Dividing Diffraction Pattern of Rotating Grating into Three Portions and Studying Each Portion Respectively

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Dividing Diffraction Pattern of Rotating Grating into Three Portions and Studying Each Portion Respectively

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Abstract

The phenomena of the rotating grating have been studied and have the varieties application. The diffraction normal order \( m_0 \) that is at the intersection of the grating’s normal vector and the screen is introduced and is the function of the incident angle of light, the grating spacing and wavelength.

We suggest the following: (1) the diffraction normal order \( m_0 \) and the zeroth order \( (m = 0) \) divide a diffraction pattern into three portions; each portion corresponds to a range of diffraction orders, \( m_I, m_{II}, m_{III} \), and is described by specific equation, respectively; (2) the number of the diffraction orders “m” in each portion is determined by the incident angle of light, the grating spacing, and wavelength; (3) the diffraction angles and positions of the diffraction orders of each portion can be presented in terms of \( m \) and \( m_0 \); (4) the diffraction pattern of a rotating grating is non-uniform.

Keywords: grating, diffraction pattern, diffraction order, rotating grating, twisting grating, inclined incident light beam, orientation-dependence of diffraction pattern, angle-dependence of diffraction pattern

1. Introduction

The phenomena of the diffraction grating are described by the grating equation that states that a grating of spacing “d” creates a diffraction pattern with diffraction orders, “m”, at discrete angles \( \theta_m \).
The diffracted angle, $\theta_m$, is measured from the grating normal. The phenomena of the grating rotating CW and CCW around one, two and three axes sequentially have been studied [1] [2].

The standard grating equation for the inclined incident light has the form

$$dsin(\theta_i) \pm dsin(\theta_m) = m\lambda$$

where the “+” and “−” are used when $\theta_i$ and $\theta_m$ are on the same side and opposite side of the grating normal. And treating all angles as positive.

However, it has been pointed out that for a single diffraction pattern of a given grating, the pattern should be divided into different portions based on the different locations of the different diffraction orders [3]. To describe a diffraction pattern, different grating equations should be used for different portions.

After introducing the diffraction normal order $m_0$, hereafter referred it as the “normal order”, that is the function of the grating spacing, wavelength and the incident angle of light, we show the followings in detail: (1) the normal order $m_0$ and the zeroth order $m = 0$ divide a diffraction pattern into three portions; each portion corresponds to a range of diffraction orders “m”, and is described by specific equations, respectively; (2) the number of the diffraction orders “m” in each portion is limited by the grating spacing, wavelength and the incident angle of light; (3) characterize the diffraction angle and positions of the diffraction orders of each portion; (4) due to (1) and (2) above, the diffraction pattern of a rotating grating is non-uniform.

2. Experimental Setup and Right-hand Rule

**Experimental Setup** Figure 1 shows the setup. The laser light propagates along negative X direction. The slits of the grating are along the Y axis.

![Figure 1. Schematic Setup](image)

**Right-Hand Rule:** Figure 2 shows the right-hand rule for determine CW and CCW rotation of the grating along an axis.
Figure 2. Right-Hand Rule for determining Rotation Direction

Next section shows how we divide the diffraction pattern of a rotated grating into three portions, and each portion is described by corresponding equation.

We have shown [1,2] that when the grating is rotated around two or three axes, the CW rotations and CCW rotations make different.

3. Grating Equations for Rotating CCW Grating Around Y axis

3.1. Normal Diffraction Order “$m_0$”

3.1.1. Definition of Normal Diffraction Order “$m_0$”

Let us define the normal diffraction order $m_0$ (hereafter denoted as “normal order”) and its position. Figure 3 is used to define the normal order $m_0$, which shows that the grating rotates CCW around the Y axis by $\theta_1$ (according to the right-hand rule, Figure 2), which is equivalent to the light beam shining on the grating at the oblique angle $\theta_1$. The line OO’ is the grating normal, the point O’ is on the screen. The “$BD = d$” is the spacing between two slits of the grating. The beam-1 and beam-2 travelling along path-1 and path-2, respectively, and meet at the point O’. After passing the grating, there is no path difference between two beams, i.e., $DO’ = BO’$. The O’ corresponds to the diffraction zeroth-order $m = 0$. The O’ corresponds to $m_0$. Denote $OO’ = L$.

We define $m_0 > 0$ and $\theta_1 > 0$ for the CCW rotation of the grating.

Figure 3. Schematic of normal order “$m_0$”
For the case of \( L \gg d \), path-1, OO' and path-2 are approximately parallel to each other. For the light beams arriving at the point O' on the grating normal, the diffraction angle \( \theta_{m_0} = 0 \). The “AB” is the path-deference between path-1 and path-2.

The standard grating equation, \( dsin(\theta_i) + dsin(\theta_m) = m\lambda \), becomes,

\[
d sin(\theta_i) = m \lambda.
\]  
(1a)

**Definition of normal order \( m_0 \):**

\[
m_0 = \frac{d}{\lambda} sin(\theta_i)
\]  
(1b)

Eq. 1b defines the normal order \( m_0 \). The \( m_0 \) is not necessary to be an integer.

