

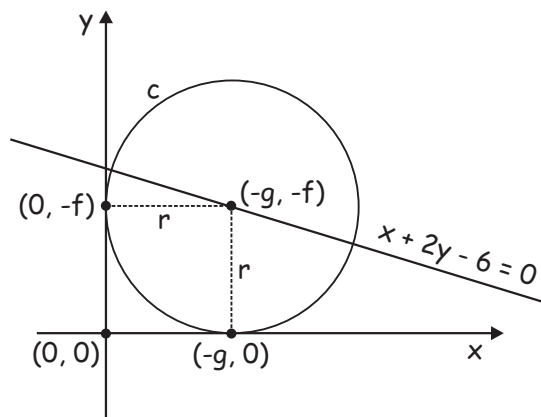
## SAMPLE PAPER 2014 (SET F): PAPER 2

### QUESTION 4 (25 MARKS)

Sketch the situation.

The circle  $c$  touches the  $x$ -axis at  $(-g, 0)$  and touches the  $y$ -axis at  $(0, -f)$ .

The line passes through the centre of the circle  $c$ .



#### FORMULAE AND TABLES BOOK

**Co-ordinate geometry: Circle** [page 19]

Given centre  $(h, k)$  and radius  $r$

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$

Radius  $\sqrt{g^2 + f^2 - c}$

$(-g, -f) \in x + 2y - 6 = 0$  [If a point is on a line, substitute the point into the equation of the line and it satisfies the equation.]

$$\therefore (-g) + 2(-f) - 6 = 0$$

$$-g - 2f - 6 = 0$$

$$g + 2f = -6 \dots\dots(1)$$

The points  $(-g, 0)$  and  $(0, -f)$  satisfy the equation of the circle.

$$(-g, 0) \in c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore (-g)^2 + (0)^2 + 2g(-g) + 2f(0) + c = 0$$

$$g^2 - 2g^2 + c = 0$$

$$c = g^2$$

$$(0, -f) \in c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore (0)^2 + (-f)^2 + 2g(0) + 2f(-f) + c = 0$$

$$f^2 - 2f^2 + c = 0$$

$$c = f^2$$

$$\therefore c = g^2 = f^2 \dots\dots(2)$$

$$\therefore g^2 = f^2 \Rightarrow g = \pm f \dots\dots(3)$$

To find the equation of each circle, choose  $g = +f$  for the first circle equation and  $g = -f$  for the second circle equation.

$$g = +f \text{ [Substitute into Eqn. (1)]}$$

$$g + 2g = -6$$

$$3g = -6$$

$$g = -2$$

$$f = -2 \text{ [Substitute into Eqn. (2)]}$$

$$c = g^2 = (-2)^2 = 4$$

$$g = -f$$

$$-f + 2f = -6$$

$$f = -6$$

$$g = 6$$

$$c = g^2 = (6)^2 = 36$$

#### EQUATION OF CIRCLES

$$x^2 + y^2 + 2(-2)x + 2(-2)y + 4 = 0 \Rightarrow x^2 + y^2 - 4x - 4y + 4 = 0$$

$$x^2 + y^2 + 2(6)x + 2(-6)y + 36 = 0 \Rightarrow x^2 + y^2 + 12x - 12y + 36 = 0$$