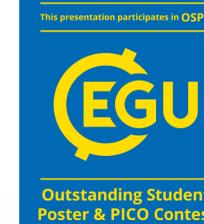




# The impact of objective function choice on the influence of data points in hydrological model calibration

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## The influence of individual data points on model performance and predictions is strongly affected by objective function choice

### 1. Aims of the study

Influence diagnostics provide valuable insights into the role of objective functions in calibration and are not widely used in hydrology.

This study:

1. **Quantifies** how objective functions affect the influence of individual data points on calibrated model performance and predictions, and
2. **Identifies** how characteristics of highly influential data vary under different objective functions.

### 2. Objective functions

**Standard least squares (SLS):** derived from the log likelihood of the normal probability density function.

**Weighted least squares (WLS):** uses the time varying standard deviation in place of the constant standard deviation in SLS.

**Logarithmic transformation (LOG):** uses logarithmic transformed flows

**Kling-Gupta Efficiency (KGE):** See [Kling et al., 2012] for more detail on the KGE.

### 3. Case-deletion influence diagnostics

Case-deletion influence diagnostics give a measure of the difference between the performance and predictions of the model calibrated with the whole data set and the model calibrated with the  $i^{th}$  point removed. We use diagnostics similar to those applied in Wright et al [2015].

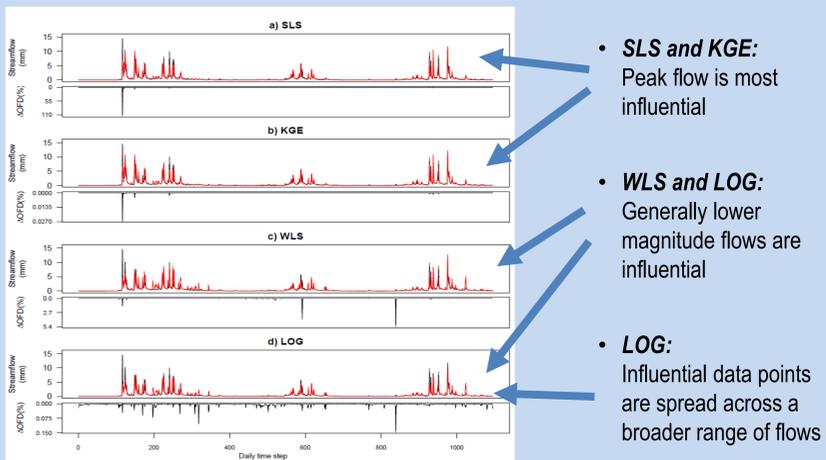
$$\Delta OF(\%)_i = \left| \frac{(OF_{-i}(\theta^{-i}) - OF_{-i}(\theta))}{OF_{-i}(\theta)} \times 100 \right| \quad (1)$$

$$\Delta \text{metric}(\%)_i = \frac{\text{metric}(Y) - \text{metric}(Y^{-i})}{\text{metric}(Y)} \times 100 \quad (2)$$

We apply (2) to three flow metrics: 1) mean flow prediction, 2) maximum flow prediction, and 3) the 10th percentile low flow by volume of the flow duration curve.

### 4. Influential flows differ under objective functions

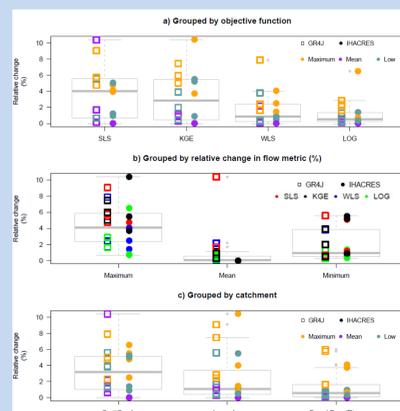
Influence diagnostics are applied to the GR4J hydrological model [Perrin 2003] and ephemeral Scott Creek catchment (SE Australia)



### 5. Factors that determine the influence of data

Influence diagnostics are applied to two hydrological models, GR4J [Perrin 2003] and IHACRES [Jakeman and Hornberger, 1993] and three catchments with varying runoff coefficients from 0.14, 0.33, and 0.57

