



Sarah SquarEd

Leading Learning in English & Mathematics

Mathematics: *Developing Arithmetical Fluency*

This document is intended to support the development of children's fluency with basic number facts, mental and written methods of calculation.

It is built on securing both children's conceptual and procedural understanding of core mathematical skills using concrete, visual and symbolic representations underpinned by accurate use of mathematical vocabulary.

These materials outline a progression from Reception through to Y6 and is underpinned by the expectations laid out within the National Curriculum 2014

Arithmetic is ... the branch of mathematics concerned with numerical calculations (knowledge of... and skill in ...)

Number Facts

Accurate and rapid recall of number bonds \uparrow 20

Accurate and rapid recall of multiplication facts \uparrow 12x12

Derive and use related number bond facts

Derive and use related multiplication facts

Mental Methods

Partition numbers in a variety of ways, calculate and recombine

Bridge to and through 10 / 100

Adjustment / Compensation

Use of doubling and halving

Use factors

Written Strategies

Written methods can:

- establish connections between CPA
- support mental calculation (jotting form)
- record and explain mental calculations
- keep track of steps in more complex calculations
- develop efficiency
- help explain thinking

Informal (expanded/compact) methods should be used for a short period of time alongside structured apparatus to enable children to understand the internal logic of formal methods

...Don't count, calculate!



Principles for teaching arithmetic

- ▶ Every day is a mental mathematics day!
- ▶ It isn't all about the speed – build **flexibility** and fluency
- ▶ Commit regular time to **teaching** mental calculation strategies
- ▶ **Teach mental strategies explicitly** but in addition invite children to explain their methods too
- ▶ Seeing mathematics through **multiple representations** supports learning
- ▶ **Talking mathematically** clarifies and refines thinking
- ▶ Learning from mistakes should build children's confidence
- ▶ If you don't use it you will end up losing it!



Reception Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods
<ul style="list-style-type: none"> ✓ Subitise up to 5 (regular, irregular and structured patterns) ✓ Recite the forward number word sequence to 30 ✓ Recite the backward number word sequence from 20 to 0 ✓ Count in 1's from any given number up to at least 30 ✓ Begin to count in 2's and 10's using visual cues to support 	<ul style="list-style-type: none"> ✓ Identify 1 more/less without counting up to 20 ✓ Recall doubles for up to 5 ✓ Recall pairs of number to 5 ✓ Recall some bonds of 10 	<ul style="list-style-type: none"> ✓ Add a single digit to a single digit number using jottings to support ✓ Reorder numbers to add efficiently 	<ul style="list-style-type: none"> ✓ Use a part whole model and describe the relationship between the parts and whole ✓ Subitise 5 and 10 on a ten-frame and use this to help calculate ✓ Understand and describe the change to a set (add/subtract) and capture this using mathematical graphics

*** Children will have regular opportunities to retrieve and practice content using a range of manipulatives in both directed and non directed activities**



Year 1 Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods
<ul style="list-style-type: none"> ✓ Subitise up to 10 by partitioning ✓ Count in 2's from any multiple of 2 forwards and backwards ✓ Count in 10's from any multiple of 10 forwards and backwards ✓ Count in 5's from any multiple of 5 forwards and backwards 	<ul style="list-style-type: none"> ✓ recall number bonds of 5, 6, 7, 8, 9 at pace ✓ subitise irregular spatial patterns using bonds to help ✓ identify 1 more/less without counting ✓ recall doubles and corresponding halves to 10 at pace 	<ul style="list-style-type: none"> ✓ Add a single digit to a teens number ✓ Subtract a single digit from a teens number ✓ Add near doubles - partition and adjust ✓ Reorder numbers to add efficiently ✓ Partition a single digit in order to add efficiently ✓ Add a multiple of 10 to a single digit without counting on in ones – 30 + 7 	<ul style="list-style-type: none"> ✓ use a part whole model and describe the relationship between the parts and whole ✓ subitise 5 and 10 on a ten-frame and use this to help calculate ✓ understand addition as aggregation and augmentation and solve problems using a range of representations (number line, PPW, ten frame, bar model, bead string etc) ✓ understand subtraction as removal from set, partitioning and difference and solve problems using a range of representations

* Children will have regular opportunities to retrieve and practice content from previous year(s)



