



# Written progression through multiplication

## PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

### MENTAL CALCULATIONS

**These are a selection of mental calculation strategies:**

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

Knowing that doubling is the same as multiplying by 2.

### Using multiplication facts

*Tables should be taught every day, either as part of the mental oral starter or other times as appropriate within the day.*

Year 3      2 times table  
               3 times table  
               4 times table  
               5 times table  
               6 times table  
               10 times table

Year 4      Derive and recall all multiplication facts up to  $10 \times 10$

Years 5 & 6 Derive and recall quickly all multiplication facts up to  $12 \times 12$ .  
 Work out products such as  $70 \times 5$ ,  $70 \times 50$  and  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value.

### Using and applying division facts

Children should be able to utilise their knowledge of multiplication to derive other facts.

E.g. If I know  $3 \times 7 = 21$ , what else do I know?

$30 \times 7 = 210$ ,  $300 \times 7 = 2100$ ,  $3000 \times 7 = 21\,000$ ,  $0.3 \times 7 = 2.1$  etc

### Use closely related facts already known

$$\begin{aligned} 13 \times 11 &= (13 \times 10) + (13 \times 1) \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

### Multiplying by 10 or 100

To know that the effect of multiplying by 10; is to shift the digits one place to the left.

To know that the effect of multiplying by 100; is to shift the digits two places to the left.

### Partitioning

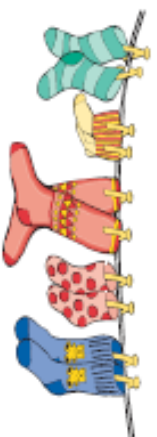
$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 102 \end{aligned}$$

### Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

**MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.**

## Key representations to support conceptual understanding of multiplication and division



$$2 + 2 + 2 + 2 + 2 = 10$$

$$2 \times 5 = 10$$

2 multiplied by 5

5 pairs

5 hops of 2



$$5 + 5 + 5 + 5 + 5 = 25$$

$$5 \times 5 = 25$$

5 multiplied by 5

5 groups of 5

5 hops of 5



$$10p + 10p + 10p + 10p + 10p = 50p$$

$$10p \times 5 = 50p$$

5 hops of 10



$$4 \times 2 = 8$$

$$2 \times 4 = 8$$

$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

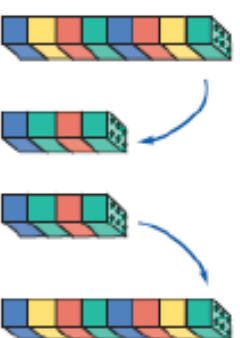
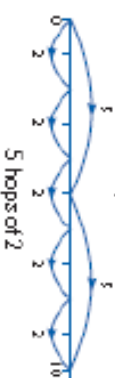
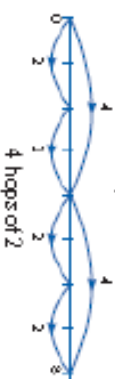
$$2 \times 5 = 10$$

$$4 \times 2 = 8$$

$$5 \times 2 = 10$$

$$2 \times 4 = 8$$

$$5 \times 2 = 10$$

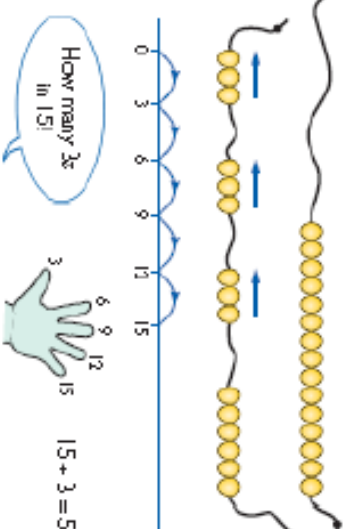


half of 8 is 4

$$8 \div 2 = 4$$

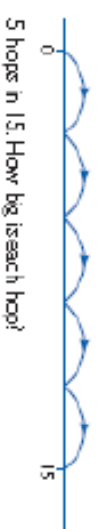
double 4 is 8

$$4 \times 2 = 8$$



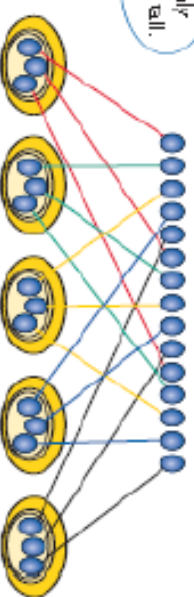
How many 3s in 15?

$$15 \div 3 = 5$$

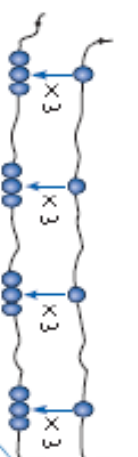


5 hops in 15. How big is each hop?

