Unification of Gauge Forces and Gravity using Tangled Vortex Knots

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Abstract. In this paper, the statistics of excitation-tangles in a postulated background ideal-superfluid field ψ_M is studied. The structure of the Standard Model is derived in terms of tangle vortex-knots and soliton. Gravity is observed in terms of torsion and curvature in the continuum. In this way, non-linear dynamics and excitations give rise to a unified field theory as well as a *Theory of Everything.* As a result of this unification, spacetime and matter are shown to be fundamentally equivalent, while gauge fields arise from reorientation and excitations of the the fundamental underlying field ψ_M.

Keywords: Theory of Everything, Gravity, Gauge Fields

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Discovering a *Theory of Everything* has been the holy grail of Physics ever since the systematic study of physical phenomena began [1–5]. Since the times of Copernicus and Leucippus of Miletus to the contemporary age, there have been attempts at tying together the disparate phenomena within a framework of consistent rules [6]. Sir Isaac Newton unified observable effects of gravity on Earth with the behaviour of celestial bodies in space [7]. James Clerk Maxwell unified electricity, magnetism and light under the umbrella of electromagnetism [8]. Albert Einstein unified space-time and mass-energy [9, 10]. Quantum field theory unified special relativity with classical field theory and quantum mechanics [11, 12], and electroweak unification was developed by Abdus Salam, Sheldon Glashow and Steven Weinberg [13–15]. The strong interaction was described by the theoretical tools founded in the theory of quantum chromodynamics [16, 17], and soon after, the Standard Model was born [18, 19]. What remains now is the unification of gravity with the quantum mechanics in a theory of quantum gravity [20]. There have been a number of candidate theories, from string theory [21–25] and loop quantum gravity [26–28] to causal sets [29] and noncommutative geometry [30]. Superfluid vacuum theory has been a candidate theory for the *Theory of Everything*, and posits that the fundamental physical vacuum is a superfluid, which accounts for all the forces of nature as excitations in the superfluid field [31–33]. This latest theory of the 'lumineferous aether' is lacking in not accounting for the multiplicity of hadrons, which it says could be accounted for using additional degrees of freedom for the underlying field.

The principle of aether has been deliberated on since the times of Plato's *Timaeus* and Aristotle [34]. Be it Ramon Llull's theory of *Quintessence*, Newton's use of the concept of aether to help match observations to mechanical rules of his conception of physics or Robert Fludd's 'subtler than light' aether, the concept fascinated scientists well into the eighteenth and nineteenth century [35–37]. Johann Bernoulli gave the idea that space was permeated by aether, which had 'excessively small whirlpools' in them, and later this influenced Huygen's wave theory of light [38, 39]. Aether, as conceived as a medium that is important for the transmission of forces, was summarily debunked by the famous Michelson-Morley experiment [40]. Though 'ether fields' may have been removed from the purview of science in the first half of the twentieth century itself, fields, in general, remained at the forefront of Physics thereafter, beginning with Louis de Broglie's wave description of elementary systems, the Born-Heisenberg-Jordan *Free Field Theory* and Dirac's quantum field theory of radiations [12, 41, 42]. Quantum Field Theory posits the emergence of localized processes in terms of excitations of quantum fields, and involves nonclassical, non-local correlations that lead to varying levels of complexity [43–53].

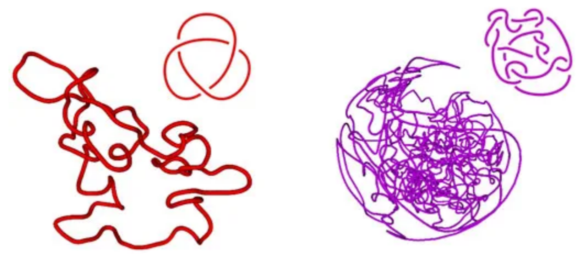

FIG. 1. Some vortex line knots in eigenfunctions, plotted alongside simpler projections of the same knot [54]

Complexity in various physical systems is often seen in subtle spatio-temporal structures and disorder [55–58]. For instance, at high energies, principles of quantum ergodicity set in for complex modes of three-dimensional domains [59]. Understanding the spatial structure of wavefunctions, especially for chaotic wave dynamics, has been a challenge [60]. If we follow the hydrodynamic

