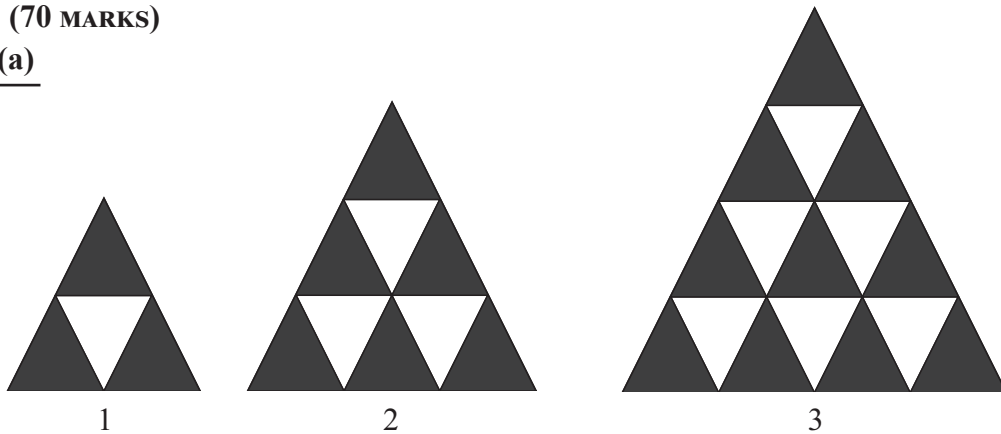


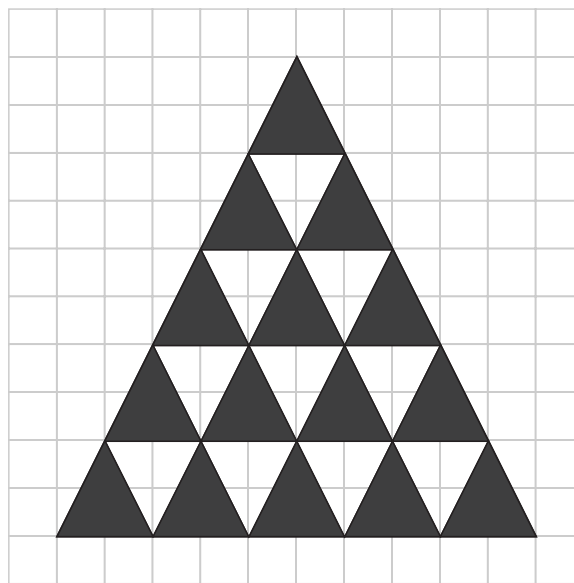
LC 2015 (SET B): PAPER 1

QUESTION 7 (70 MARKS)

Question 7 (a)



This is the fourth pattern:



MARKING SCHEME NOTES

Question 7 (a) [Scale 15B (0, 3, 15)]

- 3: • Outline triangle drawn
• Division consistent with pattern, but not 9 triangles in fifth row

Question 7 (b)

	Number of black triangles	Number of white triangles	Total Number of small triangles
Pattern 1	3	1	$T_1 = 4$
Pattern 2	6	3	$T_2 = 9$
Pattern 3	10	6	$T_3 = 16$
Pattern 4	15	10	$T_4 = 25$
Pattern 5	21	15	$T_5 = 36$

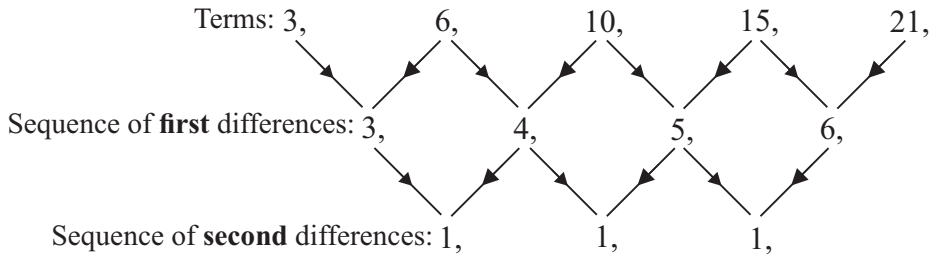
MARKING SCHEME NOTES

Question 7 (b) [Scale 15B (0, 3, 15)]

- 3: • One correct entry

Question 7 (c)

How do you show that the numbers of black triangles: 3, 6, 10, 15, 21,... forms a quadratic sequence? Subtract the terms to find the sum of first sequences: 3, 4, 5, 6,... Now subtract these terms again to find the sequence of second differences. This sequence 1, 1, 1,..., where all the terms have the same value, shows that the original sequence was quadratic.