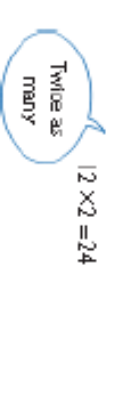
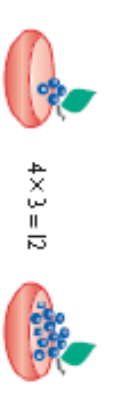
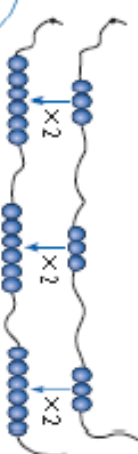
$$15 \div 5 = 3$$



15 shared between 5



Three times as many



THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF OUR CHILDREN TO ACHIEVE.

### Stage 1: Number line

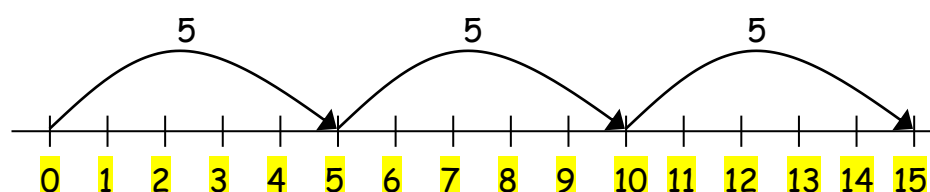
Children will develop their understanding of multiplication and use jottings to support calculation:

#### ✓ Repeated addition

$$3 \text{ times } 5 \text{ is } 3 + 3 + 3 + 3 + 3 = 15$$

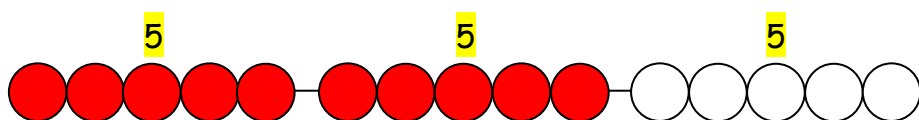
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



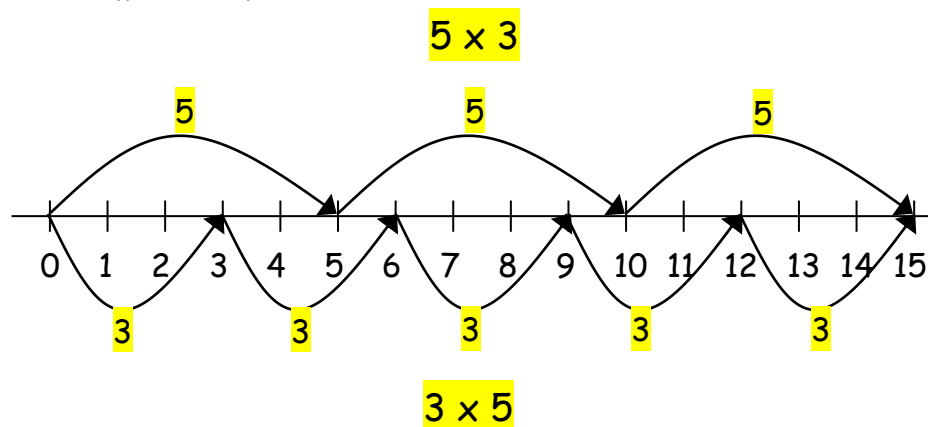
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



#### ✓ Commutativity

Children should know that  $3 \times 5$  has the same answer as  $5 \times 3$ . This can also be shown on the number line.



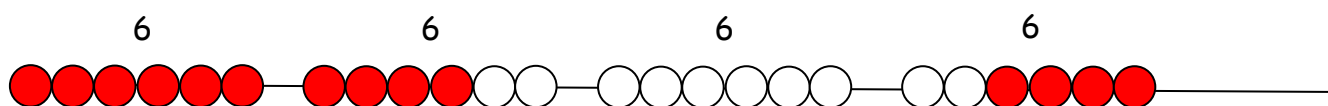
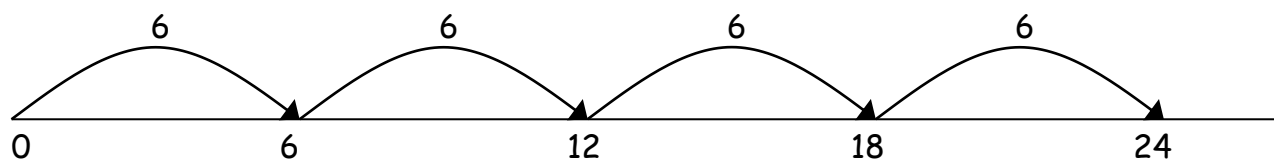
## Stage 2: Arrays

Children will continue to use:

### ✓ Repeated addition

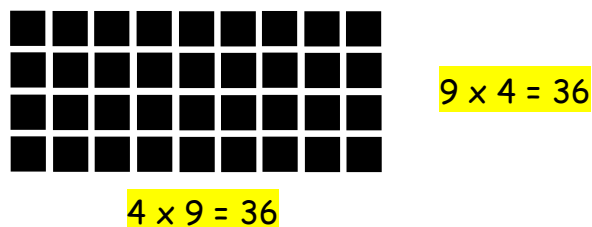
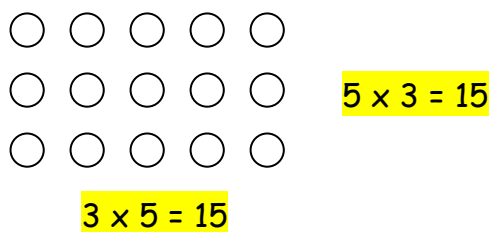
$$6 + 6 + 6 + 6 = 24 \quad \text{or} \quad 6 \times 4 \quad \text{and} \quad 4 + 4 + 4 + 4 + 4 + 4 = 24 \quad \text{or} \quad 4 \times 6$$

Children should use number lines or bead bars to support their understanding.



### ✓ Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm



20 cm

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \circ = 32$$

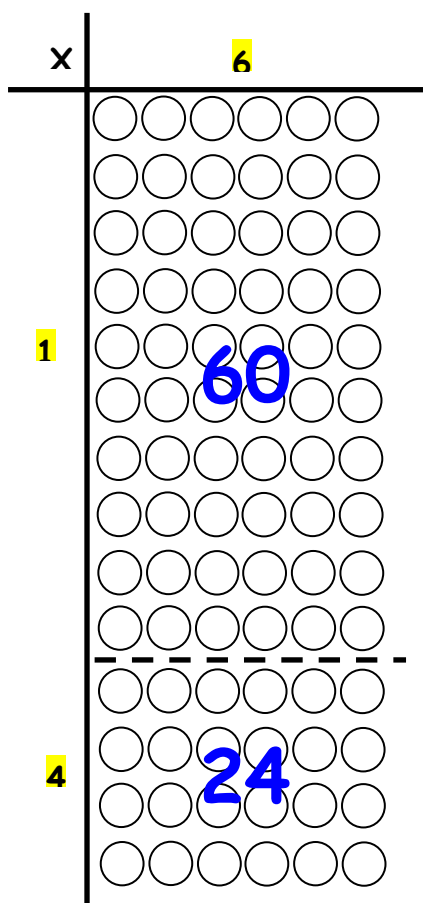
✓ **Partitioning**

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

### Stage 3: Developing the grid method

Children will continue to use arrays where appropriate leading into the grid method of multiplication.

GRID 1



$$(4 \times 6) + (10 \times 6)$$

$$24 + 60$$

$$84$$

When using the grid method for multiplication children must multiply the units first followed by the tens numbers, then the hundreds numbers if appropriate.  
4 x 6 then 10 x 6 in the example shown.

**TU × U**

(Short multiplication - multiplication by a single digit)

$14 \times 6$

Children will approximate first

$14 \times 6$  is approximately  $15 \times 6 = 90$

**GRID 2**

x	6
10	60
4	24
	84

	60
+	24
	184

**HTU × U**

(Short multiplication - multiplication by a single digit)

$346 \times 9$

Children will approximate first

$346 \times 9$  is approximately  $350 \times 10 = 3500$

**Grid 2**

x	9
300	2700
40	360
6	54
	3114
	11

It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products. Start by multiplying the units.

The next step is to move the number being multiplied to an extra row at the top.

Presenting the grid this way helps children to set out the addition of the partial products 2700, 360 and 54.

### Stage 4: Grid method 2 digit multipliers

TU x TU      72 x 38

Children will approximate first

72 x 38 is approximately 70 x 40 = 2800

x	70	2	
30	2100	60	2160
8	560	16	+ 576
			2736

1

The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 2100, 60 and 560 and 16.

	30	8	
x	70	2	
	2100	60	2160
	560	16	+ 576
			2736

1





## Stage 7: Multiplying decimals

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

E.g.  $4.92 \times 3$

Children will approximate first

$4.92 \times 3$  is approximately  $5 \times 3 = 15$

$$\begin{array}{r} 4.92 \\ \times 3 \\ \hline 14.76 \\ \phantom{1}2 \end{array}$$

### Expanded Short Multiplication

For example:

$489 \times 2.5$

Children will approximate first

$489 \times 2.5$  is approximately  $500 \times 2.5 = 1250$

$$\begin{array}{r} 489 \\ \times 2.5 \\ \hline 244.5 \\ 978.0 \\ \hline 1222.5 \end{array}$$

Children should not be made to go onto the next stage if:

- 1) They are not ready.
- 2) They are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.