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interpretation of the quantum mechanics of a single-particle, the zeros of complex-valued three-dimensional scalar fields are lines, in general, which are basically vortex filaments around which the probability current, local velocity and phase [61–64]. The amplitude is zero at the vortex core and the phase change around a vortex-line is quantised (in units of 2π). This kind of vortex topology is also seen in many-particle quantum condensates such as *Bose-Einstein Condensates* [65]. The arrangement of vortex lines in such physical systems provides a structural skeleton to the wavefunctions [62, 63, 66]. At high-energy modes, this arrangement is highly irregular and densely intertwined, often with knotting of vortex filaments. Taylor et al recently studied knotted nodal vortex lines in model systems of wave chaos, presenting interesting characteristics of vortex knots in tangled quantum eigenfunctions [54]. Even with the ubiquity of vortex filaments and tangles, in systems as disparate as cosmic string [67], quantum turbulence [68] and optical vortices [69], the systematic study of knotted structures and vortex arrangements in fields has only recently been started: theoretical studies of vortex lines in complex scalar fields, such as in superfluid flows [70], and optical vector vortex beams [71], and experimental studies with topological defects and vortices being knotted and successfully embedded in various three-dimensional fields, such as those in chiral nematic colloids [72] and liquid crystals [73], and isolated trefoil vortex knots and pairs of linked vortex rings in water [74]. Even as rigorous mathematical methods and tools to study the statistical topology of random fields are limited, there has been progress recently in exploration of these structures using computing techniques and simulations [54].

The multiplicity of tangled vortex knots provide the perfect theoretical basis for developing the principles of the *Superfluid Vacuum Theory* to present a true *Theory of Everything*, from first principles, which relies only on the non-linear dynamics of an underlying universal superfluid field ψ_M. The reason for this can be obtained by considering why the concept of the luminiferous aether as defined in classical Physics faced a number conceptual hurdles and contradictions. The primary one related to the idea that the aether had a well-defined velocity at each point in space, which contradicts the idea that all directions inside a light-cone are equivalent [75]. Dirac circumvented this by suggesting that the aether was quantum mechanical in nature, and had quantum fluctuations [76]. Applying the uncertainty principle makes us able to define the velocity of the aether at a point only upto a limited certainty. We can only define a wavefunction representing the perfect vacuum state for the aether flow, where all velocities are equiprobable. The objections to the possibility of a drag force, as posited by those contradicting the classical conception of aether, can be worked around by considered this medium to be a superfluid.

In the formulation of the *Superfluid Vacuum Theory*, the system is taken to comprise of a sea of fermions and anti-fermions, with the system's hamiltonian taken to be

$$\begin{aligned} H = \sum \epsilon_k a^\dagger_{k,\sigma_-} a_{k,\sigma_-} + \sum \epsilon_k b^\dagger_{k,\sigma_+} b_{k,\sigma_+} \\ - \sum V(k,k') a^\dagger_{k',\sigma_-} b^\dagger_{q-k',\sigma_+} b_{q-k,\sigma_+} a_{k,\sigma_-} \end{aligned} \quad (1)$$

where $(a^\dagger_{k,\sigma_-}, a_{k,\sigma_-})$ and $(b^\dagger_{k,\sigma_+}, b_{k,\sigma_+})$ represent fermion and anti-fermion (creation, destruction) operators for momentum k and spin σ_- (for fermion) and σ_+ (for anti-fermion), and single particle energy $\epsilon^{(a)}_k = \epsilon^{(b)}_k = \epsilon_k$. The interaction potential is given by $V(k,k')$ for plane-wave states with momentum k and k', and this can be decomposed into various components V_m with $m = 0, 1, 2, ...$ and spin-pairs in the quintet, singlet and triplet states. This involves generalised correlations - including two-particle and four-particle correlations. The general superfluid state state is taken as a superposition of two-particle singlet, two-particle triplet and four-particle quintet states.