**Example-3.1-1:** Figure 4 shows examples of Eq. 1b, where different spacings in terms of # slits/mm, different incident angles \( \theta_i \), and the red laser of wavelength \( \lambda = 0.65 \mu m \) is used.

![Diagram](image.png)

**Figure 4.** Correlation of \( m_0 \) (\( m_0 \)), \( d \), \( \lambda \) and incident angle \( \theta_i \) (Eq. 1b)

**Example-3.1-1a** (Figure 4): for \( d = 1000 \frac{\text{lines}}{\text{mm}} = 1 \mu m \), \( \lambda = 0.65 \mu m \), \( \theta_i = 60^\circ \), we have \( m_0 = 1.3 \).

Therefore, the first diffraction order positioned above \( m_0 \) is \( m = 2 \). There is only one diffraction order between the zeroth order \( m = 0 \) and the normal order \( m_0 \), which is \( m = 1 \).

**Example-3.1-1b** (Figure 4): for \( d = 100 \frac{\text{lines}}{\text{mm}} = 10 \mu m \), \( \lambda = 0.65 \mu m \), \( \theta_i = 60^\circ \), \( sin60^\circ = 0.866 \), \( m_0 = 13.3 \). Therefore, the first diffraction order positioned above \( m_0 \) is \( m = 14 \). There is 13 of diffraction orders between the zeroth order \( m = 0 \) and the normal order \( m_0 \), which are \( m = 1 \rightarrow 13 \).

Experiment-3.1-1 shows that the larger the spacing \( d \), the larger the value of the normal order \( m_0 \) and thus, the larger the number of the diffraction orders \( m \) between \( m \) and \( m_0 \).

**3.1.2. Position of normal order \( m_0 \) on the screen**

Let’s define the position of the normal order \( m_0 \) (Figure 3):

\[
h_0 = O'O''
\]  
(2)
The point $O'$ corresponds to the position of $m_0$. The position $h_0$ of $m_0$ is at the intersection of the grating normal and the screen. Calculating the position $h_0$, we have,

\[ h_0 = L \tan(\theta_i) \tag{3a} \]

\[ h_0 = L m_0 \frac{\lambda}{\cos \theta_i} \tag{3b} \]

![Position of normal order $h_0$ vs. incident angle $\theta_i$ (Eq. 3a)](image)

**Example 3.1-2** (Figure 5): for a relatively small incident angle, for example, $\theta_i < 30^\circ$, we approximately have a linear relation between $h_0$ and $\theta_i$: $h_0 \approx L \theta_i$.

To express $h_0$ in terms of the normal order $m_0$, Eq. (1b) gives

\[ \sin(\theta_i) = \frac{\tan(\theta_i)}{\sqrt{[\tan(\theta_i)]^2 + 1}} = m_0 \frac{\lambda}{d} \tag{4} \]

Solving Eq. (4), we obtain

\[ \tan(\theta_i) = m_0 \frac{\lambda}{\sqrt{1 - \left[m_0 \frac{\lambda}{d}\right]^2}} \tag{5} \]

Substituting Eq. (5), Eq. (3a) becomes

\[ h_0 = m_0 \frac{\lambda}{\sqrt{1 - \left[m_0 \frac{\lambda}{d}\right]^2}} \tag{3c} \]

The $h_0$ can be expressed by $m_0$ (Eq. 3c) or by $\theta_i$ (Eq. 3a) or by both $m_0$ and $\theta_i$ (Eq. 3b).

### 3.2. Dividing Diffraction Pattern into Three Portions

Now let us divide the diffraction pattern of a CCW rotating grating into three portions.

**First portion**: the first portion $m_I$ is above $m_0$, i.e., $m_I > m_0 > m$, where $m = 0$.

**Second portion**: the second portion $m_{II}$ is between $m_0$ and $m$, i.e., $m_0 > m_{II} > m$.

**Third portion**: the third portion is below $m = 0$, i.e., $m_{III} < m$ and $m_{III} < 0$.

The subscripts “I, II and III” indicate First-, Second- and Third portion respectively.

The first portion and second portion are divided by the normal order $m_0$. The second portion and third portion are divided by the zeroth-order $m = 0$. 
The plan of the article: start from (1) the first portion \( m_f \) with angle \( \theta_{mf} \); then (2) reduce \( \theta_{mf} \) to zero, i.e., go to the normal order \( m_0 \); then (3) go to second portion \( m_{II} \) with angle \( \theta_{mII} \); then (4) enlarging \( \theta_{mIII} = \theta_f \), i.e., go to the zero order; then (5) go to third portion by enlarging \( \theta_{mIII} \) to \( \theta_{mIII} \).

