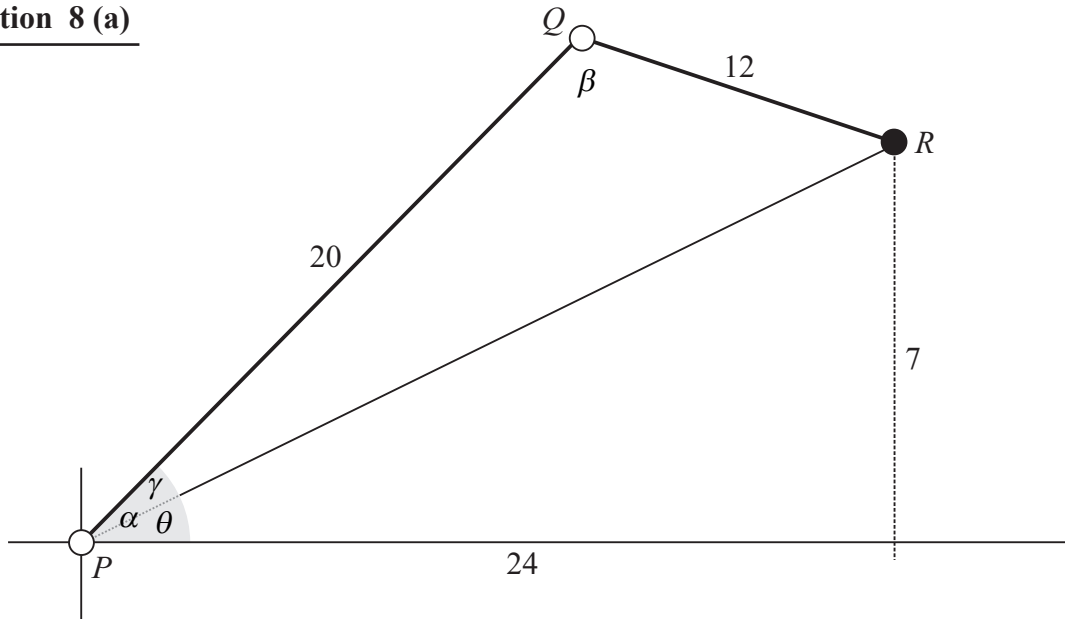


LC 2012 (SET E): PAPER 2

QUESTION 8 (75 MARKS)

Question 8 (a)



$$|PR|^2 = 24^2 + 7^2 = 625$$

$$\therefore |PR| = \sqrt{625} = 25 \text{ cm}$$

$$\tan \theta = \frac{7}{24} \Rightarrow \theta = \tan^{-1}\left(\frac{7}{24}\right) = 16.26^\circ$$

$$25^2 = 12^2 + 20^2 - 2(12)(20) \cos \beta$$

$$\cos \beta = \frac{12^2 + 20^2 - 25^2}{2(12)(20)} = \frac{-81}{480}$$

$$\therefore \beta = \cos^{-1}\left(-\frac{81}{480}\right) = 99.7^\circ \approx 100^\circ$$

$$12^2 = 20^2 + 25^2 - 2(20)(25) \cos \gamma$$

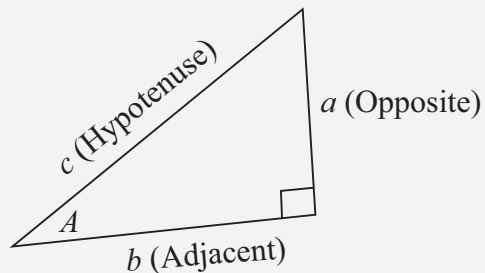
$$\cos \gamma = \frac{20^2 + 25^2 - 12^2}{2(20)(25)} = \frac{881}{1000} = 0.881$$

$$\therefore \gamma = \cos^{-1}(0.881) = 28.24^\circ$$

$$\alpha = \theta + \gamma = 16.26^\circ + 28.24^\circ = 44.4^\circ \approx 44^\circ$$