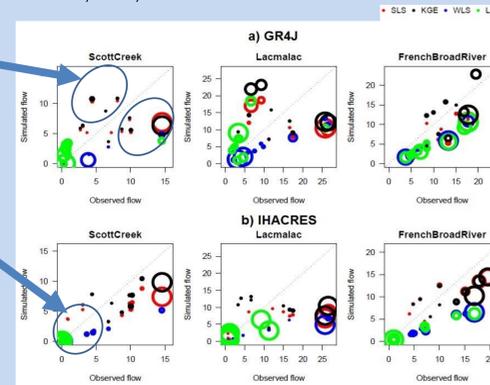
- **Objective function:** Highest influence for SLS, then KGE, WLS and LOG
- **Flow metric:** Highest influence on maximum and minimum relative change (%)
- **Catchment:** Highest influence in ephemeral catchment



### 6. Influence varies with flow magnitude

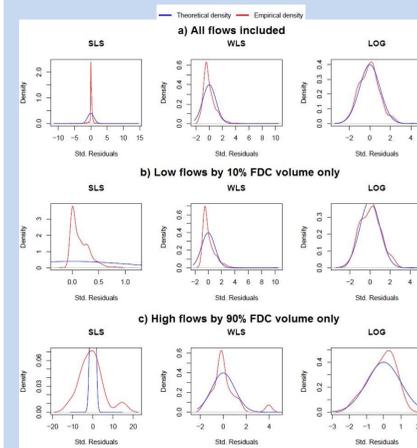
Influence diagnostics are applied to two hydrological models, GR4J [Perrin 2003] and IHACRES [Jakeman and Hornberger, 1993] and three catchments with varying runoff coefficients from 0.14, 0.33, and 0.57

- **SLS and KGE:** Higher magnitude flows and residuals are influential
- **WLS and LOG:** Lower magnitude observed and predicted flows are influential



### 7. Residual error assumptions play a key role in influence of data

We consider residual probability density functions for the GR4J model and Scott Creek catchment at three flow regimes: a) all flows, b) low flows, and c) high flows

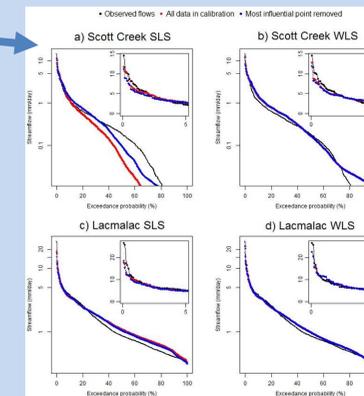


- **SLS:** Theoretical PDF does not match empirical PDF in all three flow regimes. This contributes to some data with disproportionate influence.
- **WLS and LOG:** Much closer match between PDFs for all three flow regimes. This leads to influence spread more evenly across all data points.

### 8. Influential points can have a similar impact to objective function choice

To assess the impact of a single influential point on model predictions we compare observed and predicted flow-duration curves with and without the most influential data point.

- **Single influential point has a similar magnitude impact to changing objective function from SLS to WLS**
- **Scott Creek (Ephemeral catchment):** For SLS most influential point impacts both high and low flows. For WLS most influential point impacts high flows.
- **Lacmalac (Wetter catchment):** For SLS less of an impact on flows. For WLS minimal impact on flows.



### 9. Future work: Generalised influence diagnostics and more hydrological case studies

- These results highlight the need to undertake influence assessment as a regular part of model calibration and evaluation, regardless of objective function choice.
- Generalised influence diagnostics are currently being developed to be applied to any objective function in a computationally efficient manner
- **GR4J model with 3 years of data: (~0.5 hours of computation time instead of ~40 hours for case-deletion)**
- Further application of influence diagnostics to larger case studies will yield valuable insights into model calibration

### References

Jakeman, A. J., and G. M. Hornberger (1993), How much complexity is warranted in a rainfall-runoff model?, *Water Resources Research*, 29(8), 2637-2649.  
Kling, H., M. Fuchs, and M. Paulin (2012), Runoff conditions in the upper Danube basin under an ensemble of climate change scenarios, *Journal of Hydrology*, 424-425, 264-277.  
Perrin, C., C. Michel, and V. Andreassian (2003), Improvement of a parsimonious model for streamflow simulation, *Journal of Hydrology*, 279(1-4), 275-289.  
Wright, D. P., M. Thyer, and S. Westra (2015), Influential point detection diagnostics in the context of hydrological model calibration, *Journal of Hydrology*, 527, 1161-1172.

Influence diagnostics provide a unique opportunity to quantify differences between objective functions where hydrologists previously relied on heuristics

Paper currently in review: Wright et al. [2016] Impact of objective function selection on the influence of data in hydrological modelling