

LC 2018: PAPER 1

QUESTION 4 (25 MARKS)

Question 4 (a)

Step 1: $n = 1$: Prove $[r(\cos \theta + i \sin \theta)]^1 = r^1(\cos 1\theta + i \sin 1\theta)$

Proof: $r(\cos \theta + i \sin \theta) = r(\cos \theta + i \sin \theta)$. Therefore, it is true for $n = 1$.

Step 2: $n = k$: Assume $[r(\cos \theta + i \sin \theta)]^k = r^k(\cos k\theta + i \sin k\theta)$.

Step 3: $n = k + 1$: Prove $[r(\cos \theta + i \sin \theta)]^{k+1} = r^{k+1}[\cos(k+1)\theta + i \sin(k+1)\theta]$.

$$\begin{aligned} \text{Proof : } [r(\cos \theta + i \sin \theta)]^{k+1} &= [r(\cos \theta + i \sin \theta)]^k [r(\cos \theta + i \sin \theta)] \\ &= r^k(\cos k\theta + i \sin k\theta)r(\cos \theta + i \sin \theta) \text{ [From Step 2.]} \\ &= r^{k+1}[(\cos k\theta \cos \theta - \sin k\theta \sin \theta) + i(\sin k\theta \cos \theta + \cos k\theta \sin \theta)] \\ &= r^{k+1}[\cos(k+1)\theta + i \sin(k+1)\theta] \end{aligned}$$

Therefore, assuming it is true for $n = k$ means it is true for $n = k + 1$.

So, true for $n = 1$ and true for $n = k$ implies it is true for $n = k + 1$ means it is true for all $n \in \mathbb{N}$.

MARKING SCHEME NOTES

Question 4 (a) [Scale 15D (0, 5, 7, 11, 15)]

5: • Step 1

7: • Step 2 or Step 3

11: • Uses Step 2 to prove Step 3

Note: Accept Step 1, Step 2, Step 3 in any order

Full credit -1: • Omits conclusion but otherwise correct

15: • $[r(\cos \theta + i \sin \theta)]^n = r^n(\cos(n\theta) + i \sin(n\theta))$ proved correctly

Question 4 (b)

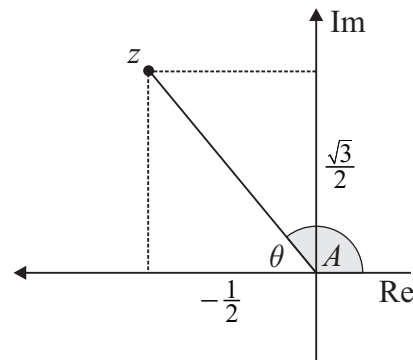
$$z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \Rightarrow r = |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\theta = \tan^{-1} \left| \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} \right| = \tan^{-1} \sqrt{3} = \frac{\pi}{3}$$

$$A = \arg z = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore z = \cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right)$$

$$z^3 = \left[\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \right]^3 = \cos 2\pi + i \sin 2\pi = 1 + 0i$$



MARKING SCHEME NOTES

Question 4 (b) [Scale 10C (0, 4, 8, 10)]

- 4: • Modulus or argument correct
• Some correct multiplication
• Apply De Moivre correctly with incorrect modulus and argument

8: • $\left(\cos(3) \frac{2\pi}{3} + i \sin(3) \frac{2\pi}{3} \right)$

- Multiplication correct but un-simplified

Full credit –1: • $\cos 2\pi + i \sin 2\pi$ or $\cos 360^\circ + i \sin 360^\circ$

Accept: Answer with reference to cube root of unity