The conception of the vacuum state in terms of fermion and anti-fermion states within the *Superfluid Vacuum Theory* seems to be ad-hoc and imposed. A more first-principles approach would involve the formulation of the elementary particles and forces in terms of topological defects and perturbations in the background fluid-field ψ_M. This is the primary invention in the formalism presented in this paper, and this begins with the idea of quantisation of the superfluid from first-principles. Fluid behavior arises in systems with microscopic constituents that are quite different from other systems that are described by field theoretic descriptions, and therefore we expect the theory to take the form of an effective field theory. We encounter fluid vortices that have arbitrarily low energy classically, irrespective of their spatial extent, in such systems. Landau was one of the first to tackle this problem, saying that vortex modes should be gapped in the quantum theory, and eventually ending up formulating the theory of superfluids [77]! While *Endlich et al* conjectured that it is not possible to quantize fluids [78], *Gripaios et al* showed that such a theory is indeed possible with the quantum fluid having a good IR and UV behavior using direct loop computations [79].

In superfluids such as helium ^4He, the classical theory of thin-core vortex filaments is valid due to the the large separation of scales between the typical distances between vortices and the vortex core radius a_0. In experiments involving turbulence and in systems such as the atomic Bose-Einstein condensates, these two length scales are comparable. In this scenario, the Gross-Pitaevskii equation (GPE) presents a realistic and appropriate model for studying these systems[80]. If the

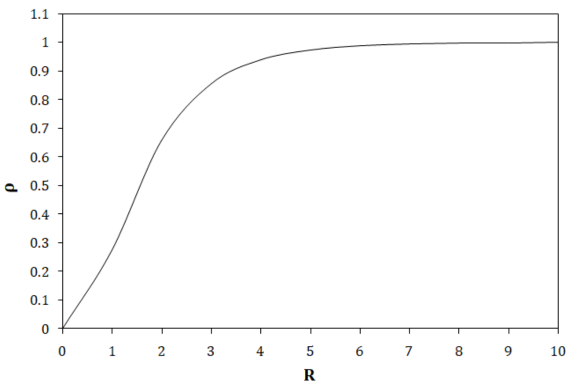

FIG. 2. The density field around a 2D axisymmetric vortex, with the radial distances being in units of the healing length

wavefunction of the background superfluid field is ψ_M, we have

$$2i\delta_t\psi_M + \nabla^2\psi_M - |\psi_M|^2\psi_M = 0 \tag{2}$$

where there is no external confining potential. The characteristic length scale of perturbations of the background superfluid field, known as the *Healing Length*, is $\xi = \frac{1}{\sqrt{\langle\rho\rangle}}$ where $\langle\rho\rangle = \frac{1}{V}\int_V |\psi_M|^2 dV$ is the mean density of the superfluid. The GPE conserves energy, the total number of particles and thereby the quantity ξ. We will be studying a system where the density field at infinity is equal to unity and perturbations are localised in a certain region of the superfluid, without loss of generality. In this system, a vortex is constructed by considering a two-dimensional S-Z plane, where S is an axis in the X-Y plane inclined to the X-axis at at angle of ϕ. A stable vortex in this superfluid is a hole in the density field around which the wave function has phase changes by $\pm 2\pi$. An accurate description of a 2D vortex centered on the origin of the S-Z plane is given by the wave function

$$\psi_V(s,z) = \sqrt{\rho(R)}e^{-i\theta(s,z)} \tag{3}$$

where $R = \sqrt{s^2+z^2}$, $\rho(R) = \frac{R^2(a_1+a_2R^2)}{1+b_1R^2+b_2R^4}$ and $\theta(s,z) = tan^{-1}(\frac{z}{s})$. The coefficients $a_1 = 0.344, a_2 = 0.029, b_1 = 0.333, b_2 = 0.029$ arise from a second-order Padé approximation [81]. We define a vortex-knot as a closed curve $T_{l,k}$ over a torus, with the toroidal radius R_0 and the poloidal radius R_1, which is determined by counting the number of toroidal wraps l and the number of poloidal wraps k. We can evaluate the wavefunctions of these vortex-knots and feed them into the Gross-Pitaevskii Equation to study the knot-dynamics.

