5th International Conference on Ocean Energy, 4 November, Halifax

A hierarchy of approaches for the optimal design of tidal turbine farms

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Abstract

From conception to construction, the process by which tidal turbine farms are scoped and designed (and even optimised – which is the focus here) is multi-layered. The industrial designer requires tools of varying fidelities working on multiple scales, depending on the task at hand. In this paper a hierarchy of modelling approaches is proposed and some examples demonstrated.

For site-scoping and resource assessment, the continuum approach enables multiple farms to be considered and optimised over a large geographic area. This is demonstrated for four farms in the Pentland Firth, Scotland. For detailed design, three-dimensional CFD codes allow flow around a turbine to be fully resolved and the physical processes closely modelled. In between, and informed by these extremes, are array design tools whereby each turbine is individually represented and the flow over the domain is calculated with the non-linear shallow water equations. In a test example, the positions of 78 turbines in a farm located in the Inner Sound of the Pentland Firth, Scotland is optimised with a resulting 25% improvement in power extracted.

A holistic approach to the design process is also presented which seeks to design with the maximisation of developer's profit – rather than power extracted - as the ultimate goal.

Keywords; Tidal farm optimisation, farm design, computational modelling, tidal turbines.

1. Introduction

Tidal turbines are devices which extract energy from tidally induced currents. Device designs vary but, in general, individual units may be rated to, at most, a few megawatts. The vision is for energy to be extracted on an industrial scale by installing many devices (dozens to hundreds) together in a 'farm' or 'array'. Suitable array sites have high peak flow rates and often complex bathymetry which, coupled with the presence of turbines, results in complex, rapidly spatially-varying and turbulent flow conditions. Since the power extracted has a cubic dependency on, and thus significant sensitivity to, the speed of that flow, determining the optimum number of turbines to place on a given site, and deciding where the individual turbines should be located are both challenging but crucial problems.

In general there are two common methodologies to the design of tidal turbine arrays. The first is to exploit the designer's experience and intuition to propose an array layout whose performance is then predicted using a high fidelity (often commercial) computational model. This approach has the benefit of accurately capturing the physical processes, but at a high computational cost. This expense means that few alternate array options can feasibly be tested and so the quality of the final design is limited by the designer's skill at proposing good layouts and iterating on

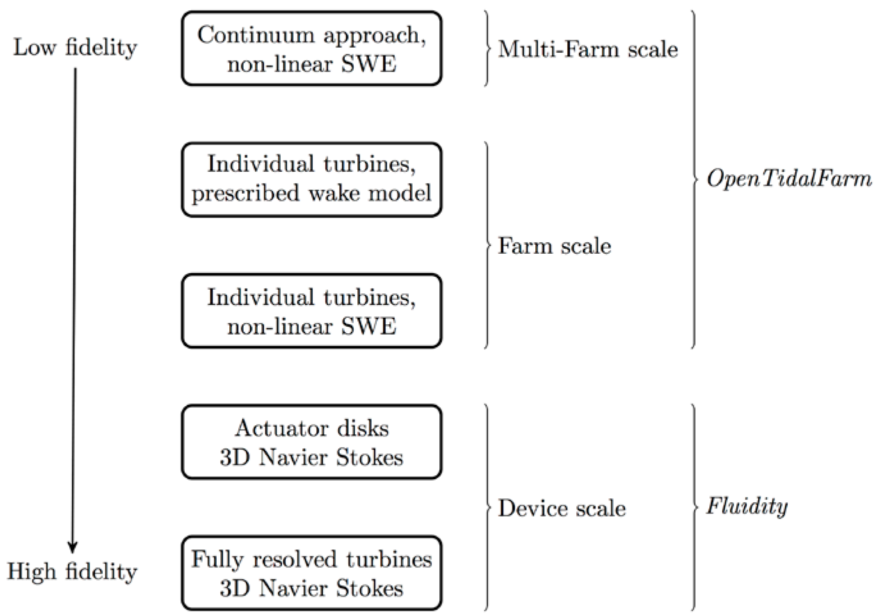

Figure 1: The hierarchy of models from lowest fidelity – for use in scoping and multi-farm scale design, to highest fidelity – where computational expense limits simulations to individual (or small numbers of) devices. In the middle of the spectrum, array designs can be efficiently optimised using a model informed by both the device-scale and broader multi-farm scale simulations. *OpenTidalFarm* and *Fluidity* are proposed codes for these applications and are discussed in section 2.1.

