

SAMPLE PAPER 6: PAPER 1

QUESTION 3 (25 MARKS)

Question 3 (a) (i)

$$z = 6 - 4i$$

$$-z = -6 + 4i$$

Question 3 (a) (ii)

$$z = a + ib \Rightarrow |z| = \sqrt{a^2 + b^2}$$

$$\begin{aligned} |z| &= |6 - 4i| = \sqrt{6^2 + (-4)^2} = \sqrt{36 + 16} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

$$\begin{aligned} |-z| &= |-6 + 4i| = \sqrt{(-6)^2 + 4^2} = \sqrt{36 + 16} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

They are both the same distance from the origin.

Question 3 (b) (i)

$$z = 3 - 2i \Rightarrow \bar{z} = 3 + 2i \quad \boxed{z = a + bi \Rightarrow \bar{z} = a - bi}$$

$$\begin{aligned} z\bar{z} &= (3 - 2i)(3 + 2i) = 9 + 6i - 6i - 4i^2 \\ &= 9 + 4 = 13 \end{aligned}$$

$$\begin{aligned} \frac{z}{\bar{z}} &= \frac{3 - 2i}{3 + 2i} \quad [\text{Multiply above and below by the conjugate of the denominator.}] \\ &= \frac{(3 - 2i)}{(3 + 2i)} \times \frac{(3 - 2i)}{(3 - 2i)} \\ &= \frac{9 - 6i - 6i + 4i^2}{13} \\ &= \frac{5 - 12i}{13} = \frac{5}{13} - \frac{12}{13}i \end{aligned}$$

Question 3 (b) (ii)

$$\left| \frac{z}{\bar{z}} \right| = \left| \frac{5}{13} - \frac{12}{13}i \right| = \sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = \sqrt{\frac{25}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = 1$$

