LC 2013 (Set D): Paper 1

Question 9 (50 marks)

Question 9 (a)

Production cost: $x = 20$

$$C(x) = 5x^2 + 750x + 3000$$

$$C(20) = 5(20)^2 + 750(20) + 3000 = €20\ 000$$

Total income: $R(x) = 1200x$

$$R(20) = 1200(20) = €24\ 000$$

Question 9 (b)

Profit: $R(20) - C(20) = €24\ 000 - €20\ 000 = €4000$

Marking Scheme Notes
Question 9 (a) [Scale 15C (0, 5, 10, 15)]
5: • Some correct substitution into either function
10: • Substantially correct substitution and calculation

Question 9 (b) [Scale 10B (0, 5, 10)]
Note: Accept candidate's work from previous section
5: • Any correct relevant step

Question 9 (c)

Call the profit function, $P(x)$. Subtract the cost function $C(x)$ from the revenue function $R(x)$ to find the profit function $P(x)$.

$$P(x) = R(x) - C(x)$$
$$= 1200x - (5x^2 + 750x + 3000) \quad \leftarrow \text{Multiply out brackets.}$$
$$= 1200x - 5x^2 - 750x - 3000 \quad \leftarrow \text{Add like terms.}$$
$$= -5x^2 + 450x - 3000$$

Marking Scheme Notes
Question 9 (c) [Scale 5B (0, 2, 5)]
2: • Relevant work such as writing $R(x) - C(x)$
• Mishandles subtraction

Question 9 (d)

Find the derivative of $P(x)$. Put the derivative equal to zero and solve for x. This value of x yields the maximum profit.

$$P(x) = -5x^2 + 450x - 3000$$
$$P'(x) = -10x + 450$$
$$P'(x) = 0 \Rightarrow -10x + 450 = 0$$
$$450 = 10x$$
$$\therefore x = 45$$

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Marking Scheme Notes
Question 9 (d) [Scale 5C (0, 2, 4, 5)]
2: • Relevant trial and improvement work
• Differentiation with error(s) and stops
4: • Correct differentiation without further work
• Substantially correct differentiation with attempt to solve $P'(x)$.

Question 9 (e)

$P(x) = -5x^2 + 450x - 3000$

$x = 45$:

$P(45) = -5(45)^2 + 450(45) - 3000$

$= €7125$

Question 9 (f)

$C(x) = 5x^2 + 750x + 3000 = 11\,000$

$5x^2 + 750x - 8000 = 0$

$x^2 + 150x - 1600 = 0$

$(x - 10)(x + 160) = 0$

$\therefore x = 10$

Marking Scheme Notes
Question 9 (e) [Scale 5B (0, 2, 5)]
Note: Accept candidate's answer from previous section
2: • Attempts to calculate $P(45)$
• Trial and improvement work, different from part (d)

Question 9 (f) [Scale 10D (0, 2, 5, 8, 10)]
2: • Recognises $C(x) = 11\,000$
• Substitutes a value for x into $C(x)$
5: • Substantially correct approach to solving quadratic equation
• Correct answer without any work
8: • Solves equation but fails to identify correct answer
• Substitutes $x = 10$ into $C(x)$ but fails to show answer is €11 000