FORMULAE AND TABLES BOOK

Trigonometry: Right-angled triangle [page 16]



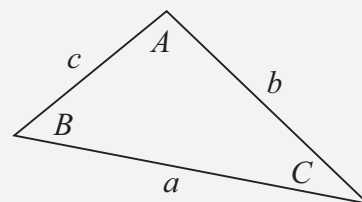
$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

FORMULAE AND TABLES BOOK

Trigonometry of the triangle:

[page 16]



$$\text{Area: } \frac{1}{2} ab \sin C$$

$$\text{Sine Rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

MARKING SCHEME NOTES

Question 8 (a) [Scale 10B (0, 5, 10)] |PR|

5: • Some use of Pythagoras

Question 8 (a) [Scale 20C* (0, 7, 18, 20)] β

7: • Cosine Rule with some substitution

18: • Cos β calculated

Question 8 (a) [Scale 25D* (0, 15, 20, 22, 25)] α

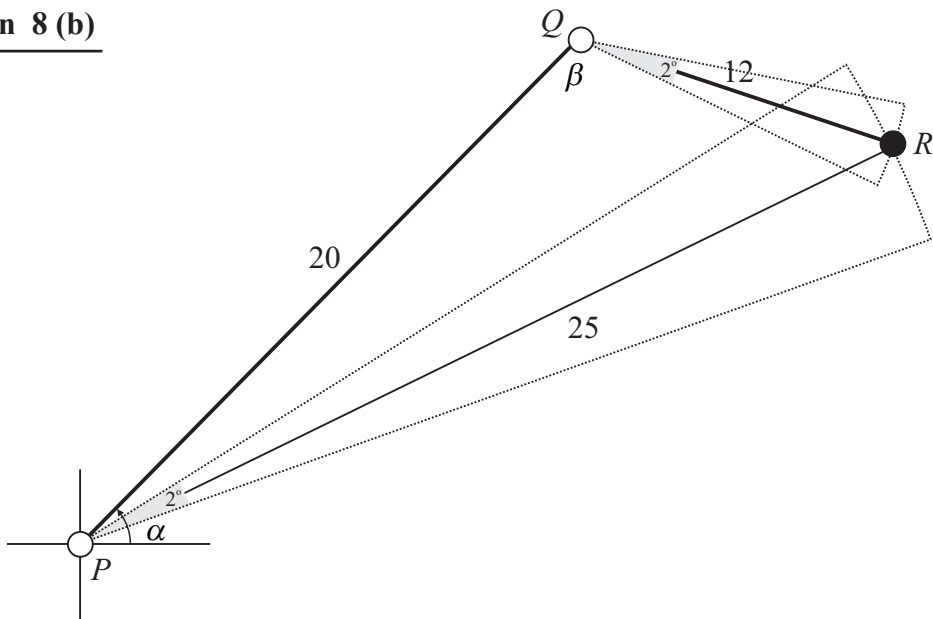
15: • Some work towards solving required angle with sine or cosine rule

• $\tan \gamma = \frac{7}{24}$ with no work towards $\alpha - \gamma$

20: • Cos ($\alpha - \gamma$) found

22: • $\alpha - \gamma$ and γ calculated but α not evaluated

Question 8 (b)



NOTE 1: An error of 1° in the angle means $\pm 1^\circ = 2^\circ$.

NOTE 2: The question states an ‘error in the location of R ’. This makes no sense. It is clearer to restate this as an ‘error in the distance moved by the point R ’.

Show the 2° movement in α acting along the line PR . As β is fixed the movement of R is the same if you move arm PQ through 2° or arm PR through 2° .

FORMULAE AND TABLES BOOK
Length and area:
Arc/sector [page 9]

$l = r\theta$
 $A = \frac{1}{2}r^2\theta$

when θ is in radians.