While this is useful to look at a finite number of topological defects, for a general effective field theory description, we must look at fluid parametrisation, in terms of a time-dependent map $\psi^i(x^j,t)$ from some space manifold M into itself. We will assume this manifold to be $\mathbb{R}^2$, and that by changing this map at short distances, we can make the map (and its inverse) smooth so that ψ is a diffeomorphism and the configuration space of this fluid is the diffeomorphism group Diff(M). We will also assume that interpenetration and cavitation of the fluid has an energy-cost and can be ignored in the field-theoretic description so that the map is bijective. Unlike for Lie Groups for which an exponential maps takes one from tangent space to the Lie group, since the configuration space of ψ is infinite-dimensional and not a Lie Group in the usual sense, the exponential map is seen to be inadequate for its description [79]. We therefore use the parametrisation $\psi = x + v$, where x is the identity map on M and v denotes the transverse 'vortex' mode. The action for the fluid, which is taken to be perfect for a consistent quantum description that requires non-dissipative conditions, can be derived by requiring that the theory be invariant under Poincare transformations of x and area-preserving diffeomorphisms of ψ [78], and has been known for quite some time [82]. In (2+1)D, the lagrangian is $\mathcal{L} = \alpha_0 f(\sqrt{det(\delta_\mu\psi^i\delta^\mu\psi^j)})$, with α setting the overall phase and $f'(1) = 1$. This satisfies the conservation of the energy-momentum tensor $T_{\mu\nu} = (\rho+p)u_\mu u_\nu + p\eta_{\nu\mu}$ for $\rho = -\alpha_0 f$, $p = -\alpha_0(\sqrt{det(\delta_\mu\psi^i\delta^\mu\psi^j)}f' - f)$ and $u^\mu = \frac{1}{2\sqrt{det(\delta_\mu\psi^i\delta^\mu\psi^j)}}\epsilon^{\mu\alpha\beta}\epsilon_{ij}\delta_\alpha\psi^i\delta_\beta\psi^j$.

Considering $\psi^i = x^i + v^i$, we have the Lagrangian [79, 83]

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}(\dot{v}^2 - c^2[\delta v]^2) - \frac{(3c^2+f_3)}{6}[\delta v]^3 + \frac{c^2}{2}[\delta v][\delta v^2] \\ &+ \frac{(c^2+1)}{2}[\delta v]\dot{v}^2 - \dot{v}.\delta v.\dot{v} - \frac{(f_4+3c^2+6f_3)}{24}[\delta v]^4 \\ &+ \frac{(c^2+f_3)}{4}[\delta v]^2[\delta v^2] - \frac{c^2}{8}[\delta v^2]^2 + \frac{(1-c^2)}{8}\dot{v}^4 \\ &- c^2[\delta v].\dot{v}.\delta v.\dot{v} - \frac{(1-3c^2-f_3)}{4}[\delta v]^2\dot{v}^2 \\ &+ \frac{1-c^2}{4}[\delta v^2]\dot{v}^2 + \frac{1}{2}\dot{v}.\delta v.\delta v^T.\dot{v} + ...\end{aligned} \tag{4}$$

where $f_n = \frac{\delta^n f}{\delta\sqrt{det(\delta_\mu\psi^i\delta^\mu\psi^j)}^n}|_{det(\delta_\mu\psi^i\delta^\mu\psi^j)=1}$, $c = \sqrt{f_2}$ is the speed of sound and $[\delta v]$ is the trace of the matrix $\delta^i v^j$.

While this is informative of the dynamics of the vortices, this is not endowed with the physical resources to be able to describe all the forces of nature. A way to get around this is to involve higher dimensional defects and interaction-knots. Recently it was found that one could use entanglement between vortex-membranes to study the Standard Model [84, 85], in which knot dynamics on 3D vortex-membranes in 5D superfluid systems were studied, with a new theory called Knot Physics being derived from there to characterise the evolution of entanglement of the 3D leapfrogging vortex-membranes. We can derive the unified theory of the Standard Knot by considering the low energy effective theory for the composite knot with (N = 4, M = 3), where N denotes the number of vortex-membranes and M denotes the number of levels [86]. This is, however, only valid for one-generation of particles: the up quark, the down quark, electron and neutrino. Given the restrictions of the construction, we cannot change the half-winding numbers to create different generations of particles.

