**NB In all 4 operations it is crucial for children to be secure in problem solving and applying their knowledge at each stage before progressing.**

Written stages in Addition

1st Stage: Use numberlines and practical resources to *support* calculation. Teachers *demonstrate* the use of the numberline. Bead strings can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.
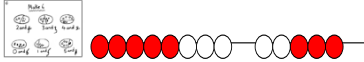
Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.
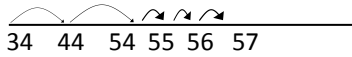

8+5=13
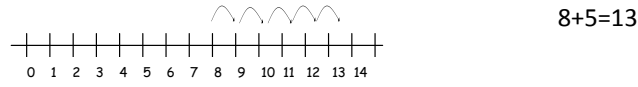
Children are introduced to hundred squares. To move to the right when adding ones and to move down to add tens.

2nd Stage: Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.
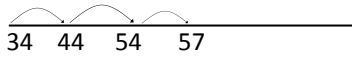
First counting on in tens and ones.

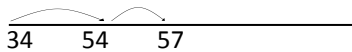+10 +10 +1 +1 +1   34+23=57

34 44 54 55 56 57

Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

+10 +10   +3   34+23=57

34 44   54   57

*Followed by adding the tens in one jump and the units in one jump.
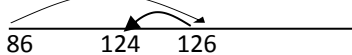
+20   +3   34+23=57

34   54   57

3rd Stage: Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

Compensation

+40   -2   38+86=124

86   124   126

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Partitioning – record mental methods using partitioning.

47 +76 = 40 +70 +7+ 6 = 110 +13 = 123

Leading to

 40 + 7
 70 + 6
110+13 = 123

4th Stage: in Key Stage 2 Expanded column addition Using 2 and 3 digit numbers

```
  2 6 7
+   8 5
    1 2 (7+5)
  1 4 0 (60+80)
  2 0 0
  3 5 2
```

From this, children will begin to carry below the line, including decimals.

```
 3 6 7              3.59
+  8 5            +  .78
 4 5 2             4.37
 1 1               1 1
```

*It is important to know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

5th Stage: Children should extend the carrying method to numbers with at least four digits, including decimals. Children will begin adding fractions with similar denominators.

6th Stage: Children should extend the carrying method to numbers with any number of digits, including decimals. Children will begin adding fractions with different denominators.

Children are encouraged to check their answers using the inverse calculation. Inverse operations are opposite operations that undo each other. Addition and subtraction are inverse operations.

Written stages in Subtraction

1ˢᵗ Stage: Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc. They use numberlines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

$$-1 \quad -1 \quad -1 \qquad\qquad 6-3=3$$

0 1 2 3 4 5 6 7 8 9 10
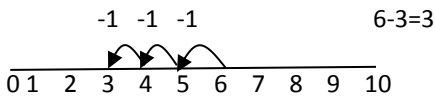
The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and the _difference in value_ between the numbers.

Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.
13 – 4 = 9

0 1 2 3 4 5 6 7 8 9 10 11 12 13
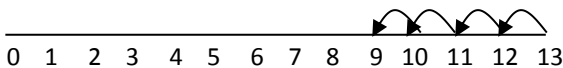
Children also use hundred squares, learning to subtract ones by moving left and to subtract tens by moving up.
2ⁿᵈ Stage Children will begin to use empty number lines to support calculations.
First counting back in tens and ones

$$-1 \quad -1 \quad -1 \quad -10 \quad -10 \qquad 47-23=24$$

24 25 26 27   37   47
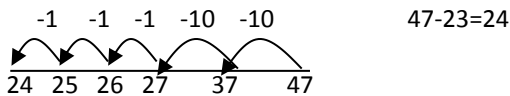
Children should then begin to subtract the units in one jump and the tens in one jump (similar to addition).
Counting on: If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on (find the difference).
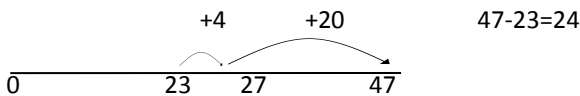
$$+4 \qquad\quad +20 \qquad\qquad 47-23=24$$

0   23   27   47

3ʳᵈ Stage: Children will continue to use empty number lines with increasingly large numbers.
Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Where the numbers in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used. A numberline can also be used for subtracting decimals.

