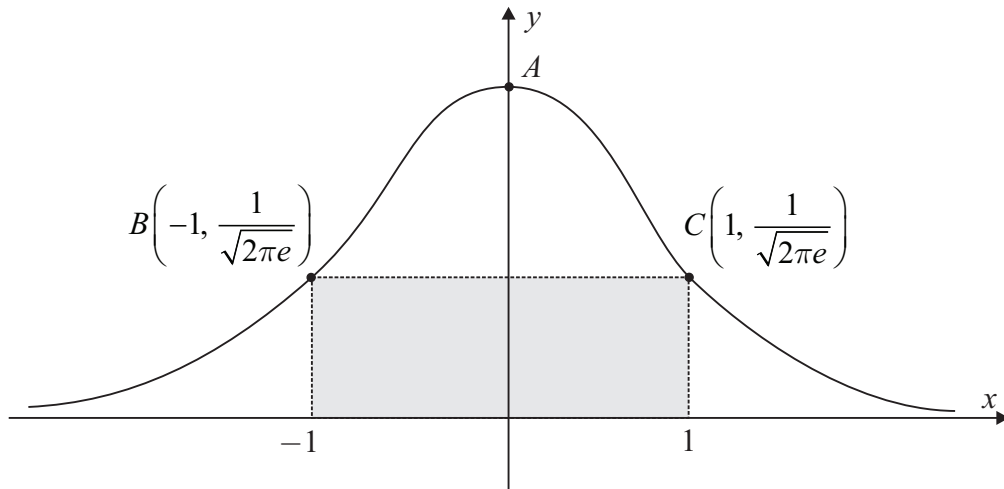


LC 2018: PAPER 1

QUESTION 8 (40 MARKS)



Question 8 (a)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$x = 0: f(0) = \frac{1}{\sqrt{2\pi}} e^0 = \frac{1}{\sqrt{2\pi}}$$

$$\therefore A\left(0, \frac{1}{\sqrt{2\pi}}\right)$$

Question 8 (b)

$$\text{Area} = 2 \times \frac{1}{\sqrt{2\pi e}} = 0.484$$

MARKING SCHEME NOTES

Question 8 (a) [Scale 10C (0, 4, 8, 10)]

- 4: • $x = 0$
 • Value for x substituted into $f(x)$

8: • $\frac{1}{\sqrt{2\pi}}$

Full credit – 1: • (0, 0.3989)

Question 8 (b) [Scale 10C (0, 4, 8, 10)]

- 0: • Integrating original function

- 4: • length = 2
 • Width = [y co-ordinate]

8: • $\left[1\left(\frac{1}{\sqrt{2\pi e}}\right)\right]$

Full credit – 1: • Area = 0.484

Question 8 (c)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \times (-x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$f'(1) = -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}} = -\frac{1}{\sqrt{2\pi e}} < 0$$

$\therefore f(x)$ is decreasing at C

MARKING SCHEME NOTES

Question 8 (c) [Scale 10C (0, 4, 8, 10)]

- 4: • $x = 1$ identified
 • Some correct differentiation

- Indicates significance of $\frac{dy}{dx} < 0$

- 8: • Derivative found

Question 8 (d)

$$f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \times (-x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$\begin{aligned} f''(x) &= -\frac{1}{\sqrt{2\pi}} [x e^{-\frac{1}{2}x^2} \times (-x) + e^{-\frac{1}{2}x^2}] \\ &= -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} [1 - x^2] \end{aligned}$$

$$\text{Point of inflection: } f''(x) = 0 \Rightarrow -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} [1 - x^2] = 0$$

$$1 - x^2 = 0 \Rightarrow x^2 = 1$$

$$\therefore x = \pm 1$$

\therefore Point of inflection at C where $x = -1$.

MARKING SCHEME NOTES**Question 8 (d) [Scale 10D (0, 3, 5, 8, 10)]**

3: • $f'(x)$ transferred or found

- Mention of $f''(x)$
- Identifies $x = -1$

5: • $f''(x)$ identified and some correct differentiation

8: • $f''(x)$ found

Note: If the product rule and chain rule are not applied in finding $f''(x)$ then the candidate can be awarded mid-partial credit at most