Degrees to radians: $\times \frac{\pi}{180^\circ}$
 Radians to degrees: $\times \frac{180^\circ}{\pi}$

Arc through which R moves for a 2° movement in alpha: $s \approx 25 \times 2 \times \frac{\pi}{180} = 0.87$ cm

Arc through which R moves for a 2° movement in beta: $s \approx 12 \times 2 \times \frac{\pi}{180} = 0.42$ cm

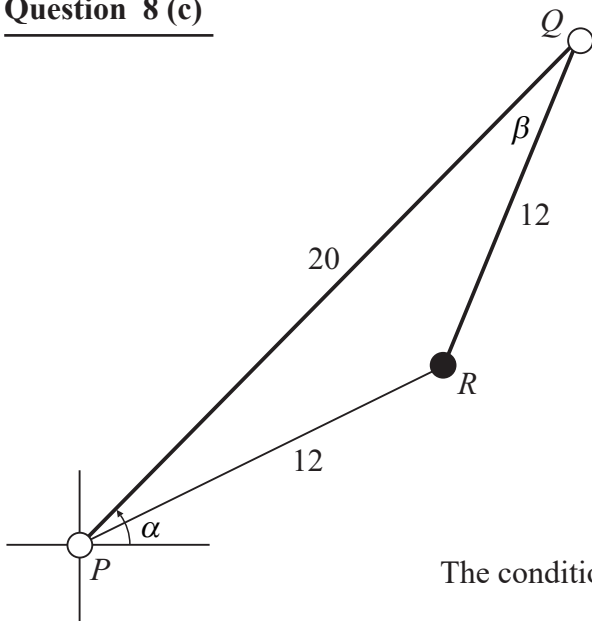
There is obviously a greater error in the distance moved by the point R for the longer arm. An error of 1° in α causes a greater error in the distance moved by R .

MARKING SCHEME NOTES

Question 8 (b) [Scale 5C (0, 3, 4, 5)]

- 3: • Effort at working out values for angles
• Correct answer without justification
- 4: • Correct answer without being fully justified

Question 8 (c)



The distance moved by R is more sensitive to errors in α when $|PR| > 12$ and is more sensitive to errors in β when $|PR| < 12$. The position of the robotic arm shown above is the borderline case.

$$12^2 = 12^2 + 20^2 - 2(12)(20) \cos \beta$$

$$2(12)(20) \cos \beta = 20^2$$

$$24 \cos \beta = 20$$

$$\cos \beta = \frac{20}{24} = \frac{5}{6}$$

$$\therefore \beta = \cos^{-1}\left(\frac{5}{6}\right)$$

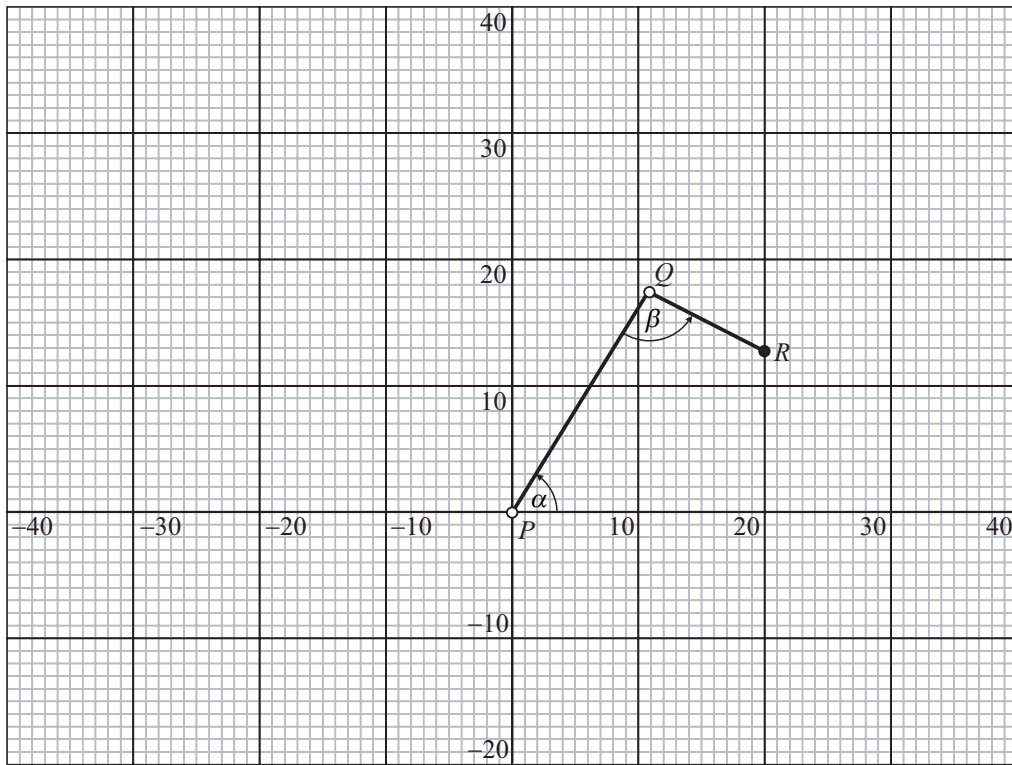
The condition $|PR| > 12$ is true whenever $\beta > \cos^{-1}\left(\frac{5}{6}\right) \approx 33.6^\circ$.