Instead of taking the system to be a 5D system with interaction-knots, if we take a 3+1D space with the particles being made of vortical solitons that move around each other, we have an interesting system of study. Instead of the two-knot systems in the vortex-membrane model, here coupling of more than two vortical solitons can be allowed, whose dynamics can be solved using nonlinear dispersive partial differential equations. *Mandelstam* showed that finite-length vortices in an SU(n) Nielsen-Olesen model need the explicit introduction of monopoles, which are confined in multiples of n by the Meissner effect, and thereby the model could offer a natural explanation of quark confinement [87]. A key element that I introduce is that of self-similarity: *vortices within vortices*. The hierarchical nature of vortex tubes in turbulence has been studied previously [88]. *Leykam et al* studied the existence, dynamics and stability of multivortex discrete solitons [89]. The important topological quantity in this framework is the vortex topological charge (TC), which is the phase change in units of 2π accumulated around the vortex core. In the past there have been attempts at looking at specific gauge fields and their connection to superfluid systems, such as that between Quantum Chromodynamics and Superfluid Helium-II (particularly with an emphasis on the topological susceptibility in the former and winding number susceptibility in the latter) [90].

The circulation of the vortex tangle into knots produces the various particles. For instance, the Trefoil knot could produce a proton with the branches of the knot symbolising the quarks. In the simple vortex soliton model, if the charge is the net topological circulation number, the mass being the speed of circulation (which also relates to the flavor in this formalism) and the relative orientation of solitons in 3D space (also seen from the idea of coloured knots [91]) giving an idea of the colour-charge, we can build the various particles accordingly. In the tangled vortex model, we have tangle knots that require energy for the tangle-knots to be formed. The physical attributes of the particles can be naturally determined using a model that associates structure and form to the atomic constituents, and explores their interactions, mostly by their combinations. A reaction or a physical process, in this picture, is a stage of combination (fusion) or separation (fission) of composite tangle-knots. Two of the most interesting results of this model are: absence of the $SU(3)$ gauge theory associated with quantum chromodynamics and the absence of the Higgs Boson. The reason for the former is the absence

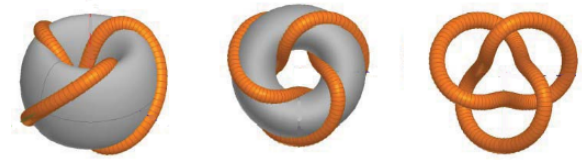

FIG. 3. Trefoil knot, shown winding around a torus, and by-itself

of quarks beyond just as tangle-elements, along with the absence of gluons to account for confinement, which is a natural result of the knot structure of the quark-composites. Mass, on the other hand, emerges in terms of solitonic distortions of spacetime. The idea of considering structures with half-twists as fundamental elements of this theory, be it within an unknot or a trefoil knot, comes from the idea that if one considers that the twist in a Mobius strip can be thought of as a half-twist between strands, if the perimeter of the strip is regarded as constituting the strand(s). A fermion can be represented geometrically by a Mobius strip since the quantum mechanical wavefunction of a spin $\frac{1}{2}$ particle is represented by a spinor that requires two full rotations to come back to its original state, and this gives a further motivation to pursue a half-twist as the fundamental element in our formulation. The half-strand, in this model, emerges in the form of crossings in a generalised Mobius form, which can be wound on a torus, as shown in *Figure 1.* From the perspective of topology and algebraic geometry, a torus is homeomorphic to the Cartesian product of two circles: $S_1 \times S_1$. As a group the circle has the U(1)/SO(2) topological structure, and therefore, locally, the toroidal topology has a U(1) substructure. At the same time, a torus in SU(2) yields a torus in SO(3) [92]. The U(1) and SU(2) Lie groups will form the geometric basis for the formulation of the Vortex-knot unification theory.

Formally, we can define the vortex-knot as a closed, non-self intersecting curve embedded in a torus that cuts a meridian at $p > 1$ points and a longitude at $q > 1$ points (p and q relatively prime integers), is a non-trivial knot $T_{p,q}$, with winding number $w = q/p$. If we only focus at a crossing point, as shown in Figure 4, we can define the charge as 'Up' if moving under from right to left and 'Down' if moving under from left to right. For a three-particle system, we can have the following combinations with their associated number of half-twists (n_{HT}), with n_{HT} of 'u' being +1 and n_{HT} of 'd' being '-1': 'uuu' (n_{HT} = +3), 'uud' (n_{HT} = +1), 'udd' (n_{HT} = -1) and 'ddd' (n_{HT} = -3). For empirically found fermions, we know that a Proton (with 'uud') has charge +1 and a Neutron (with 'udd') has no charge. Therefore, charge of 'u' is $+\frac{2}{3}$ and the charge of 'd' is $-\frac{1}{3}$. We have an interesting formula between the charge of the constituent particles (q), average charge of the

