

## SAMPLE PAPER 7: PAPER 1

### QUESTION 5 (25 MARKS)

#### Question 5 (a)

$$y = f(x) = x^3 - 6x^2 + 9x$$

$$\frac{dy}{dx} = f'(x) = 3x^2 - 12x + 9$$

$$\frac{d^2y}{dx^2} = f''(x) = 6x - 12$$

$$\frac{dy}{dx} = f'(x) = 3x^2 - 12x + 9$$

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$\therefore x = 1, 3$$

Turning Points (TP): Put  $\frac{dy}{dx} = 0$  and solve for  $x$ .

$$\left(\frac{d^2y}{dx^2}\right)_{\text{TP}} < 0 \Rightarrow \text{Local maximum}$$

$$\left(\frac{d^2y}{dx^2}\right)_{\text{TP}} > 0 \Rightarrow \text{Local minimum}$$

$$f(x) = x^3 - 6x^2 + 9x$$

$$f(1) = (1)^3 - 6(1)^2 + 9(1) = 4 \Rightarrow (1, 4) \text{ is a TP.}$$

$$f(3) = (3)^3 - 6(3)^2 + 9(3) = 0 \Rightarrow (3, 0) \text{ is a TP.}$$

Deciding which of the turning points is a local maximum or local minimum:

**METHOD 1:** Look at the  $y$ -coordinates in each turning point.

$(1, 4)$  is a local maximum and  $(3, 0)$  is a local minimum, as 4 is greater than 0.

**METHOD 2:** Use calculus.

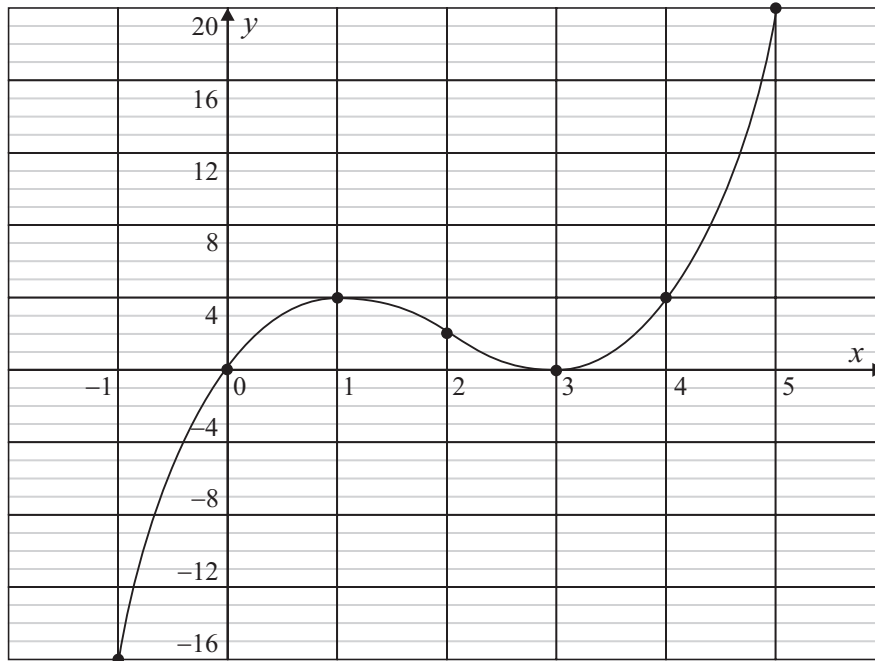
$$\frac{d^2y}{dx^2} = f''(x) = 6x - 12$$

$$f''(1) = 6(1) - 12 = -6 \Rightarrow (1, 4) \text{ is a local maximum.}$$

$$f''(3) = 6(3) - 12 = 6 \Rightarrow (3, 0) \text{ is a local minimum.}$$

#### Question 5 (b)

$x$	-1	0	1	2	3	4	5
$y$	-16	0	4	2	0	4	20

**Question 5 (c)**

**Question 5 (d)**

$$ax + by + c = 0 \Rightarrow \text{Slope: } m = -\frac{a}{b}$$

$$t: 9x - y + 3 = 0$$

$$m = 9$$

$$f(x) = x^3 - 6x^2 + 9x$$

$$f'(x) = \frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\frac{dy}{dx} = 9 \Rightarrow 3x^2 - 12x + 9 = 9$$

$$3x^2 - 12x = 0$$

$$x(x - 4) = 0$$

$$\therefore x = 0, 4$$

$$f(x) = x^3 - 6x^2 + 9x$$

$$f(0) = (0)^3 - 6(0)^2 + 9(0) = 0$$

$$f(4) = (4)^3 - 6(4)^2 + 9(4) = 64 - 96 + 36 = 4$$

(0, 0) and (4, 4) are the points of contact of these two tangents.

$$t_1: m = 9, \text{ point}(x_1, y_1) = (0, 0)$$

$$y - 0 = 9(x - 0)$$

$$y = 9x$$

$$9x - y = 0$$

$$t_2: m = 9, \text{ point}(x_1, y_1) = (4, 4)$$

$$y - 4 = 9(x - 4)$$

$$y - 4 = 9x - 36$$

$$9x - y - 32 = 0$$