4ᵗʰ Stage: Used in Key Stage 2
Partitioning and decomposition   754-86

```
    700  +  50  +  4
-           80  +  6
    700  +  40  + 14    adjust from T to U
-           80  +  6
    600  +  140 + 14    adjust from H to T
_           80 +  6
    600  +  60  +  8  = 668
```

Decomposition with shortened method

```
  614 1              7 9 10                 1
   7̶5̶4̶                8̶0̶0̶                 1̶2.45
 -  86              -  377              -  3.34
   668                423                  9.11
```

5ᵗʰ Stage: Children should extend the decomposition method to numbers with any number of digits, including decimals. Numberlines and mental methods should also be used, if more efficient than decomposition. Children will start subtraction fractions with similar and then different denominators.

Children are encouraged to check their answers using the inverse calculation. Inverse operations are opposite operations that undo each other. Addition and subtraction are inverse operations.

Written stages in Multiplication

1st Stage: Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.
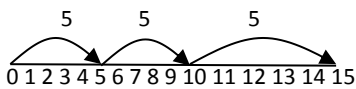
2nd Stage: Children will develop their understanding of multiplication and use jottings to support calculation:
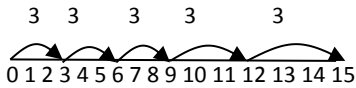
Repeated addition

3 times 5   is   5 + 5 + 5 = 15   or   3 lots of 5   or   5 x 3

Repeated addition can be shown on a number line:

5 x 3 = 5 + 5 + 5

```
      5       5           5
    ⌒   ⌒   ⌒
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.

```
   3    3    3    3       3
  ⌒ ⌒ ⌒ ⌒   ⌒
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

Arrays: Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

○○○○○
○○○○○   5x3=15
○○○○○
3x5=15
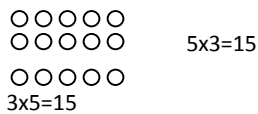
3rd Stage: Partitioning

38 x 5 = (30 x 5) + (8 x 5)

$\qquad$ = 150 + 40

$\qquad$ = 190

Children need to have a good understanding with times tables, as well as understanding place value when numbers are multiplied by x10 and x100 before moving on to 4th stage.
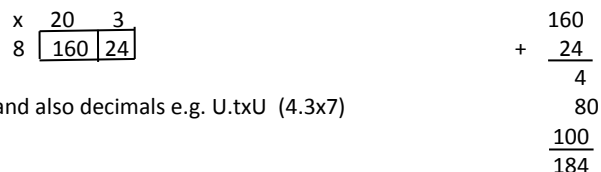
4th Stage: Children will continue to use arrays where appropriate leading into the grid method of multiplication.

Grid method: TU x U

(Short multiplication – multiplication by a single digit)

23 x 8 (Children will approximate first)

23 x 8 is approximately 25 x 8 = 200

x	20	3
8	160	24

```
    160
+    24
      4
     80
    100
    184
```

5th Stage: Grid method: Extended to HTUxU and TUxTU and also decimals e.g. U.txU (4.3x7)

Grid method
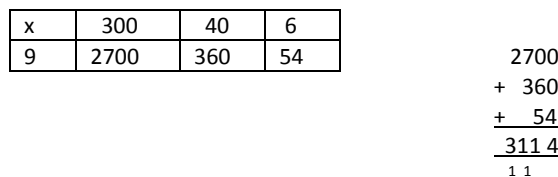
HTU x U

(Short multiplication – multiplication by a single digit)