these. An alternate approach is to use a global optimisation algorithm (such as a genetic algorithm) to test multiple alternate array designs (generally in the order of 1000's to 100,000's). Due to the large number of array designs that must be compared, each model evaluation (that is the flow solve from which the performance of the array design is calculated) must be computationally cheap to make the approach practically feasible. The result is that physical processes are modelled in a much simplified manner (Barnett et al. 2014).

In this paper, a hierarchy of approaches to the array design problem is proposed whereby models of varying fidelity are used in different stages and for different aspects of the design process. Optimisation of the number of turbines (array size) and their locations (micro-siting) lies at the heart of this framework and enables the industrial designer to maximise measures such as the power extracted by the array or the financial profit that it will generate.

2. A hierarchical approach

The proposed hierarchy of modelling approaches is illustrated by figure 1. At the multiple-farm scale, the designer is concerned with identifying and scoping potential tidal stream power sites. Here, rather than resolving individual turbines, the entire array is represented by a continuum or ‘smeared’ bottom friction field. The non-linear shallow water equations are solved over the domain in question (likely a relatively coarse resolution mesh of an entire coastal region) and the amount, as well as the spatial distribution of friction is optimised to maximise the power extracted from the flow, normalised for the amount of friction deployed.

At the other extreme, CFD codes such as *Fluidity* (Piggott et al. 2008) are capable of modelling fully resolved turbines in three dimensions over short time spans. *Fluidity* allows for dynamic mesh adaptivity, which helps ensure that the hydrodynamic structure of the wake is well resolved, and this enables an understanding of how a specific device will impact upon the flow once in situ. Such simulations are computationally expensive to run and difficult to validate (as real-world data is largely unavailable). Therefore an additional level of abstraction is often introduced in which the turbines are represented as actuator disks, or blade element momentum models, the flow is then simulated as before; in three-dimensions with the Navier-Stokes equations. It then becomes computationally tractable to model small groups of turbines and analyse how they interact with each other. This type of run can be compared against laboratory results, enabling validation of the model.

In the middle of this spectrum is an approach which is informed by the observations of the detailed run, but is sufficiently idealised that, as with the continuum approach, multiple layout options may be evaluated and thus the array design may be optimised. In the simple case, the ambient flow over the domain is computed, then the wakes of the turbines in the array (as found by applying a prescribed wake model) are superimposed (Thomson et al. 2011; Barnett et al. 2014). This gives an approximation of the resultant flow field and enables the power extracted by the array to be estimated. Such a simulation is computationally relatively cheap, and therefore this approach works well with *global* optimisation algorithms. The layout returned by the global optimisation can then be used as a starting point for a local, *gradient-based* optimisation algorithm (Barnett et al. 2014).

Super-position of wakes on the ambient flow does not adequately capture many of the physical processes (e.g. blockage effects with large arrays) which may govern the power extracted by the array. Consequently, Funke et al. (2014) formulate the turbine layout problem as an optimisation problem constrained by the shallow water equations. At each optimisation iteration the performance of the turbine layout is evaluated by solving a finite element shallow water model. Individual turbines are represented as individual areas of increased friction. Since these now need to be individually resolved, a finer mesh is required and so each flow solve is more expensive than with the continuum approach. The gradient of the performance is determined through solving the associated adjoint equations and the turbine locations are optimised through efficient gradient-based optimisation methods. Such methods require fewer iterations, so at each iteration, a more computationally expensive model may be used.

The final, optimised array layout, can then be modelled in higher fidelity using, for example, the three-dimensional actuator disk method.