### 3.3. Grating Equations for First Portion: \( m_f > m_0 > 0 \)

#### 3.3.1. Grating Equations

When \( m_f > m_0 > 0 \), \( Y_{mf} \) is above \( O' \), i.e., the incident angle \( \theta_f \) and diffraction angle \( \theta_{mf} \) are on the same side of grating normal (Figure 7). Where \( Y_{mf} \) is the position of the diffraction order \( m_f \). The path-difference is,

\[
AB + BC.
\]

For far field, path-1, \( OY_{mf} \) and path-2 can be treated as approximately parallel to each other. Where

\[
AB = d \sin(\theta_f) \quad \text{and} \quad BC = d \sin(\theta_{mf}).
\]
\[ d \sin(\theta_i) + d \sin(\theta_{m_l}) = m_l \lambda. \]  \hspace{1cm} (6)

\[ \theta_{m_l} = \arcsin \left[ m_l \frac{\lambda}{d} - \sin(\theta_i) \right]. \]  \hspace{1cm} (7a)

Substituting Eq. (1b) into Eq. (6), we obtain the grating equations in terms of \( m_l \) and \( m_0 \) (Figure 8).

\[ d \sin(\theta_{m_l}) = \lambda[m_l - m_0] \]  \hspace{1cm} (8)

\[ \theta_{m_l} = \arcsin \left[ \frac{\lambda}{d} (m_l - m_0) \right] \]  \hspace{1cm} (7b)

**Example 3.3-1** (Figure 8): let us consider an example: \( \lambda = 0.65 \) \( \mu \)m, the incident angle \( \theta_i = 45^\circ \).

For \( d_{1000}, m_0 = 1.09, m_I = 2 \)

For \( d_{500}, m_0 = 2.2, m_I = 3, 4, 5 \)

For \( d_{300}, m_0 = \frac{3.33 \times 0.707}{0.65} = 3.62, m_I = 4, 5, 6, 7, 8 \)

For \( d_{200}, m_0 = \frac{5 \times 0.707}{0.65} = 5.4, m_I = 6, 7, 8, 9, 10, 11, 12, 13 \)

For \( d_{100}, m_0 = \frac{10 \times 0.707}{0.65} = 10.8, m_I = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26 \)

![Figure 8 Correlation of diffraction angle \( \theta_{m_I}, m_I, m_0, d, \lambda \) (650 nm), at \( \theta_i = 45^\circ \).](attachment:image.png)

### 3.3.2. Limitation of Number of Diffraction Orders \( m_I \)

Eq. (7a) and Eq. (7b) limit the value of the integer diffraction orders \( m \).

Since \( \sin(\theta_{m_I}) \leq 1 \), thus Eq. 8 gives

\[ \frac{\lambda}{d}[m_I - m_0] \leq 1, \]

namely

\[ m_I \leq \frac{d}{\lambda} + m_0 \]  \hspace{1cm} (9a)

Substituting Eq. (1b) into Eq. (9a), the number of diffraction orders “\( m_I \)” is proportional to spacing \( d \) and satisfy
\( m_l \leq \frac{d}{\lambda} [1 + \sin \theta_i] \) \hspace{1cm} (9b)

**Example-3.3-2** (Figure 9): let us consider an example: \( \lambda = 0.65 \mu m \), the incident angles \( \theta_i = 30^\circ, 45^\circ, 60^\circ, 75^\circ \), and the spacing of the grating \( d \) is 1mm/1000 lines, 1mm/300 lines, 1mm/200 lines, 1mm/100 lines and 1mm/50 lines respectively.

![Figure 9 - Ranges of diffraction order \( m_l \) are determined by \( \theta_i \), \( d \) and \( \lambda \) (Eq. 9b)](image)

Eq. 9b (Figure 9) shows the maximum values of the diffraction orders \( m \) of the grating with different densities of slits and different incident angles. The grating with low density of slits has larger value of diffraction orders \( m \). On the contrary, the grating with high density of slits has a smaller value of diffraction orders.

### 3.3.3. Position of Diffraction Orders \( m_l \) on Screen

Next, let us calculate the positions of the diffraction orders \( m_l \) on the screen (Figure 10).

![Figure 10 - Schematic of Position of diffraction order \( m_l \)](image)

For \( L \gg d \), the three rays \( r \), \( r_1 \) and \( r_2 \) are essentially treated as being parallel, thus, \( \theta \approx \theta_{ml} \).

Denoting \( h_{ml} = Y_m O'' \), we have
\[ h_{ml} = L \tan(\theta_i + \theta_{ml}) \]  

(10)

The spacing between two adjacent diffraction orders is

\[ \Delta h \equiv h_{ml+1} - h_{ml} = L \left[ \tan(\theta_i + \theta_{ml+1}) - \tan(\theta_i + \theta_{ml}) \right] \]  

(11)

**Example-3.3-3**: For a given grating of \( d = 500 \text{ lines/mm} = 2 \text{ } \mu\text{m} \), \( \lambda = 0.65 \text{ } \mu\text{m} \), the incident angle is \( \theta_i = 30^\circ \), then we have \( m_o = \frac{d}{\lambda} \sin(\theta_i) = \frac{2}{0.65} \times 0.5 = 1.54 \).

