

Conjunction Inference Using the Bayesian Interpretation of the Positive False Discovery Rate (pFDR)

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Introduction

Functional neuroimaging often requires a test for the conjunction of several effects. For example, task A and B may each involve working memory but use different modalities; it is of interest to find brain regions where both task A and B are significantly activated.

In separate work (see poster by Brett et al), we show that conjunction inference based on the minimum statistic test (Worsley & Friston, 2000; SPM99; SPM2) does not control the relevant false positive rate. That is, a significant P-value for a minimum statistic only means that one or more of the effects are significant, *not* that all effects are significant.

In this work we propose an approach to conjunction inference which overcomes this fundamental limitations of the minimum statistic test. We use the Bayesian interpretation of the Positive False Discovery Rate (pFDR) (Storey, 2003). A pFDR "q-value" is also the posterior probability of the (random) null hypothesis given the extremity of the data. It is precisely the Bayesian complement of the P-value (a P-value is the probability of the extremity of the data given the null hypothesis). Using posterior probabilities, it is easy to control the relevant conjunction false positive rate, the probability of "not all effects true".

We introduce pFDR, describe our method, and apply it to a real dataset.

Gentle Methods

1. Create images of pFDR q-values. 2. Sum the q-value images to be conjoined. 3. Reject the conjunction null where sum image is less than 0.05.

Statistical Methods

pFDR. For one statistic image and a given threshold u , the FDR is the expected proportion of false positives among suprathreshold voxels. The pFDR is same *except* that it conditions on there being at least one suprathreshold voxel. pFDR has been described as "the rate at which discoveries are false".

Bayes & pFDR. Let T_i be the statistic value at voxel i and let H_i be the null hypothesis; $H_i = 0$ if the null is true or 1 otherwise. Let H_i be *random*. Then for a threshold u , $\text{pFDR}(u) = P(H_i = 0 \mid T_i \geq u)$ (Storey 2003). This is the posterior probability of the null given that voxel i is as or more extreme than u .

Conjunction. Now consider conjoining K statistic images. The conjunction null is $U_k\{H_i^k=0\}$, the state of one or more nulls being true. We control the posterior conjunction null probability:

$$P(U_k\{H_i^k=0\} \mid \Pi_k\{T_i^k > t_i\}) \\ \leq \sum P(H_i^k=0 \mid \Pi_k\{T_i^k > t_i\})$$

$$= \sum_k P(H_i^k=0|T_i^k>t_i)$$

The inequality uses Bonferroni and the equality uses independence over the K statistics at each voxel (not spatial independence). The last summation is just a summation of pFDR q-values.

Results

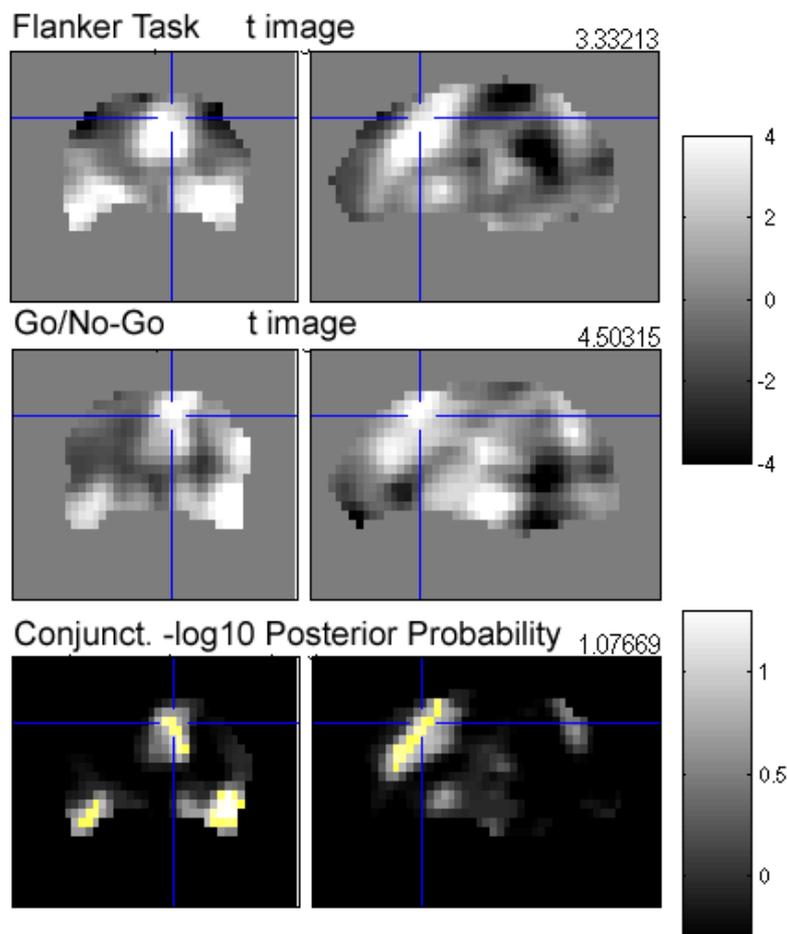
We applied this method to a study of response inhibition. We considered $K=2$ tasks, a go-no-go task and a flanker task. See Figure.

Conclusion

Using pFDR we have proposed a new method for conjunction inference. Our method is easy to apply, yet controls the appropriate conjunction false positive rate.

References

Storey JD. (2003) The positive false discovery rate: A Bayesian interpretation and the q-value. *Annals of Statistics*, 31: 2013-2035



Conjunction of two tasks involving response inhibition. Top two panels show t statistic images. Bottom shows -log10 posterior probability of either or both effects being null. Yellow regions correspond to posterior probability less than 0.1; 3 voxels are less than 0.05.