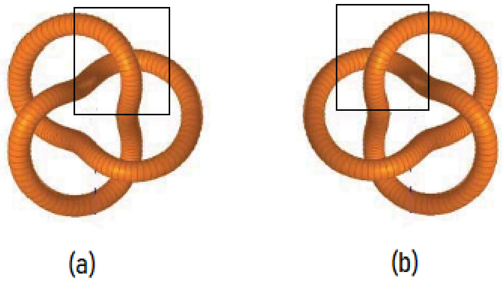

FIG. 4. Charge based on crossing orientation (a) Down - d, (b) Up - u

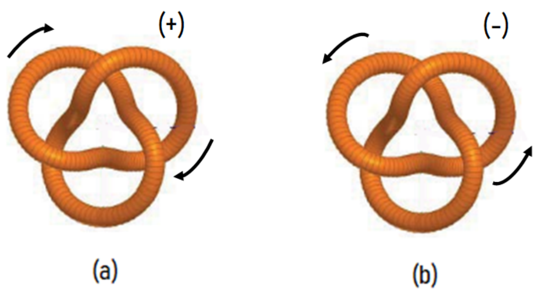

FIG. 5. Illustration of circulation of (a) Antiparticle with (+) parity, (b) Particle with (-) parity

composite particles ($\overline{Q}$) and the n_{HT}:

$$q = \frac{n_{HT}}{2} + \overline{Q} \tag{5}$$

This is similar to the well-known formula: $q = I_3 + \frac{Y}{2}$, where I_3 is the third component of the isospin and Y is the hypercharge. We can thus make the assignment:

$$I_3 = \frac{n_{HT}}{2}, \overline{Q} = \frac{Y}{2} \tag{6}$$

An important point of discussion here is the manner of defining the anti-particles within this formalism. This is straightforward, due to the fluidic nature of the constituent strands. The net direction of traversing in the knot, when reversed, gives us the antiparticle. We can define parity here as the orientation of the direction of traversing: clockwise is (-) and counterclockwise is (+).

The trefoil knots can combine in various combinations by fusing at one of the half-twist points, if and only if the direction of traversal and the half-twist are compatible, as best defined by Hopf Algebra. We can use a tensor-formalism, with composite fermion tangle-knots represented as a covariant tensor with three indices (M_{abc}) and the corresponding composite antifermion tangle-knot as contravariant tensor with three indices (N^{def}). These indices can be 'u', 'd' or even 's', if the formalism is extended to incorporate *strangeness*. Implementing fusion is then expressed by equating a subscript of M and a superscript of N using the Kronecker delta function,

$$\delta^m_n M_{abc} N^{def} = K^{def/n}_{abc/m} \tag{7}$$

where K denotes the composite resultant particle. The notation abc/m indicates the subscripts not selected by the choice of m and the notation def/n indicates the superscripts not selected by the choice of n. To help define the nature of the fusion, we define a state function for the connections in the tangential and normal direction to the flux-tangle, at the point of fusion. Let the normal-connection be $\langle\eta_\alpha\rangle$ and the tangential-connection be $\langle\tau_\alpha\rangle$, where the value of α is 'u', 'd' or 's'. We will define $a^\dagger$ and a as the creation and destruction operators for a connection. We can then write a general fusion as

$$\langle\epsilon^f_\alpha\rangle = a\langle\tau_\alpha\rangle + a^\dagger\langle\eta_\alpha\rangle \tag{8}$$

and a general fission as

$$\langle\epsilon^{fi}_\alpha\rangle = a\langle\eta_\alpha\rangle + a^\dagger\langle\tau_\alpha\rangle \tag{9}$$

We define the value of a $u-\overline{u}$ coupling as x, $d-\overline{d}$ coupling as y and $s-\overline{s}$ coupling as z. Generally, $\langle\epsilon^f_\alpha\rangle = (p_\alpha)$ with $p_u = x$, $p_d = y$ and $p_s = z$. If we integrate this into (7), we have

$$\delta^m_n \langle\epsilon^f_{\nu(m)}\rangle M_{abc} N^{def} = M_{abc/m} N^{def/n} (p_{\nu(m)} \tag{10}$$

This gives us the mathematical tools to look at how particles can fuse. An important aspect of elementary particles is *spin*, and this too can be assigned using the number of half-twists here: vortex-knots with odd values of n_{HT} that represent fermions are characterized by odd multiples of spin 1/2 while those with even values of n_{HT} that represent bosons are characterized by even multiples of spin 1/2.