MARKING SCHEME NOTES

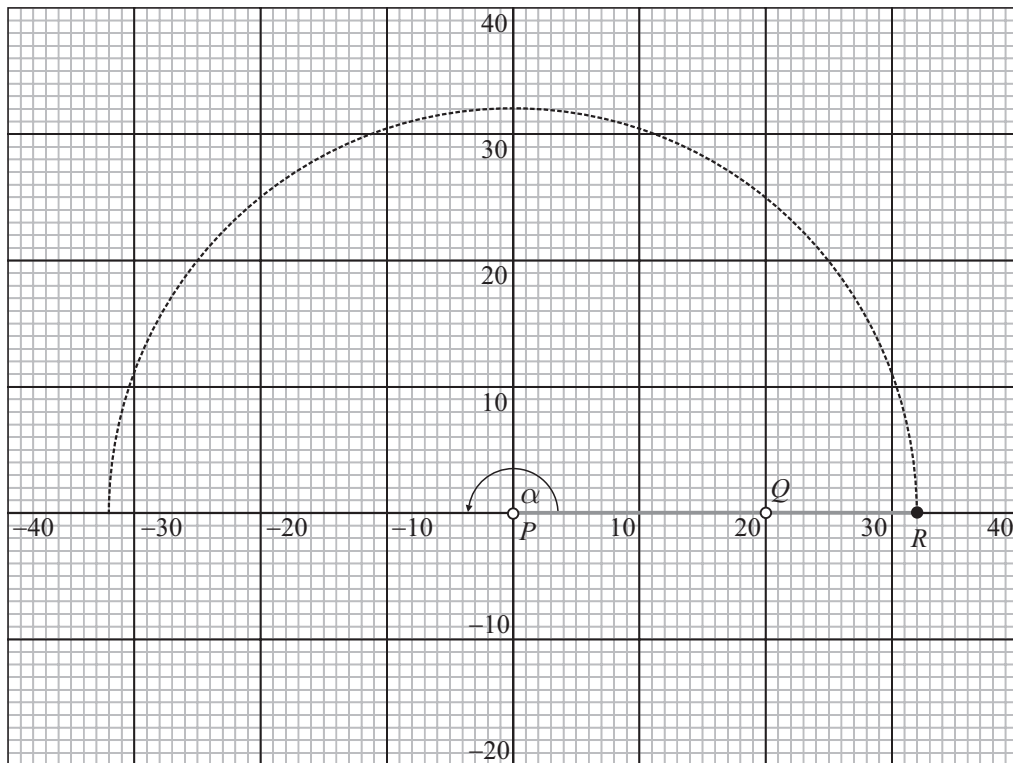
Question 8 (c) [Scale 5C (0, 3, 4, 5)]