MARKING SCHEME NOTES

Question 7 (c) [Scale 5D (0, 2, 3, 4, 5)]

- 2: • Correct sequence of numbers
- 3: • First difference established
- 4: • Second difference established

Question 7 (d)

	Number of black triangles	Number of white triangles	Total Number of small triangles
Pattern 1	3	1	$T_1 = 4$
Pattern 2	6	3	$T_2 = 9$
Pattern 3	10	6	$T_3 = 16$
Pattern 4	15	10	$T_4 = 25$
Pattern 5	21	15	$T_5 = 36$
Pattern 6	28	21	$T_6 = 49$
Pattern 7	36	28	$T_7 = 64$
Pattern 8	45	36	$T_8 = 81$
Pattern 9	55	45	$T_9 = 100$

- (i) There are 55 **black** triangles in the 9th pattern.
- (ii) There are 45 **white** triangles in the 9th pattern
- (iii) There are 100 **small** triangles, in total, in the 9th pattern

MARKING SCHEME NOTES

Question 7 (d) (i) (ii) (iii) [Scale 15C (0, 3, 10, 15)]

- 3: • One correct answer
- 10: • Two correct answers

Question 7 (e)

Total number of triangles: 4, 9, 16, 25, 36, 49...

$$T_1 = (1 + 1)^2 = 2^2 = 4$$

$$T_2 = (2 + 1)^2 = 3^2 = 9$$

$$T_3 = (3 + 1)^2 = 4^2 = 16$$

$$T_4 = (4 + 1)^2 = 5^2 = 25$$

$$T_5 = (5 + 1)^2 = 6^2 = 36$$

$$T_6 = (6 + 1)^2 = 7^2 = 49$$

Therefore, in general the total number of triangles $T_n = (n + 1)^2$

MARKING SCHEME NOTES**Question 7 (e) [Scale 5B (0, 3, 5)]**

- 3: • n^2 as answer
• Writes $an^2 + bn + c$

Question 7 (f)

B_n is the number of black triangles. The sequence of black triangles is given by: 3, 6, 10, 15,...

$$B_n = \frac{1}{2}n^2 + \frac{3}{2}n + c$$

$B_1 = 3 \leftarrow B_1$, the first term in the sequence, has a value of 3.

$$\therefore \frac{1}{2}(1)^2 + \frac{3}{2}(1) + c = 3 \leftarrow \text{Replace } n \text{ by } 1 \text{ in the formula to find } B_1.$$

$$\frac{1}{2} + \frac{3}{2} + c = 3 \leftarrow \text{Multiply each term by } 2.$$

$$2 + c = 3$$

$$\therefore c = 1$$

$$\therefore B_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$$

MARKING SCHEME NOTES**Question 7 (f) [Scale 5B (0, 3, 5)]**

- 3: • Any relevant substitution into B_n

Question 7 (g)

Total number of tile $T_n =$ Number of black tiles $B_n +$ Number of white tiles W_n

$$T_n = B_n + W_n \Rightarrow W_n = T_n - B_n$$

$$W_n = (n + 1)^2 - \left(\frac{1}{2}n^2 + \frac{3}{2}n + 1\right)$$

$$= n^2 + 2n + 1 - \frac{1}{2}n^2 - \frac{3}{2}n - 1$$

$$= \frac{1}{2}n^2 + \frac{1}{2}n$$

MARKING SCHEME NOTES**Question 7 (g) [Scale 5C (0, 3, 4, 5)]**

- 3: • Formula for the number of black and/or T_n from (e) written down
4: • $W_n + B_n = (n + 1)^2$

Question 7 (h)

$$T_n = (n+1)^2 = 625 \leftarrow \text{Total number of tiles } T_n \text{ is 625. Solve for } n.$$

$$n+1 = \sqrt{625} \leftarrow \text{Take the square root of both sides.}$$

$$n+1 = 25$$

$$\therefore n = 24 \leftarrow \text{This is the 24th pattern.}$$

$$B_{24} = \frac{1}{2}(24)^2 + \frac{3}{2}(24) + 1 = 325 \leftarrow \text{Number of black tiles in the 24th pattern } B_{24}$$

$$W_{24} = \frac{1}{2}(24)^2 + \frac{1}{2}(24) = 300 \leftarrow \text{Number of white tiles in the 24th pattern } W_{24}$$

MARKING SCHEME NOTES**Question 7 (h) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Black = 625 – White or White = 625 – Black
 - Pattern extended
 - Correct answer without work
- 4:**
- Finds n , or number of black, or number of white