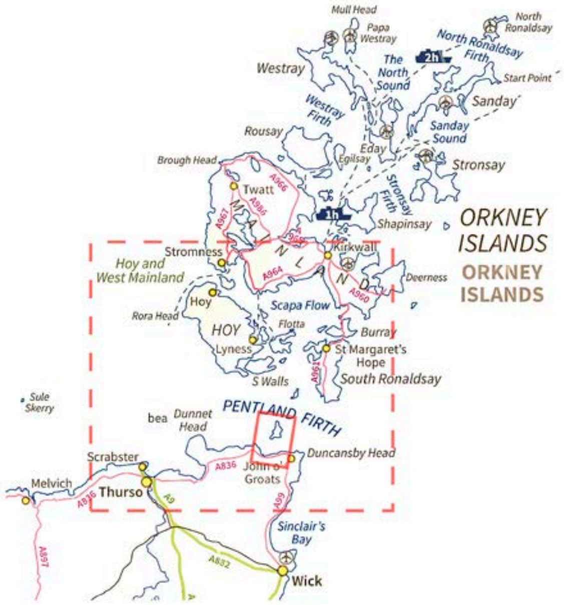

Figure 2: Map of the Orkney Islands, Scotland. Domain for continuum, multi-farm optimisation shown by dashed box. Domain for individually resolved turbine array optimisation (section 5.2) shown by solid box. Map from edina.ac.uk/digimap.

2.1 Computer Models

OpenTidalFarm (Funke et al. 2014) (www.opentidalfarm.org) is a code which solves the shallow-water equations using the finite-element method. It is built on a code generation framework which facilitates efficient computation, including access to the gradient using the adjoint approach, and is specifically designed for the optimisation of turbine micro-siting (whether using the continuum approach described in section 3 or resolving individual turbines as in section 5).

Whilst several CFD codes are capable of fully resolved three-dimensional turbine modelling, *Fluidity* (Piggott et al. 2008) (www.fluidity-project.org) has been identified for several reasons. *Fluidity* has the ability to model at the oceanic scale as well as the turbine scale and, by using dynamic mesh adaptivity, *Fluidity* is able to run complex models with more computational economy. *Fluidity* is being developed alongside *OpenTidalFarm* by the authors, and the long term vision is to merge the capabilities of the two projects so that the entire hierarchy will fall within the scope of a single package.

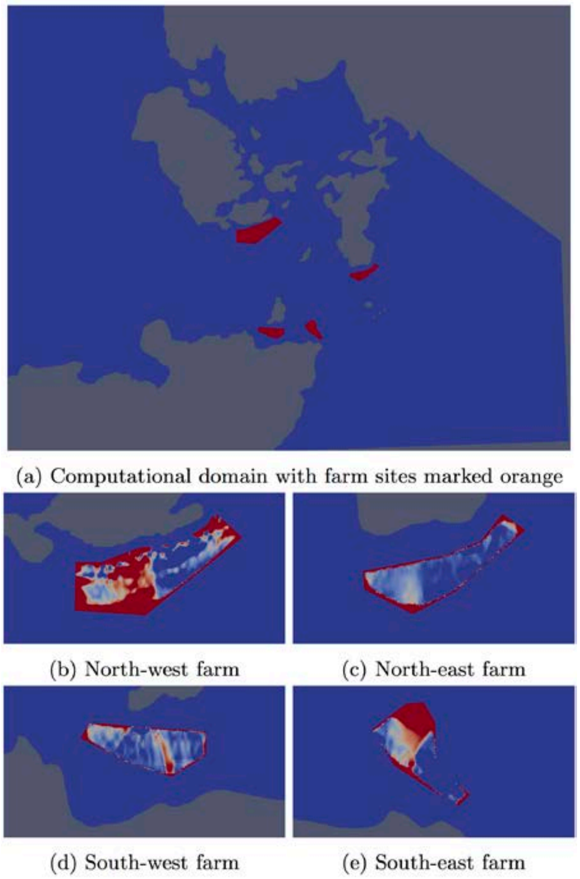

Figure 3: A preliminary continuum farm optimisation applied to four farms around the Orkney Islands, Scotland. Figures (b)-(d) show the optimised turbine density function for the four farms. Red indicates areas of densely deployed turbines, blue indicates areas with no turbines.

3. Continuum farm optimisation

The continuum farm approach is the lowest fidelity model presented in this paper, and is primarily suited for resource assessment, and during the scoping and early conception stages of the design and planning process of tidal farms.

This approach does not resolve individual turbines, but instead models a continuum of turbines using a turbine density function, and applies bottom friction proportional to this. In this setup, an area with high turbine density values conforms to an area with densely packed tidal turbines. An area with zero turbine density conforms to an area with no turbines.