Year 2 Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods (as set out in school policy)
<ul style="list-style-type: none"> ✓ Subitise up to 10 by partitioning ✓ Count across tens boundaries accurately in 1s (ie 65 to 75) ✓ Count in 5's from any multiple of 5 ✓ Count in 3's from any multiple of 3 ✓ Count in $\frac{1}{2}$ s up to 10 	<ul style="list-style-type: none"> ✓ use recall of bond to 10 to derive those to 20 at pace ✓ Use and recall bonds of 10 to derive those of 100 at pace ✓ recall multiplication facts for 2/5/10 at pace in any order ✓ What need to be added to any 2 digit number to make the next multiple of ten ✓ Apply doubles knowledge to double any 2 digit number 	<ul style="list-style-type: none"> ✓ add any single digit number to a multiple of 10 ✓ Add and subtract a multiple of ten to and from any given 2 digit number ✓ Add 9 by adding 10 and adjusting ✓ Add 11 by adding 10 and adjusting ✓ Partition and combine tens and ones (no regrouping required) ✓ Add near 2 digit doubles 23 + 24 	<ul style="list-style-type: none"> ✓ use a part whole model and describe the relationship between the parts and whole ✓ add/subtract a 2-digit number and ones using apparatus and written column method alongside ✓ add/subtract two, 2-digit numbers without renaming, using PV understanding ✓ add/subtract two, 2 digit number with renaming, using PV understanding ✓ use a number line to calculate a small difference by counting on, using ten as a staging post where applicable ✓ understand that multiplication is commutative but that division is not ✓ use known facts (2x, 5x, 10x) to help solve problems – does not default to counting strategies when working with these tables

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Year 3 Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods (as set out in school policy)
<ul style="list-style-type: none"> ✓ skip count in multiples of 3, 4 and 8 ✓ count in 50s ✓ count in 100's ✓ count up and down in tenths 	<ul style="list-style-type: none"> ✓ recall multiplication and division facts for the 3 ✓ recall multiplication and division facts for the 4 ✓ recall multiplication and division facts for the 8 ✓ use known facts to derive related ($3 \times 2 = 6$ then $3 \times 20 = 60$) ✓ Sums and differences of multiples of 10 ($50 + 80 =$, $120 - 90 =$) ✓ Doubles for multiples of ten ✓ Halve any multiple of ten up to 200 (eg 170) 	<ul style="list-style-type: none"> ✓ partition to add and subtract and then recombine ✓ Add/subtract a multiple of 10 to any given 3 digit number ✓ Add a multiple of 100 to any given 3 digit number ✓ Partition +/- 10 then adjust ✓ Partition – double then recombine ✓ Find a small difference by counting on – partition (HTO then add on, OR using ten as a staging post) ✓ Multiply a number by 10 ✓ add and subtract fractions with the same denominator within one whole 	<ul style="list-style-type: none"> ✓ add and subtract numbers with the same number of digits, where regrouping/renaming is required, using formal written methods ✓ add and subtract numbers with up to three digits, where regrouping/renaming is required, using formal written methods ✓ add and subtract numbers with up to three digits where the numbers contain '0' as a placeholder, using formal written methods ✓ use the part whole model to help partition number to make them easier to multiply $3 \times 24 = (3 \times 20) + (3 \times 4) =$ ✓ to multiply a 2-digit number and ones (using PV to support) using formal method with apparatus alongside ✓ to divide a 2-digit number by 1 digit number using formal method of recording with apparatus alongside

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Year 4 Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods (as set out in school policy)
<ul style="list-style-type: none"> ✓ skip count in multiples of 6, 7, 9 ✓ count in 1000's from any given number ✓ count in 25's from 0 ✓ count backwards through zero to include negative numbers ✓ count up and down in hundredths 	<ul style="list-style-type: none"> ✓ recall multiplication and division facts for all multiplication tables ✓ use known facts to derive related ($3 \times 2 = 6$ then $3 \times 20 = 60$) ✓ know 1000 more/less than a given number at pace ✓ \times by 0 and 1 ✓ $\div 1$ ✓ Pairs of fractions that total 1 ✓ Halve any even number to 200 	<ul style="list-style-type: none"> ✓ Use double facts to partition, double then recombine 3 digit numbers ✓ What needs to be added to a 3 digit number to make the next multiple of a hundred ✓ Add/subtract a near multiple by rounding and adjusting ✓ use factor pairs for known facts ✓ \times/\div any given number by 10/100 ✓ Double any multiple of 100 ✓ Multiply multiple of ten by a single digit (eg, 30×4) ✓ Use partitioning and distributive law to multiply ($13 \times 4 = (10+3) \times 4$) ✓ add and subtract fractions with the same denominator 	<ul style="list-style-type: none"> ✓ add and subtract numbers with up to four digits, where regrouping/renaming is required, using formal written methods ✓ add and subtract numbers with up to four digits where the numbers contain '0' as a placeholder, using formal written methods ✓ multiply two-digit and three-digit numbers by a one-digit number using formal written layout