1) Using Eq. 9b, \( m_i \leq \frac{d}{\lambda} [1 + \sin \theta_i] \), to determine \( m_f \). \( m_i \leq \frac{2}{0.65} (1 + 0.5) = 4.6 \), so we have three diffraction orders \( m_i = 2, 3, 4 \).

2) Using Eq. 7a, \( \theta_{ml/2} = \arcsin \left[ \frac{m_i^2}{d} - \sin(\theta_i) \right] = \arcsin \left[ \frac{2 \times 0.65}{2} - 0.5 \right] = 8.6^\circ \).

3) \( \theta_{ml/3} = \arcsin \left[ \frac{3 \times 0.65}{2} - 0.5 \right] = 28.4^\circ \)

4) \( \theta_{ml/4} = \arcsin \left[ \frac{4 \times 0.65}{2} - 0.5 \right] = 53.1^\circ \)

5) Applying Eq. 11, let \( L = 1 \), we have.

Distance between diffraction order 2 and 3 is:

\[ \Delta h_{m3-2} \equiv h_{m3} - h_{m2} = L \left[ \tan(\theta_i + \theta_{m3}) - \tan(\theta_i + \theta_{m2}) \right] = 0.83 \]

Distance between diffraction order 3 and 4 is

\[ \Delta h_{m4-3} \equiv h_{m4} - h_{m3} = L \left[ \tan(\theta_i + \theta_{m4}) - \tan(\theta_i + \theta_{m3}) \right] = 6.63 \]

Therefore, the distances between two adjacent diffraction orders are non-uniform.

### 3.4. Grating Equations for Second Portion: \( m_0 > m_f > 0 \)

#### 3.4.1. Grating Equations

When \( 0 < m_f < m_0 \), \( Y_m \) is in between the point \( O' \) and the point \( O'' \), i.e., the incident angle \( \theta_i \) and diffraction angle \( \theta_{mlf} \) are on the opposite sides of the grating normal, and \( \theta_i > \theta_{mlf} \) (Figure 11).

![Figure 11. Schematic of Second Portion of CCW rotating grating](image)

We have the path difference,

\[ AB - DC \text{ and } AB > DC. \]

Where \( AB = d \sin(\theta_i) \), \( DC = d \sin(\angle CBD) = d \sin(\theta_{mlf}) \), \( \theta_i > \theta_{mlf} \) and \( m_f > 0 \).
The grating equation for this situation in terms of \( \theta_i \) and \( \theta_{mII} \) is

\[
d \sin(\theta_i) - d \sin(\theta_{mII}) = m_{II} \lambda. \tag{12}
\]

Substituting Eq. (1b) into Eq. (12), we have the grating equation in terms of \( m_{II} \) and \( m_0 \),

\[
\sin(\theta_{mII}) = \frac{1}{d} \left[ m_0 - m_{II} \right] \tag{13a}
\]

\[
\theta_{mII} = \arcsin \left[ \frac{1}{d} \left( m_0 - m_{II} \right) \right] \tag{13b}
\]

The larger the value of the integer “\( m_{II} \)”, i.e., \( Y_{mII} \) moves to O’, the smaller the diffraction angle \( \theta_{mII} \).

**Example 3.4-1** (Figure 12): let us consider an example: \( \lambda = 0.65 \text{ \( \mu \)m} \), the incident angle \( \theta_i = 45^\circ \).

For \( d_{1000} = \frac{1 \text{mm}}{1000 \text{ lines/mm}} = 1 \text{ \( \mu \)m} \), \( m_0 = 1.09, m_{II} = 1 \)

For \( d_{500}, m_0 = 2.18, m_{II} = 1.2 \)

For \( d_{330}, m_0 = \frac{3.33 \times 0.707}{0.65} = 3.62, m_{II} = 1.2,3 \)

For \( d_{200}, m_0 = \frac{5 \times 0.707}{0.65} = 5.44, m_{II} = 1.2,3,4,5 \)

For \( d_{100}, m_0 = \frac{10 \times 0.707}{0.65} = 10.88, m_{II} = 1.2,3,4,5,6,7,8,9,10 \)

Figure 12 shows the diffraction angles vs. the combination of diffraction orders \( m_i \) and \( m_{II} \).

Figure 12. Diffraction angles vs. the combination of diffraction orders \( m_i \) and \( m_{II} \)

When a diffraction order is closer to the normal order \( m_0 \), its diffraction angle is smaller.

**3.4.2. Limitation of Number of Diffraction Orders \( m_{II} \)**

Note that Eq. (13a) limits the value of “\( m_{II} \)”. Since \( 0 < \sin(\theta_{mII}) < 1 \), thus Eq. 13a gives

\[
0 < \frac{1}{d} \left[ m_0 - m_{II} \right] < 1, \text{ namely, the integer diffraction orders “} m_{II} \text{” satisfy}
\]

\[
m_{II} > m_0 - \frac{d}{\lambda} \tag{14a}
\]

\[
m_0 > m_{II} \tag{14b}
\]

Substituting Eq. (1b) into Eq. (14a), the integer diffraction orders “\( m_{II} \)” has to satisfy

\[
m_{II} > \frac{d}{\lambda} \left( \sin(\theta_i) - 1 \right) \tag{14c}
\]

By definition, the integer \( m_{II} > 0 \), Eq. 14c is satisfied, which is what we discussed in Section 3.3, i.e.,
0 < m_{II} < m_0. \quad (14d)

### 3.4.3. Position of Diffraction Orders $m_{II}$ on Screen

Next let us determine the position of the diffraction order $m_{II}$.