- 3: • Some reference to distance between P and R
• Treats as percentage error in angles, rather than absolute error in location. e.g. "If α is smaller than β , then a 1° error in α is a bigger percentage error than a 1° error in β ."
- 4: • One situation only dealt with correctly
• Clearly understands concept that the radius of the rotation is the determining factor, but makes error(s) in explanation (e.g. mixes up distances involved)

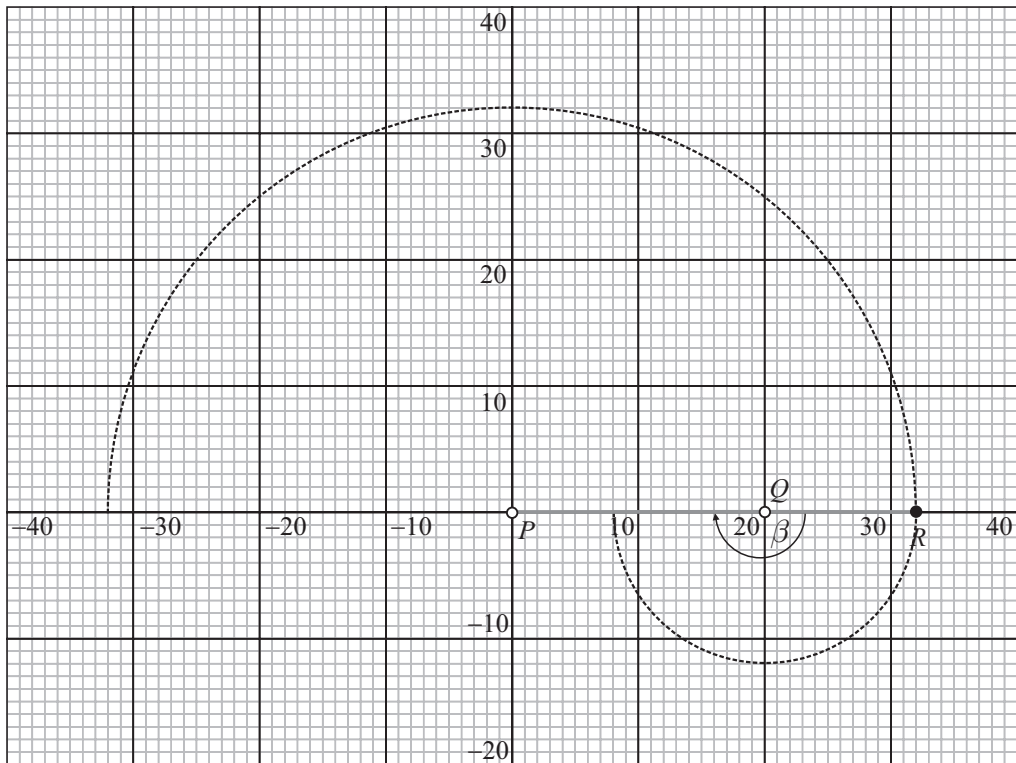
Question 8 (d)