346 x 9

Children will approximate first

346 x 9 is approximately 350 x 10 = 3500

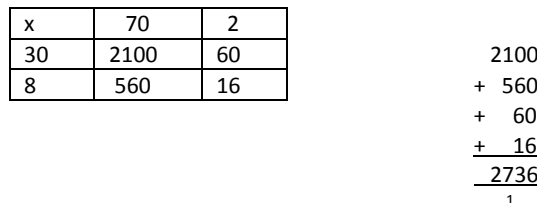| x	300	40	6
9	2700	360	54

```
   2700
+   360
+    54
   311 4
    1 1
```

TU x TU

(Long multiplication – multiplication by more than a single digit)

72 x 38

Children will approximate first

72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16

```
   2100
+   560
+    60
+    16
   2736
     1
```

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other

6th Stage: Vertical expanded method:

```
     38
   X  7
     56   (8 x 7)
    210   (8 x 30)
    266
```

Lastly on to compact method.

$$\begin{array}{r} 38 \\ \times\ 7 \\ \hline 266 \end{array}$$

In addition children begin to multiply fractions.

Children are encouraged to check their answers using the inverse calculation. Inverse operations are opposite operations that undo each other. Multiplication and division are inverse operations.
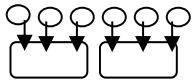
Written stages in Division
1st Stage: Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.
2nd Stage: Children will develop their understanding of division and use jottings to support calculation
Sharing equally
6 sweets shared between 2 people, how many do they each get?

Grouping or repeated subtraction:
There are 6 sweets. How many people can have 2 sweets each?
OO / OO / OO
Repeated subtraction using a number line or bead bar
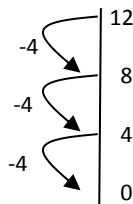 -3   -3   -3   -3   12÷3=4

0 1 2 3 4 5 6 7 8 9 10 11 12
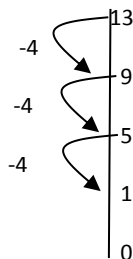**3rd Stage:**
Repeated subtraction using a vertical number line
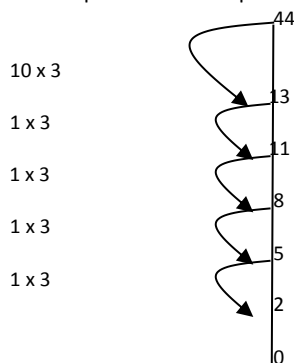Children will use an empty number line to support their calculation And arrays where appropriate.
12÷4=3

Children should also move onto calculations involving remainders.
 13÷4=3 r1

4th Stage: Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. 44÷3=14r2

10 x 3

1 x 3

1 x 3

1 x 3

1 x 3

This method will be used for calculations with or without a remainder. Use alongside arrays if necessary.
Then onto the vertical method:
Short division TU ÷ U   72÷3=

$$\begin{array}{r} 7\ 2 \\ -\ 3\ 0\quad 3 \times 10 \\ \hline 4\ 2 \\ -\ 3\ 0\quad 3 \times 10 \\ \hline 1\ 2 \\ -\ 1\ 2\quad 3 \times 4 \\ \hline 0 \end{array}$$

          = 24

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2. Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

5th Stage:
Expanded short division HTU ÷ U   196 ÷ 6

```
              32 r 4
        6 ) 196
            - 180   6x 30
              16
            -  12   6x 2
               4
```

Answer :   32   remainder 4 or   32 r 4 or 32 4/6 or 32 2/3   Children are encouraged to check their answers using the inverse calculation.(multiplication)

Children should learn how to write the remainder as an integer and as a fraction.

6th Stage:
Compact short division HTU ÷ U

Children should continue writing the remainder as an integer and as a fraction. In addition they should learn how to write it as a decimal. Therefore they are using this method for dividing decimals.