The key advantage of the continuum approach is that individual turbines do not need to be resolved in the numerical model. Consequently, compared to the higher fidelity models, coarser meshes may be used. As such, each flow solve is computationally cheaper. This approach is, therefore, ideally suited for resource assessment, and to simultaneously optimise multiple farms with potentially hundreds of turbines and complex constraints over large geographical areas. Such tasks would be computationally infeasible with higher fidelity models.

The mathematical formulation and numerical discretization of this problem follows closely that presented in Funke et al. (2014). The tidal current prediction is based on the shallow water equations. The turbine density function enters the equations as a contribution to the bottom friction (in addition to the natural bottom friction). For added computational efficiency, one tidal cycle can be approximated by two steady-state flood/ebb solves. To solve the resulting equations in this preliminary example, an unrealistically high viscosity of 250 m^2s^{-1} was required, and hence the presented results should be considered as highly idealistic. The bathymetry is assembled from the General Bathymetry Chart of the Oceans (GEBCO 2008), the Digimap bathymetry maps, and data from the Scottish government. The discretization of this setup leads to roughly 2.2×10^5 triangles of 50 m size inside and up to 759 m outside the farms.

The optimisation is set up to maximise the farm performance by varying the turbine density function. Minimum and maximum bound constraints ensure that no turbines are deployed outside the farm areas and that density threshold values are not exceeded. The performance of the farms is measured as a combination of the total power produced and the turbine costs. The cost is estimated from the number of turbines which can be derived from the turbine density function. The resulting optimisation problem was solved using a gradient-based PDE-constrained algorithm using the adjoint approach. For more details, refer to Funke et al. (2014).

The results are shown in figures 3(b) – 3(d). The optimal turbine density function indicates the preferred areas for turbine installations (red areas). In contrast to the *combined* optimisation of the farms, if each farm is optimised *individually* (i.e. neglecting the presence of the other farms), the combined power output of the four farms is overestimated by nearly 15 %. This indicates that turbine developers might overestimate their site potential if the installation of competitor farms are not taken into account. Indeed, there is also the implication that it may be beneficial for developers of proximate sites to coordinate the design of their respective arrays with each other for their mutual benefit.

The continuum approach can also be used to quickly generate an initial estimate for the optimal number and positions of turbines in a farm. This estimate can then be verified by a higher fidelity model, or used as a starting point and further improved upon through micro-siting optimisation. Such simulations can be used as a tool for resource assessment. In contrast to currently used large-scale resource assessment methods that rely purely on the flow current without any turbines installed, the continuum farm approach incorporates the effects of the turbine installations on the tidal flow.

4. Three-dimensional modelling

Any computational model is an abstraction of the real-world physical processes. The ideal goal is to model these processes with the highest accuracy in order to recreate the process with the greatest precision. A three-dimensional model in which the turbine and its blades are fully resolved, and the flow is computed by solving the full Navier-Stokes equations is possible for individual turbines over short time spans with LES or RANS turbulence models, e.g. Batten et al. (2013); McNaughton et al. (2014); Churchfield et al. (2013). However, for more than a single or very small number of turbines, this approach becomes computationally prohibitive, and essentially impossible within an optimisation framework.

Various alternative simplified/ parameterised models exist, for example actuator disks or simulations based on blade element momentum or actuator line theory. The actuator disk model is a method whereby turbines can be approximately modelled as porous-like disks at far coarser overall mesh resolutions. These approaches have been used to simulate relatively small numbers of turbines, where turbine-wake interactions have been studied along with considerations of inter- and intra- row spacing in order to maximise array power, e.g. Bai et al. (2009); Yang et al. (2014); Lee et al. (2010).

To date, a major focus with all of these studies of 3D turbine, as well as turbulence, models has been their validation against scaled laboratory data. Accelerating the collection of turbulence data in the real marine environment, and in the presence of prototype devices will enable the calibration and validation of 3D numerical models. This will be a crucial on-going activity over coming years, as the first tidal sites begin to be developed and devices deployed. The relative abilities of, and thus the requirements for, 2D depth-averaged versus full 3D models in the accurate representation of turbine-wake-turbine interactions within an array is an on-going research question. This is clearly required, however, in order to have confidence in the results of numerical array optimisation studies and associated power or profit estimates.