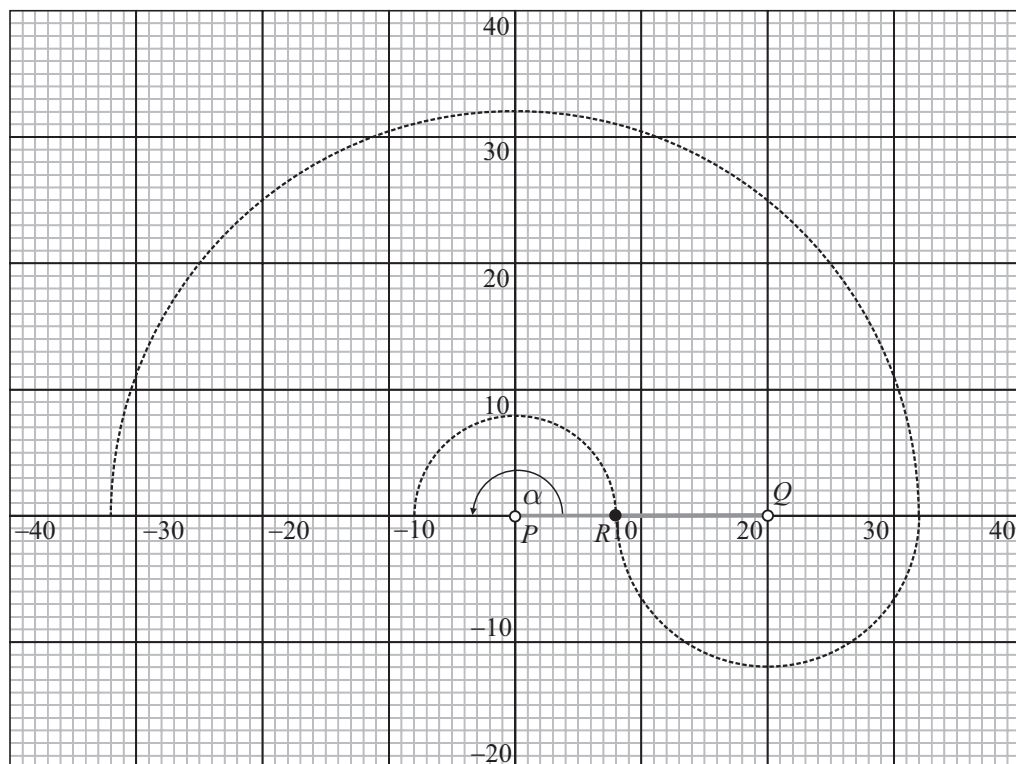
Stretch the arm PR to its full length of 32 cm. Rotate the arm about P from 0° to 180° . R traces the circle shown.



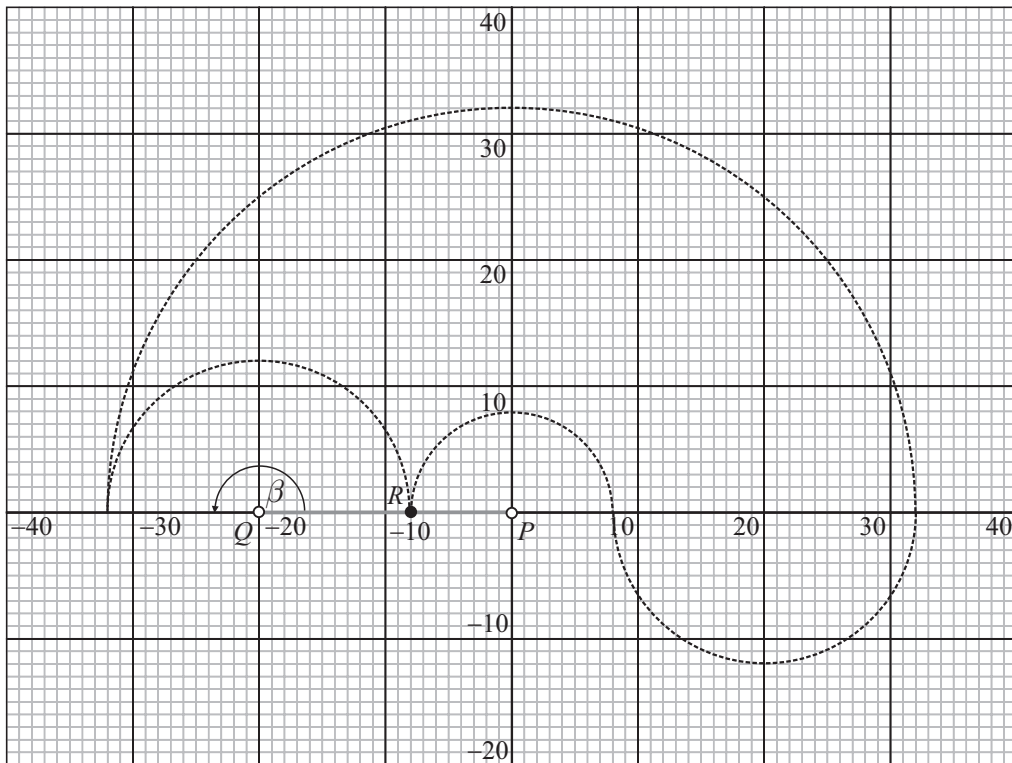
With the arm in the position shown rotate QR about Q from 0° to 180° . R traces the circle shown.