For $L \gg d$, the three rays $Y_{m_{II}}O', Y_{m_{II}}D$ and $Y_{m_{II}}B$ are essentially treated as being parallel, thus,

$$\angle Y_{m_{II}}OO' \approx \theta_{m_{II}}. \quad (15)$$

Denoting $O'Y_{m_{II}} = h_{m_{II}}$, we have

$$h_{m_{II}} = L \tan (\theta_i - \theta_{m_{II}}) \quad (16)$$

The spacing between two adjacent diffraction orders is

$$\Delta h_{m_{II}} = L [\tan(\theta_i - \theta_{m_{II}+1}) - \tan(\theta_i - \theta_{m_{II}})]. \quad (17)$$

Therefore, the diffraction pattern of the rotating grating is nonuniform.

**Example 3.4.2:** For a given grating of $d = 500 \ \text{lines/mm} = 2 \ \mu m$, $\lambda = 0.65 \ \mu m$, the incident angle is $\theta_i = 30^\circ$, then we have $m_0 = \frac{d}{\lambda} \sin(\theta_i) = \frac{2}{0.65} \times 0.5 = 1.54$.

1. Using Eq. 14c to determine $m_{II}$. We have $m_{II} = 1$.
2. Using Eq. 13b, $\theta_{m_{II}} = \arcsin \left[ \frac{1}{d} (m_0 - m_{II}) \right] = \arcsin \left[ \frac{0.65}{2} (1.54 - 1) \right] = 10.1^\circ$.
3. $h_{m_{II}} = L \tan(\theta_i - \theta_{m_{II}}) = L \tan 19.9^\circ = 0.36L$

### 3.5. Grating Equations for $m = 0$

The path difference before passing through the grating is $AB = dsin\theta_i$. The path difference after passing through the grating is $CD = dsin\theta$.

![Figure 13. Schematic of Zeroth Order of CCW rotating grating](image-url)

Figure 13 shows that $\theta_{m0} = \theta = \theta_i$, so

$AB = CD$ and $AB - CD = 0$,

i.e., the net path difference is zero. Thus, the grating equation is

$$dsin\theta_i - dsin\theta_{m0} = 0 = m\lambda, \quad (18)$$

$m = 0$. 
The diffraction order \( m = 0 \) separates the Second portion (described in Section 3.4) and third portion.

3.6. Grating Equations for Third Portion: \( m_{III} < 0 \)

3.6.1. Grating Equations

When \( Y_m \) is below the point \( O'' \) \((m = 0)\), we define \( m_{III} < 0 \). The incident angle \( \theta_i \) and diffraction angle \( \theta_{m_{III}} \) are on the opposite sides of the grating normal, and \( \theta_{m_{III}} \geq \theta_i \) (Figure 14). The diffraction angle for the third portion starts from \( \theta_{m_{III}} = \theta_i \), which leads to the zeroth order \( m = 0 \), and then gradually increase, such that \( m_{III} = -1, -2, \ldots \).

We have the path difference,

\[ AB - DC \text{ and } AB < DC. \]

Where \( AB = d \sin(\theta_i) \) and \( DC = d \sin(\angle CBD) = d \sin(\theta_{m_{III}}) \).

![Figure 14. Schematic of Third Portion of CCW rotating grating](image)

The grating equation for this situation in terms of \( \theta_i \) and \( \theta_{m_{III}} \) is

\[ d \sin(\theta_i) - d \sin(\theta_{m_{III}}) = m_{III} \lambda. \]

Substituting Eq. (1b) into Eq. (19), we have the grating equation in terms of \( m_{III} \) and \( m_0 \),

\begin{align*}
\sin(\theta_{m_{III}}) &= \frac{\lambda}{d} [m_0 - m_{III}] \quad (20a) \\
\theta_{m_{III}} &= \arcsin \left[ \frac{\lambda}{d} (m_0 - m_{III}) \right] \quad (20b)
\end{align*}

Note that the third portion starts from \( O'' \) and contain the diffraction orders below \( O'' \).

3.6.2. Limitation of Number of Diffraction Orders \( m_{III} \)

Note that Eq. (20a) limits the value of “\( m_{III} \)”. Since \( 0 \leq \sin(\theta_{m_{III}}) \leq 1 \), thus Eq. 20a gives

\begin{align*}
0 &\leq \frac{\lambda}{d} [m_0 - m_{III}] \leq 1. \quad (21a) \\
m_{III} &\geq m_0 - \frac{\lambda}{d}. \quad (21b) \\
m_{III} &\geq \frac{d}{\lambda} [\sin(\theta_i) - 1]. \quad (21c)
\end{align*}

**Example-3.6-1:** For \( d_{500}, \theta_i = 30^0. m_0 = 1.53, m_{III} \geq m_0 - \frac{\lambda}{d} = -1.53. \) Thus \( m_{III} = -1 \).