5. Array optimisation

The computational demands of high resolution three-dimensional models make them unsuitable as a basis of an array optimisation. Conversely, the continuum approach is too low fidelity to satisfactorily specify a micro-siting design by itself. As such, the knowledge gained through detailed modelling at the turbine scale and coarser

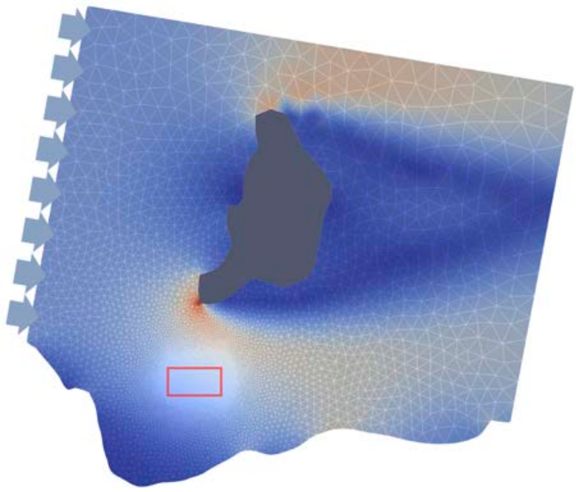

Figure 4: Domain for Inner Sound of the Pentland Firth array optimisation with individual turbines example. Mesh elements (shown) range from 50 m down to 2 m in turbine area (marked red). Inflow to the domain for peak flood tide is shown by arrows and resulting ambient flow field (without turbines installed) is shown by colour plot.

modelling at the coastal scale is fed into and applied at the array scale (see figure 1). The idea behind the two array-scale approaches discussed below is that they are sufficiently high fidelity to capture the important physical processes well, but cheap enough to facilitate array optimisation.

5.1 Prescribed wake models

One convenient and economical way in which individual turbines may be modelled on an array scale, is to calculate the ambient velocity over the domain and superimpose the effects of the turbines at their locations. Flow through a turbine causes a velocity deficit behind the turbine and acceleration around the sides. These changes in velocity can be estimated analytically through use of a 'wake model' and efficiently incorporated with the ambient flow to estimate the flow over the domain and hence approximate the power extracted by the turbines - which is proportional to the cube of the velocity at the turbine (Barnett et al., 2014).

This process can be achieved with comparatively little computational effort, and consequently an estimation of the power extracted by a certain array formation can be made very quickly. This facilitates the use of *global* optimisation methods, such as genetic algorithms (Thomson et al., 2011; Barnett et al., 2014). Since the function linking the power extracted by the turbines and locations of those turbines is non-linear and may have many local maxima, such global optimisation techniques provide a good search of the potential array layouts.

The key simplifying assumption made in the prescribed wake model approach is that the computation of the flow is decoupled from the effects of the turbine. This means that some of the complex interactions between turbines can be missed, along with further afield changes such as flow redirection caused by array-scale blockage. Consequently there is a need for an array optimisation approach based upon a fully coupled flow model as discussed below.

5.2 Non-linear shallow water equation

The methodology of the fully coupled model is to represent turbines as independently resolved areas of increased friction and compute the flow field as the solution to the non-linear shallow water equations. The turbines act as realistic momentum sinks which may be parametrised through pre-defined thrust curves and the power extracted is found through use of a pre-defined power curve. The adjoint approach (Funke, 2012) is used to efficiently compute the gradient of the array's performance with respect to the location of each turbine. Using this information, a gradient-based optimisation algorithm improves the layout of the turbines for the next iteration.