To look at mesons, the idea presented by Fermi et al [93] describing a meson as a combination of a nucleon and an anti-nucleon is explored. If we denote the combinations 'ddd' as 'A', 'udd' as 'B', 'uud' as 'C' and 'uuu' as 'D', we can write the mesons and baryons as combinations of these and their conjugates: Δ^- is BC*B, Δ^0 is BC*C, Δ^+ is CB*B, Δ^{++} is CB*C, π^- is CB*, π^- is BC* and π^0 being either BB* or CC*. The way we ascertain the structure of these mesons is using empirically found reactions, such as

$$n + \pi^- \rightarrow \Delta^- \tag{11}$$

$$n + \pi^0 \rightarrow \Delta^0 \tag{12}$$

$$n + \pi^+ \rightarrow \Delta^+ \tag{13}$$

$$p + \pi^- \rightarrow \Delta^0 \tag{14}$$

$$p + \pi^0 \to \Delta^+ \tag{15}$$

$$p + \pi^+ \to \Delta^{++} \tag{16}$$

Since, neutron n has form B and proton p form C, and given the charge of the pions, we can say that the pions will be of a form constituted by a pairing between any two of B, B*, C and C*. We can check for the charges and each combination to determine the exact form. We can similarly work out the form of other baryons. However, to extend it to the three generations of quarks and other baryons, we have to look at the fundamental difference between generations. This is primarily with regards to mass and flavour. To differentiate between them therefore, we need to look at how mass arises in this formalism. As shown by Avrin [94], if we study the behavior of a torus vortex-knot under the influence of General Relativity, we will see that it obeys the Sine- Gordon equation

$$\frac{d^2\theta}{ds^2} + \eta^2 sin\theta = 0 \tag{17}$$

where $\eta = \sqrt{\frac{R+rcos\theta}{\mu r((R+rcos\theta)^2+\mu^2 r^2)}} \approx \frac{1}{\sqrt{\mu R r}}$, with r being the inner radius of the torus, R being the outer radius of the torus and $\mu = \frac{n}{m}$ for an (m,n) torus knot. This can be put into a dynamic $F = ma$ form, with the mass being $m = \frac{c^2}{4\pi G}\sqrt{\mu R r}$. Thus the mass varies due to the winding number, the inner and the outer radii of the torus-knot. If we consider the same unknot/trefoil configuration (thereby not altering the winding number), using the variation in the radii of the knot, we can obtain particles of different masses. In this manner, we can form the half-twist that embodies the strange-quark, and thereby all composite-particles comprising the strange-quark. This completes the baryon and meson sectors of the Standard Model. With regards to bosons, they are envisioned as composite particles as well, with their characteristic masses. We know that a photon is formed with the annihilation of a particle with its antiparticle, and therefore a photon has the form AA*, BB*, CC* or DD*. However, since we do not see an annihilation by neutrons or delta baryons into photons empirically, we only consider AA*, with both radii R and r tending to 0, for vanishing photon mass. The gauge bosons for the weak interaction are $W^+ =$ BA*, $W^- =$ B*A and $Z^0 =$ AA* and BB*, with masses $80.379 \pm 0.012 GeV/c^2$ (for the W bosons) and $91.1876 \pm 0.0021 GeV/c^2$ (for the Z boson) respectively.

Conclusion. Using the formulated vortex-knot model of unification, we can characterizes the various particles in the Standard Model in terms of vortical-knot solitons in spacetime-superfluid. The inherent ambiguity in that a given label (corresponding to a trefoil or unknot) may represent multiple fundamental particles of the Standard Model depending on the interaction that we are modeling is resolved using the variation in mass due to the radii of vortex-flow around the torus-knot. In this way, using a combinatorial framework, toroidal topology and first-principles-based properties matched with empirical data, we have a complete unified theory of Physics, which is based on an underlying spacetime-superfluid with excitations in the form of vortex tangles and knots that embody curvature and torsion in an otherwise featureless continuum. In terms of predictions, the occurrence of other combinations of the basic half-twists in this model can be tested.

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