The total number of the diffraction orders is the total numbers of \( m_{III} + m_I + m_I + m(= 0) = 6 \):
\[ m_i \leq \frac{d}{\lambda} [1 + \sin \theta_i] = 4.6 \quad m_i = 2, m_i = 3, m_i = 4, \]
\[ m_{II} > \frac{d}{\lambda} [\sin (\theta_i) - 1], \quad m_{II} = 1 \]
\[ m_{III} \geq \frac{d}{\lambda} [\sin \theta_i - 1]. \quad m_{III} = -1 \]

Zeroth order:
\[ m = 0 \]

Figure 15. Example-3.6-1: Picture of total number of diffraction orders

Note that the order \( m_i = 4 \) did not shown, we believe it would be shown by using higher sensitive detector.

3.6.3. Position of Diffraction Orders \( m_{III} \) on Screen

We denote the position of diffraction orders \( m_{III} \) on screen \( O''Y_{m_{III}} = h_{m_{III}} \). Figure 16 shows, for \( L \gg d \), path-1, \( OY_{m_{III}} \) and path-2 can be treated as parallel, and thus
\[
\angle Y_{m_{III}} O' O'' = \theta_{m_{III}} - \theta_i
\]

Figure 16. Schematic of deriving position of diffraction orders \( m_{III} \)

Thus, the position of diffraction orders \( m_{III} \) is
\[
h_{m_{III}} = L \tan (\theta_{m_{III}} - \theta_i). \tag{22}\]

Example-3.6-1: For a given grating of \( d = 500 \text{ lines/mm} = 2 \mu m \), \( \lambda = 0.65 \mu m \), the incident angle is \( \theta_i = 30^\circ \), then we have \( m_0 = \frac{d}{\lambda} \sin (\theta_i) = 1.54 \).

1. Using Eq. 21c to determine \( m_{III} \). We have \( m_{III} = -1 \).
2. Using Eq. 20b, \( \theta_{m_{III}} = \arcsin \left[ \frac{1}{d} (m_0 - m_{III}) \right] = 55.6^\circ \).
(3) Using Eq. 22, \( h_{mll} = L \tan(\theta_{mll} - \theta_l) = 0.48L \)

4. Grating Equations for Rotating CW Grating Around Y axis

When the grating is rotating around one axis, say the Y axis, there is no difference between CW and CCW rotations. However, we have shown that when the grating is rotated around two or three axes sequentially, the CW and CCW rotations make different.

4.1. Normal Diffraction Order “\( m_0 \)”

4.1.1. Definition of Normal Diffraction Order “\( m_0 \)” for CW rotating Grating

Figure 17 shows that the grating rotates CW around the Y axis by \( \theta_l \), which is equivalent to the light beam shining on the grating at the oblique angle \( \theta_l \). The line OO’ is the grating normal. The spacing between two slits is “d”. The beam-1 and beam-2 along path-1 and path-2, respectively, meet at the point \( O’ \). After passing the grating, there is no path deference between two beams, i.e., \( DO’ = BO’ \). The O” corresponds to the zeroth order \( m = 0 \). The O’ corresponds to \( m_0 \).

For the case of \( L \gg d \), path-1, OO’ and path-2 are approximately parallel to each other. For the light beams arriving at the point O’ on the grating normal, the diffraction angle \( \theta_{m0} = 0 \).

\[
\text{Figure 17. Schematic of normal diffraction order “} m_0 \text{” of CW rotating grating}
\]

The “AB” is the path-deference of light propagating along path-1 and path-2, \( \angle O’OO'' = \theta_l \), we have

\[
d \sin(\theta_l) = m_0 \lambda. \tag{23a}
\]

**Definition of the normal order \( m_0 \):**

\[
m_0 = \frac{d}{\lambda} \sin(\theta_l). \tag{23b}
\]

We define \( m_0 < 0 \) for the CW rotation of the grating. Thus \( \theta_l < 0 \).

Eq. 23b has the same form as Eq. 1b. However, \( m_0 > 0 \) in Eq. 1b for the CCW rotating grating.

4.1.2. Position of normal order \( m_0 \) on screen
Defining $O'O'' = h_0$, then we have

$$h_0 = L \tan(\theta_i). \quad (24a)$$

The $h_0$ is determined by the $\theta_i$ and $h_0 < 0$.

Eq. 24a has the same form as Eq. 3a. However, $h_0 > 0$ in Eq. 3a for the CCW rotating grating.