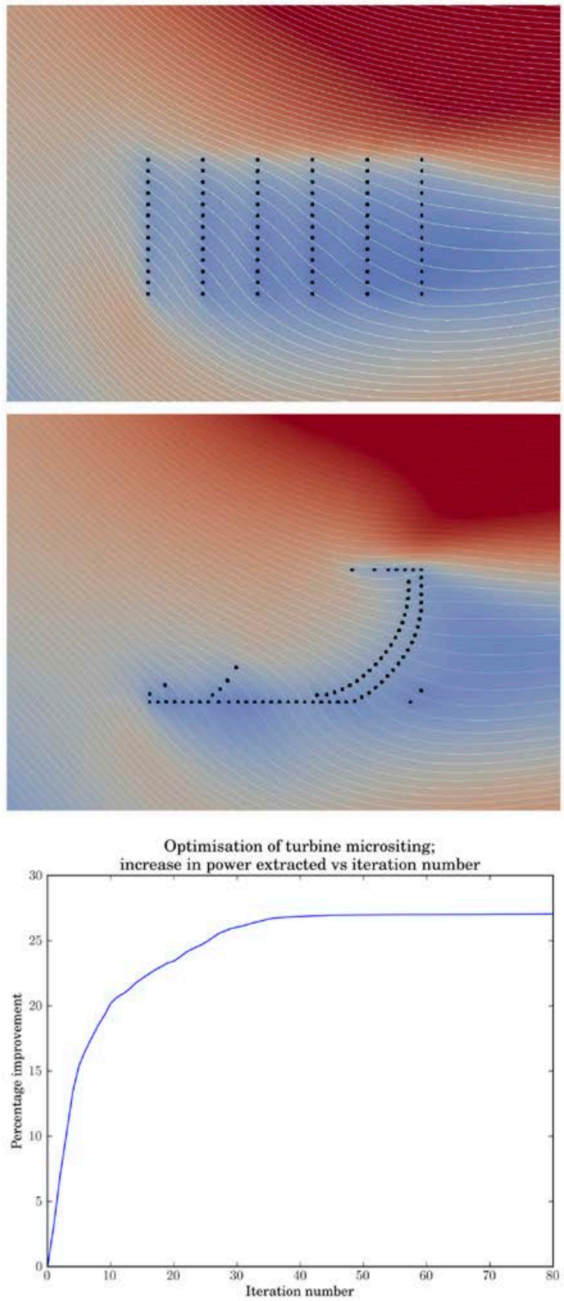

Figure 5: Top; Starting arrangement of 78 turbines (represented as black dots) with velocity field and streamlines. Middle; optimised array micro-siting design. Bottom; graph showing the improvement in power extracted by the array over the initial configuration, as the micro-siting is optimised.

Gradient-based optimisation of tidal turbine farm layouts requires in the order of hundreds of iterations, rather than the thousands to tens of thousands needed by global optimisation approaches such as genetic algorithms. Fewer iterations means that a more expensive/realistic model may be used each time, so the computational effort is focused on capturing the physical processes in as high fidelity as possible.

The main advantage of this approach is that the computational cost of determining the gradient is independent of the number of turbines and the number of iterations required for convergence has also been observed to be independent of the array size (Funke et al., 2014). This makes the approach inherently scalable, meaning it can efficiently determine optimised turbine layouts even on the restricted computational budget of a desktop machine.

Figure 4 shows the domain and mesh for an optimisation example based upon the Inner Sound of the Pentland Firth, Scotland. A turbine area (shown in red) is designated, representing the area in which turbines may be deployed. The mesh size within this area is 2 m, enough to resolve the individual turbines well. The optimisation was run for a steady-state peak flood, flow enters the domain as shown in figure 4, leaving across the opposite boundary. All other boundaries are modelled as frictionless. The depth is assumed constant at 30 m, with constant 0.0025 roughness. The viscosity was set to 30 m^2s^{-1}.

The turbines are initialised in a grid formation (figure 5), at each iteration the shallow water equations are solved, and the power extracted over the farm is calculated, as is the gradient of the power with respect to the x and y coordinates of each turbine. The locations of the turbines are adjusted for the next iteration. In the example, the farm was optimised in 80 iterations for an improvement of 25 % in the power extracted.

One drawback of this methodology is use of a gradient-based, *local* optimisation algorithm. While this does converge in orders of magnitude fewer iterations (as compared to global techniques) they can return *local* rather than *global* optima. Barnett et al., (2014) has demonstrated how the two techniques may be combined with associated improvements to the optimal array design that is found.

