

LC 2018: PAPER 1**QUESTION 7 (55 MARKS)****Question 7 (a)**

$$t(x) = k \left[\ln \left(1 - \frac{x}{80} \right) \right]$$

$$x = 35 \text{ wpm}, t(35) = 35 \cdot 96 \text{ days}$$

$$k = \frac{t(x)}{\left[\ln \left(1 - \frac{x}{80} \right) \right]} = \frac{35 \cdot 96}{\left[\ln \left(1 - \frac{35}{80} \right) \right]} = -62 \cdot 5$$

Question 7 (b)

$$t(x) = 100 \text{ days: } 100 = -62 \cdot 5 \left[\ln \left(1 - \frac{x}{80} \right) \right]$$

$$-\frac{100}{62 \cdot 5} = \ln \left(1 - \frac{x}{80} \right)$$

$$e^{-\frac{100}{62 \cdot 5}} = 1 - \frac{x}{80}$$

$$\frac{x}{80} = 1 - e^{-\frac{100}{62 \cdot 5}}$$

$$x = 80(1 - e^{-\frac{100}{62 \cdot 5}}) \approx 64 \text{ wpm}$$

MARKING SCHEME NOTES**Question 7 (a) [Scale 15C (0, 5, 10, 15)]**

5: • Effort at transposing

- Some substitution into function
- Full substitution and stops

10: • Function written in terms of k and fully substituted

- One incorrect substitution worked correctly and with some reference to $k \neq -62 \cdot 5$

Question 7 (b) [Scale 5C (0, 3, 4, 5)]

3: • Some substitution into function

- Trial and improvement (more than 1 iteration)
- Correct answer without work

4: • $e^{-\frac{100}{62 \cdot 5}} = 1 - \frac{x}{80}$

- Equation rewritten in terms of x or $\frac{x}{80} =$

Question 7 (c)

x (wpm)	0	19	20	30	40	50	60	70
$t(x)$ (days)	0	8	18	29	43	61	87	130

MARKING SCHEME NOTES**Question 7 (c) [Scale 20D (0, 5, 10, 15, 20)]**

5: • One entry correct

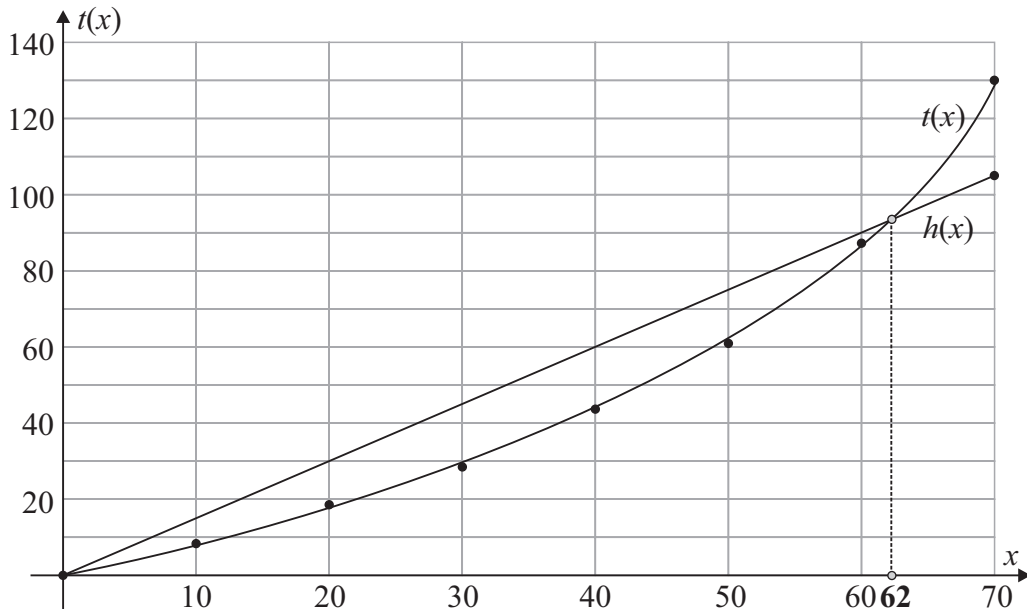
- One plot (from candidates table) correct

10: • 4 entries correct and 4 plots of table values

15: • All plots consistent with candidates table values (with at least 1 correct value)

- Table correct but incorrect plots

Question 7 (d)



$$p(x) = 1.5x$$

$$p(0) = 1.5(0) = 0$$

$$p(70) = 1.5(70) = 105$$

MARKING SCHEME NOTES

Question 7 (d) [Scale 5C (0, 3, 4, 5)]

- 3: • One point on line identified
 • One point (not origin) plotted
- 4: • 2 points on line identified and plotted

Question 7 (e)

$$h(x) = p(x) - t(x)$$

(i) $h(x) = 0 \Rightarrow p(x) = t(x)$
 $x \approx 62$

(ii) $h(x) = 1.5x + 62.5 \left[\ln \left(1 - \frac{x}{80} \right) \right]$

$$\frac{dh}{dx} = 1.5 + 62.5 \times \frac{1}{\left(1 - \frac{x}{80} \right)} \times \left(-\frac{1}{80} \right)$$

$$= 1.5 - \frac{62.5}{(80-x)}$$

$$\frac{dh}{dx} = 0 : 1.5 = \frac{62.5}{(80-x)} \Rightarrow 1.5(80-x) = 62.5$$

$$120 - 1.5x = 62.5 \Rightarrow 57.5 = 1.5x$$

$$\therefore x = \frac{115}{3}$$

$$h\left(\frac{115}{3}\right) = 1.5\left(\frac{115}{3}\right) + 62.5 \left[\ln \left(1 - \frac{\left(\frac{115}{3}\right)}{80} \right) \right]$$

≈ 17 days

MARKING SCHEME NOTES

Question 7 (e) (i) [Scale 5B (0, 2, 5)]

- 2: • Point of intersection indicated on graph
 • $h(x)$ written in terms of x

Tolerance: ± 2 wpm

Question 7 (e) (ii) [Scale 5B (0, 3, 4, 5)]

- 3: • Any correct differentiation
 • $h(x) = 1.5x + 62.5 \ln \left(1 - \frac{x}{80} \right)$
 • $h'(x) = 0$
- 4: • Differentiation correct but un-simplified
 • Value for x and stops

$$y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)$$