To find $\tan(\theta_i)$ in terms of the diffraction order $m$ and $m_0$, Eq. (23a) gives

$$\tan(\theta_i) = m_0 \frac{\lambda}{d \sqrt{1 - [m_0^2]}}, \quad (25)$$

Using Eq. (25), Eq. (24a) becomes

$$h_0 = m_0 \frac{L}{d} \frac{\lambda}{\sqrt{1 - [m_0^2]}} \quad (24b)$$

Note that for CCW rotating grating, the normal order $m_0$ is on the positive side of the zeroth order $m = 0$, while for CW rotating grating, the normal order $m_0$ is on the negative side of the zeroth order.

### 4.2. Dividing Diffraction Pattern into Three Portions

Now let us divide the diffraction pattern of a CW rotating grating into three portions (Figure 18).

**First portion:** the first portion is above $O''$, i.e., $m_I > m$, where $m = 0$, thus $m_I > 0$.

**Second portion:** the second portion is between $m_0$ and $m = 0$, i.e., $m > m_{II}$, $m_0 = m_{II} < 0$.

**Third portion:** the third portion is below $m_0$, i.e., $m_{III} < m_0 < 0$.

The subscripts “I, II and III” indicate First-, Second- and Third portion respectively.
The first portion and second portion are divided by the zeroth order \( m = 0 \). The second portion and third portion are divided by the normal-order \( m_0 \).

### 4.3. Grating Equations for First Portion: \( m_l > 0 > m_0 \)

When \( m_l > 0 > m_0 \), \( Y_{m_l} \) is above \( O' \), i.e., \( \theta_{m_l} > \theta_i \) (Figure 19). The incident angle \( \theta_i \) and diffraction angle \( \theta_{m_l} \) are on the opposite sides of the grating normal. The incident angle \( \theta_i \) and diffraction angle \( \theta_{m_l} \) are on the opposite sides of the grating normal, and \( \theta_{m_l} \geq \theta_i \) (Figure 19). The diffraction angle for the third portion starts from \( \theta_{m_l} = \theta_i \), which leads to the zeroth order \( m = 0 \), and then gradually increase, such that \( m_l = 1, 2, ... \).

We have the path difference,

\[ DC - AB \text{ and } DC > AB \]

where \( AB = d \sin(\theta_i) \) and \( DC = d \sin(CBD) = d \sin(\theta_{m_l}) \). \( h_{m_l} = 0'' Y_{m_l} \).

![Figure 19. Schematic of First Portion of CW rotating grating](image)

The diffraction equations are,

\[
d \sin(\theta_{m_l}) - d \sin(\theta_i) = m_l \lambda.
\] (26)

\[
sin(\theta_{m_l}) = \frac{\lambda}{d} (m_l + m_0)
\] (27a)

\[
\theta_{m_l} = \arcsin \left[ \frac{\lambda}{d} (m_l + m_0) \right]
\] (27b)

\[
h_{m_l} = L \tan (\theta_{m_l} - \theta_l)
\] (28)

The larger the value of the integer “\( m_l \)”, the larger the diffraction angle \( \theta_{m_l} \) and the larger the position “\( h_{m_l} \)”.

Eq. 26 and Eq. (27a) limit the value of “\( m_l \)”, since \( \sin(\theta_{m_l}) \leq 1 \), thus,

\[
m_l \leq \frac{d}{\lambda} [1 - \sin(\theta_i)]
\] (29a)

\[
m_l \leq \frac{d}{\lambda} - m_0
\] (29b)

### 4.4. Grating Equation for: \( m = 0 \)
Lowering $Y_{ml}$ until reaching $O$". The line OO" is the zeroth order that dividing the first portion and the second portion (Figure 20).

The diffraction angle is equal to the incident angle, $\theta_{ml} = \theta_i$.

4.5. Grating Equation for Second Portion: $0 > m_{ll} > m_0$

When $0 > m_{ll} > m_0$, $Y_{ml}$ is between O" and O", i.e., $\theta_{ml} < \theta$). The incident angle $\theta_i$ and diffraction angle $\theta_{ml}$ are on the opposite sides of the grating normal. We have the path difference, $DC - AB$,

where $AB = dsin(\theta_i)$ and $DC = dsin(\angle CBD) = dsin(\theta_{ml})$, $h_{ml} = O"Y_{ml}$ (Figure 21).

The diffraction equations are,

$$dsin(\theta_i) - dsin(\theta_{ml}) = m_{ll}\lambda.$$  \hfill (30)

$$sin(\theta_{ml}) = \frac{\lambda}{d}[m_0 - m_{ll}]$$  \hfill (31a)

$$\theta_{ml} = \arcsin \left[\frac{\lambda}{d}(m_0 - m_{ll})\right]$$  \hfill (31b)

$$h_{ml} = L \tan (\theta_i - \theta_{ml})$$  \hfill (32)

Eq. (31a) limits the value of “$m_{ll}$”, since $|sin(\theta_{ml})| \leq 1$, thus, $\frac{\lambda}{d}[m_0 - m_{ll}] \leq 1$.  \hfill (33)
4.4. Grating Equation for Third Portion: \(0 > m_0 > m_{III}\)