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Year 5 Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods (as set out in school policy)
<ul style="list-style-type: none"> ✓ count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 ✓ count forwards and backwards with positive and negative whole numbers, including through zero ✓ recognise and describe linear number sequences ✓ count forwards and backwards in simple fractions 	<ul style="list-style-type: none"> ✓ recall multiplication and division facts for all multiplication tables ✓ recall prime numbers up to 19 ✓ recognise and use square numbers and cube numbers ✓ \times by 0 and 1 ✓ $\div 1$ ✓ recall decimal complements of 1 ✓ Double and halve decimals ✓ What must be added to any 4 digit number to make the next multiple of 1000 ✓ What must be added to a decimal (1dp) to make next whole number ✓ Factor pairs to 100 	<ul style="list-style-type: none"> ✓ Add/subtract a near multiple of 10/100/1000 to any given number ✓ Partition and recombine to add/subtract, including decimals ✓ Add/subtract multiple of 10/100/1000 and adjust ✓ Partition to double, including decimals ✓ $\times 50 - \times 100$ then halve ✓ $\times 25 - \times 100$, halve and halve ✓ \times / \div decimals by $\times 10/100$ ✓ Multiply pairs of multiples of 10 (60×20) ✓ Multiply multiple of 100 by single digit (3×700) ✓ Divide multiple of 10 by a single digit ($270 \div 3$) ✓ \times / \div by $4/8$ by doubling/halving ✓ add and subtract fractions with the same denominator and denominators that are multiples of the same number ✓ multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams 	<ul style="list-style-type: none"> ✓ add and subtract whole numbers with more than 4 digits, including using formal written methods ✓ add and subtract numbers with a mixed number of digits where the numbers contain '0' as a placeholder, using formal written methods ✓ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ✓ divide numbers up to 4 digits by a one-digit number using the formal written method of short division

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Year 6 Progression Chart

Counting	Fact Recall	Mental Skills & Methods	Written Calculation Methods (as set out in school policy)
	<ul style="list-style-type: none"> ✓ recall multiplication and division facts for all multiplication tables with place value to calculate mentally ✓ recall prime numbers up to 19 and use this to derive prime number to 100 ✓ recognise and use square numbers and cube numbers ✓ x by 0 and 1 ✓ $\div 1$ ✓ Double and halve decimals ✓ What must be added to a decimal with 2dp to make the next whole number 	<ul style="list-style-type: none"> ✓ multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places ✓ multiply one-digit numbers with up to two decimal places by whole numbers ✓ use written division methods in cases where the answer has up to two decimal places ✓ Partition: double/halve ✓ Round and adjust/compensate ✓ Percentage strategies – 10% - half/double, 50% half for 25% etc 	<ul style="list-style-type: none"> ✓ decide if a known fact, mental or written method is required ✓ solve calculations involving a mixed number of digits and operations ✓ solve multi-step context-free and context-based problems ✓ use their knowledge of the order of operations to carry out calculations involving the four operations ✓ add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions ✓ multiply simple pairs of proper fractions, writing the answer in its simplest form ✓ divide proper fractions by whole numbers

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Supporting Materials



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National Curriculum 2014 Progression through the PoS: +/-

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ☑ read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs ☑ represent and use number bonds and related subtraction facts within 20 ☑ add and subtract one-digit and two-digit numbers to 20, including zero ☑ solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> • solve problems with addition and subtraction: <ul style="list-style-type: none"> ☑ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ☑ applying their increasing knowledge of mental and written methods ☑ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> ☑ a two-digit number and ones ☑ a two-digit number and tens ☑ two two-digit numbers ☑ adding three one-digit numbers ☑ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> • add and subtract numbers mentally, including: <ul style="list-style-type: none"> ☑ a three-digit number and ones ☑ a three-digit number and tens ☑ a three-digit number and hundreds • add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> • add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> • add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> • use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why



National Curriculum 2014 Progression through the PoS: \times/\div

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<p><i>Pupils should be taught to:</i></p> <ul style="list-style-type: none"> • solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays <i>with the support of the teacher</i> 	<p><i>Pupils should be taught to:</i></p> <ul style="list-style-type: none"> • recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context 	<p><i>Pupils should be taught to:</i></p> <ul style="list-style-type: none"> • recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. 	<p><i>Pupils should be taught to:</i></p> <ul style="list-style-type: none"> • recall multiplication and division facts for multiplication tables up to 12×12 • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. 	<p><i>Pupils should be taught to:</i></p> <ul style="list-style-type: none"> • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 • solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. 	<p><i>Pupils should be taught to:</i></p> <ul style="list-style-type: none"> • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • use their knowledge of the order of operations to carry out calculations involving the four operations • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