6. A holistic design methodology

While the power extracted from the flow is a key metric by which to judge the quality of turbine array designs, in projects led by business-for-profit perhaps a more useful metric is to think of the business case for the installation. By adding financial cost models, the designer can consider the comparative expense of one array design over another and weigh this against the additional power that one may extract as compared to the other. This provides a more holistic approach to the array design problem.

The costs affecting an array design can be broadly categorised as those which depend on the number of turbines (size-dependent costs) and those dependent on the location of the turbines (location-dependent costs).

Determining the optimal array size is a key question facing developers scoping a site, and is largely unexplored in scientific literature. Using the continuum approach is a relatively straightforward approach, since this converts the discrete problem - how many turbines - into a continuous one, whereby the integral of the friction function may be used to provide an estimate of the number of turbines that should be installed.

Determining the size of an array using one of the higher fidelity models in which the turbines are individually resolved is a somewhat more difficult problem. As has been established, the amount of energy that is extracted at a given site by a certain number of turbines is highly dependent upon the micro-siting arrangement of those turbines. Therefore determining how the power output of the array will vary with the number of turbines requires a micro-siting optimisation for every array size that is of interest.

One method of optimising the array size is to treat the micro-siting as an ‘inner’ optimisation finding an upper limit to the power output of a given number of turbines by optimising their micro-siting layout. The ‘outer’ optimisation treats this inner optimisation as a computationally expensive black-box and uses Gaussian Process Regression (GPR) to maximise the utility of each time it is run for a given number of turbines. This enables an efficient optimisation of the number of turbines which should be installed on a given site, as demonstrated by Culley et al. (2014b).

For size-dependent costs, the cost functions are often fairly simple (costs being proportional to the number of turbines or the amount of power being extracted). However, location-dependent costs can be more problematic, since the costs may not always be calculated so directly. For example, the cost of undersea cabling will be linked to the length of cabling required, which is dependent not only on the location of the turbines, but also on the design of the cable network (or 'routing') which connects them. The shortest routing for a given arrangement of turbines must also be found through an optimisation process (Culley et al., 2014a) Therefore, in the course of optimising the array micro-siting, at each iteration the optimal cable routing must be found. The financial cost of the cable along with the financial income from the power extracted by the layout is calculated by means of a financial model. The gradient of financial return with respect to the turbine locations is also found and the gradient-based optimisation seeks to move the turbines so as to maximise this financial return (Culley et al., 2014a).

7. Conclusion

In this paper, a hierarchy of modelling approaches has been proposed for the design of tidal turbine arrays. These approaches ranged from low-fidelity, depth-averaged representations of whole coastal regions with multiple farms, to fully resolved three-dimensional turbines in much smaller domains.

Different stages in the design process call for different tools. Rapid, computationally cheap models are ideal for resource estimation and the scoping of initial sites. Conversely, high-resolution, three-dimensional techniques are imperative to investigate how the wake structure will be governed by factors such as the blade, pylon and nacelle shapes of the proposed turbines, as well as background turbulence levels. In the middle of the spectrum are array optimisation tools based on either a fully coupled shallow-water model with gradient-based optimisation of turbine position, or the uncoupled prescribed wake model approach with global optimisation methods. Such optimisations can, and should, be informed by the results of both the higher and lower fidelity models in the hierarchy.

A simple example of a multi-farm optimisation using the continuum approach was given, which demonstrated that if array designers neglect the presence of near-by turbine arrays in their models, they may over-estimate energy yield.

Similarly, array micro-siting optimisation was demonstrated in which the turbines were individually resolved and the flow field was the solution to the shallow-water equations found using the finite element method. The location of the turbines - the micro-siting design - was then optimised using a gradient-based method for an improvement of circa 25 % over 80 iterations.

The concept of design optimisation to optimise the financial return (as opposed to the energy yield) was also discussed, with methodologies presented to account for both size-dependent and location-dependent costs. This facilitates a more holistic approach to the design of tidal turbine arrays as it allows the benefits of different array designs to be considered alongside the costs of realising those benefits.

Acknowledgements

The authors would like to acknowledge Imperial College London, the Engineering and Physical Sciences Research Council (projects: EP/J010065/1 and EP/L000407/1) and a Center of Excellence grant from the Research Council of Norway to the Center for Biomedical Computing at Simula Research Laboratory for funding which supported this work.

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