When \(m_0 > m_{III}\), \(Y_{m_{III}}\) is below \(O'\), i.e., the incident angle \(\theta_i\) and diffraction angle \(\theta_m\) are on the same side of the grating normal. Defining \(O'y_{m_{III}} = h_{III}\). We have the path difference,

\[ AB + BC, \]

where \(AB = d\sin(\theta_i)\) and \(BC = d\sin(\theta_{m_{III}})\).

\[ dsin(\theta_i) + dsin(\theta_{m_{III}}) = m_{III}A. \]  \(\text{(34)}\)

Substituting Eq. (23b) into Eq. (34), we obtain

\[
\sin(\theta_{m_{III}}) = \frac{\lambda}{d} [m_{III} - m_0] \] \(\text{(35a)}\)

\[ \theta_{m_{III}} = \arcsin \left( \frac{\lambda}{d} (m_{III} - m_0) \right) \] \(\text{(35b)}\)

\[ h_{m_{III}} = L \tan(\theta_i + \theta_{m_{III}}) \] \(\text{(36)}\)

Eq. (35a) limits the value of “\(m_{III}\)”, since \(|\sin(\theta_{m_{III}})| \leq 1\), thus

\[
\frac{\lambda}{d} |m_{III} - m_0| \leq 1
\] \(\text{(37)}\)

5. Summary and Discussion

For the diffraction pattern of a grating rotating CW and CCW around one axis that parallels the slits of the grating, we break the pattern into 3 portions. The three portions are divided by the normal order \(m_0\) and the zeroth-order, \(m = 0\), respectively, then describe each portion respectively.

We show that (1) Each portion is described by specific equations; (2) The distances between two adjacent diffraction orders are non-uniform; (3) The number of the diffraction orders is limited by the spacing of the grating, wavelength and the incident angle.

Let us summarize the contents of this article below.

1) **Zero order \(m = 0\): CCW and CW rotations**
The equations describing the diffraction orders of both the CW and CCW rotation of the grating are the same.

2) \( m_0 \): normal diffraction order (normal order)

CCW rotation: \( m_0 > 0 \),

\[
m_0 = \frac{d}{\lambda} \sin(\theta_i),
\]

\[
h_0 = m_0 \frac{\lambda}{\sqrt{1 - [m_0 d]^2}}
\]

CW rotation: \( m_0 < 0 \)

\[
m_0 = \frac{d}{\lambda} \sin(\theta_i)
\]

\[
h_0 = m_0 \frac{\lambda}{\sqrt{1 - [m_0 d]^2}}
\]

3) First portion:

CCW rotation: \( m_i > m_0 > 0 \):

\[
d\sin(\theta_{m_i}) + d\sin(\theta_i) = m_i \lambda,
\]

\[
\theta_{m_i} = \arcsin \left( \frac{\lambda}{d} (m_i - m_0) \right).
\]

CW rotation: \( m_i > 0 > m_0 \)

\[
d\sin(\theta_{m_i}) - d\sin(\theta_i) = m_i \lambda
\]

\[
\theta_{m_i} = \arcsin \left( \frac{\lambda}{d} (m_i + m_0) \right)
\]
\[ h_{ml} = L \tan(\theta_i + \theta_{ml}), \]
\[ m_i \leq \frac{d}{\lambda} + m_0, \]

4) **Second portion:**
CCW rotation: \( m_o > m_{II} > 0, \)

\[ d \sin(\theta_i) - d \sin(\theta_{mII}) = m_{II} \lambda \]
\[ \theta_{mII} = \arcsin \left( \frac{\lambda}{d} (m_0 - m_{II}) \right) \]
\[ h_{mII} = L \tan(\theta_i - \theta_{mII}) \]
\[ m_{II} > m_0 - \frac{d}{\lambda} \]

CW rotation: \( 0 > m_{II} > m_o \)

\[ d \sin(\theta_i) - d \sin(\theta_{mII}) = m_{II} \lambda \]
\[ \theta_{mII} = \arcsin \left( \frac{\lambda}{d} (m_0 - m_{II}) \right) \]
\[ h_{mII} = L \tan(\theta_i - \theta_{mII}) \]
\[ \frac{\lambda}{d} |m_0 - m_{II}| \leq 1 \]

5) **Third portion**
CCW rotation: \( m_o > 0 > m_{III}, \)

\[ d \sin(\theta_i) - d \sin(\theta_{mIII}) = m_{III} \lambda \]
\[ \theta_{mIII} = \arcsin \left( \frac{\lambda}{d} (m_0 - m_{III}) \right) \]
\[ h_{mIII} = L \tan(\theta_i - \theta_{mIII}) \]
\[ m_{III} \geq m_0 - \frac{d}{\lambda} \]

CW rotation: \( 0 > m_0 > m_{III} \)

\[ d \sin(\theta_i) + d \sin(\theta_{mIII}) = m_{III} \lambda \]
\[ \theta_{mIII} = \arcsin \left( \frac{\lambda}{d} (m_{III} - m_0) \right) \]
\[ h_{mIII} = L \tan(\theta_i + \theta_{mIII}) \]
\[ \frac{\lambda}{d} |m_{III} - m_0| \leq 1 \]

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Reference