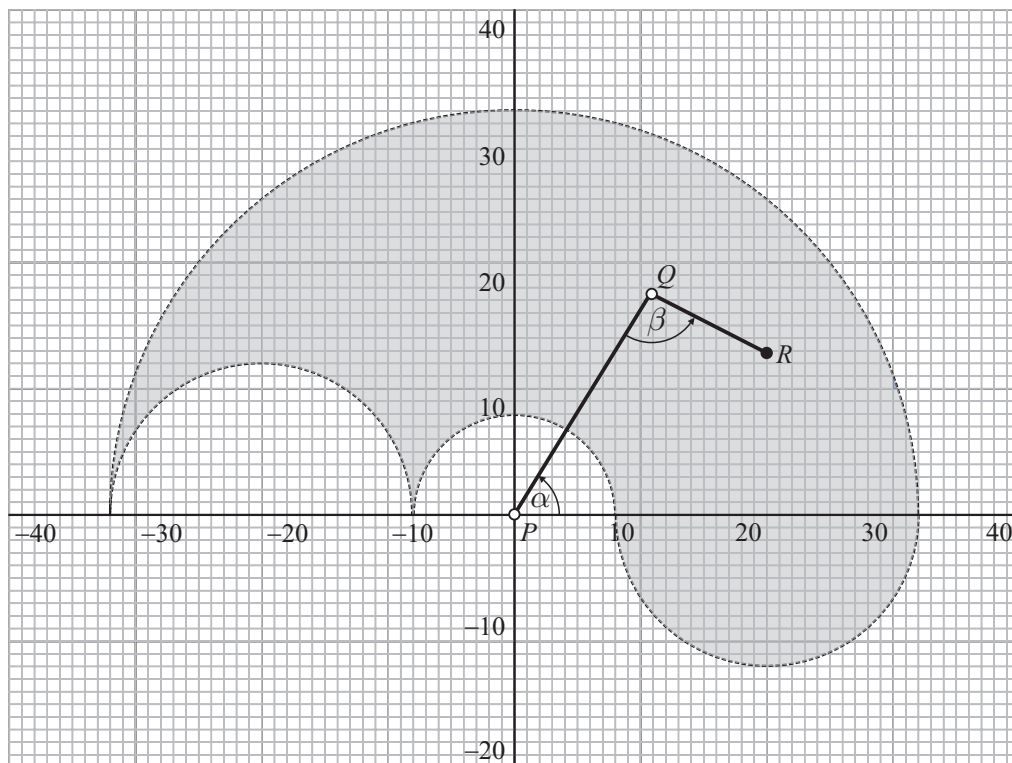
With the arm in the position shown rotate PQ about P from 0° to 180° . R traces the circle shown.



With the arm in the position shown rotate RQ about Q from 0° to 180° . R traces the circle shown.



The circles drawn were for the extreme positions of the arm. All points inside these curves can be reached by R .



MARKING SCHEME NOTES

Question 8 (d) [Scale 10C (0, 4, 8, 10)]

- 3: • Any relevant semi circle sketched or implied
- 4: • Any correct semi circle inserted in addition to semicircle centre P with radius 32.