

## LC 2018: PAPER 2

### QUESTION 8 (60 MARKS)

#### Question 8 (a)

(i) Population mean:

Population:  $\mu = 4.64, \sigma = 0.12$

Sample:  $n = 10, \bar{\mu} = 4.64, \bar{\sigma} = \frac{0.12}{\sqrt{10}} = 0.038$

$\bar{x} = 4.6: \bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}} = \frac{4.6 - 4.64}{0.038} = -1.05$

$\bar{x} = 4.7: \bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}} = \frac{4.8 - 4.64}{0.038} = 1.58$

$P(4.6 < \bar{x} < 4.7) = P(-1.05 < \bar{z} < 1.58)$

$= P(\bar{z} < 1.58) - P(\bar{z} < -1.05)$

$= P(\bar{z} < 1.58) - P(\bar{z} > 1.05)$

$= P(\bar{z} < 1.58) - [1 - P(\bar{z} < 1.05)]$

$= P(\bar{z} < 1.58) - 1 + P(\bar{z} < 1.05)$

$= 0.9429 - 1 + 0.8531 = 0.796$

(ii) Population sample:

$n = 400, \hat{p} = \frac{324}{400} = 0.81$

$\hat{\sigma} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.81(1-0.81)}{400}} = 0.0196$

Confidence interval:  $\hat{p} - 1.96\hat{\sigma} \leftrightarrow \hat{p} + 1.96\hat{\sigma}$

$0.81 - 1.96(0.0196) \leftrightarrow 0.81 + 1.96(0.0196)$

$0.77 \leftrightarrow 0.85$

The company is 95% confident that between 77% and 85% of the population like the new bar.

#### MARKING SCHEME NOTES

##### Question 8 (a) (i) [Scale 20D (0, 5, 10, 15, 20)]

5: •  $z_1$  formulated with some correct substitution

•  $z_2$  formulated with some correct substitution

10: •  $z_1$  and  $z_2$  fully substituted

15: •  $-1.05$  and  $1.58$  or equivalents

##### Question 8 (a) (ii) [Scale 15C (0, 4, 11, 15)]

4: • CI formulated with some correct substitution

• 1.96

•  $\hat{p} \pm \frac{1}{\sqrt{n}}$  incomplete or completed

11: • CI fully substituted

#### Question 8 (b)

(i) Table of statements:

Statement	Always True	Sometimes True	Never True
1. When forming confidence intervals (for fixed $n$ and $\hat{p}$ ), an increased confidence level implies a wider interval.	✓		
2. As the value of $\hat{p}$ increases (for fixed $n$ ), the estimated standard error of the population proportion increases.		✓	
3. As the value of $\hat{p}(1-\hat{p})$ increases (for fixed $n$ ), the estimated standard error of the population proportion increases.	✓		
4. As $n$ , the number of people sampled, increases (for fixed $\hat{p}$ ), the estimated standard error of the population proportion increases.			✓

(ii) Let  $p = \hat{p}(1 - \hat{p}) = \hat{p} - \hat{p}^2$

$$\frac{dp}{d\hat{p}} = 1 - 2\hat{p}$$

$$\frac{dp}{d\hat{p}} = 0 \Rightarrow 1 - 2\hat{p} = 0$$

$$\therefore \hat{p} = 0.5$$

Maximum value:  $p = \hat{p}(1 - \hat{p}) = 0.5(1 - 0.5) = 0.25$

(iii)  $1.96 \times \hat{\sigma} = 1.96 \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 1.96 \times \sqrt{\frac{0.25}{800}} = 0.0346$

**NOTE:** The radius of the confidence interval is the margin of error =  $1.96 \times \hat{\sigma}$ .

**MARKING SCHEME NOTES**

**Question 8 (b) (i) [Scale 5C (0, 2, 4, 5)]**

2: • Any 1 correct

4: • Any 2 correct

**Question 8 (b) (ii) + (iii) [Scale 10D (0, 3, 5, 8, 10)]**

3: • Any relevant calculus

- $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{800}}$

- Effort at completing the square

5: •  $\hat{p} = \frac{1}{2}$  or equivalent

8: • Maximum value

**Question 8 (c)**

$$\begin{aligned} \text{Fund} &= 20\,000 + \frac{20\,000 \times 1.01}{1.024} + \frac{20\,000 \times 1.01^2}{1.024^2} + \dots + \frac{20\,000 \times 1.01^{25}}{1.024^{25}} \\ &= 20\,000 \left[ 1 + \frac{1.01}{1.024} + \left( \frac{1.01}{1.024} \right)^2 + \dots + \left( \frac{1.01}{1.024} \right)^{25} \right] \quad \text{Note: } a = 1, r = \frac{1.01}{1.024}, n = 26 \\ \text{Fund} &= 20\,000 \times \frac{1 \left( 1 - \left( \frac{1.01}{1.024} \right)^{26} \right)}{1 - \frac{1.01}{1.024}} = \text{€}440\,132.40 \end{aligned}$$

**MARKING SCHEME NOTES**

**Question 8 (c) [Scale 10D (0, 3, 5, 8, 10)]**

3: •  $20000(1.01)$  or  $\frac{20\,000}{1.024}$

5: •  $\frac{20\,000(1.01)}{1.024}$  or similar term

- Correctly handles inflation element or completes correctly present values element and finishes

8: • GP with  $a, r$  and  $n$  identified

**Note:** Treat  $n = 25$  as a misreading