

Supporting Information
For
Lone-Pair Electron Driven Thermoelectrics at Room Temperature

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Stability: The structures and the corresponding phase diagrams of Tl_3TaSe_4 , Tl_3VS_4 , In_3TaSe_4 and In_3VS_4 were predicted using Materials Project¹. While Tl_3TaSe_4 and Tl_3VS_4 give stable ternary phase diagrams (as shown in Fig. S1), In_3TaSe_4 and In_3VS_4 are found to be thermodynamically unstable at 0K.

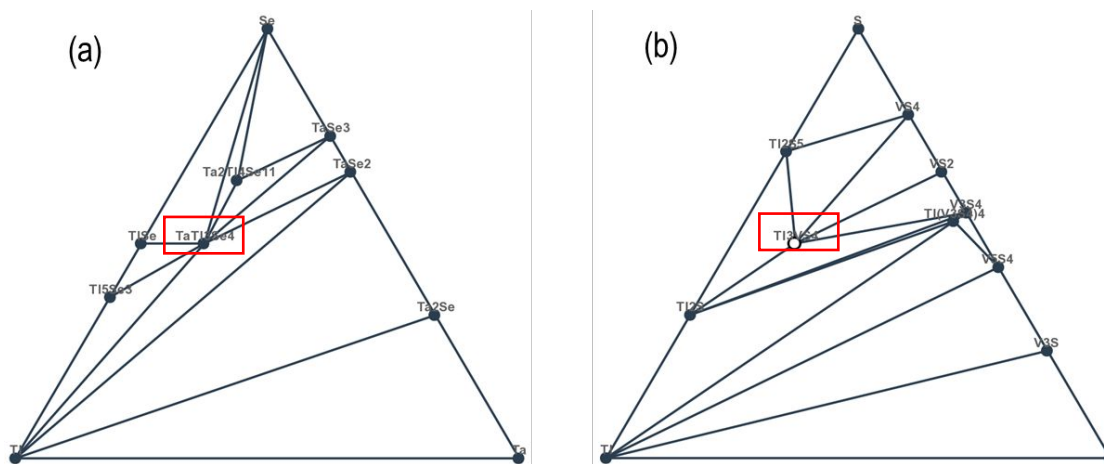


Figure S1: Phase diagrams for (a) Tl_3TaSe_4 and (b) Tl_3VS_4 from Materials Project¹.

Mean Square Displacement (MSD): Atomic displacement (u) of the j^{th} atom in the l^{th} unit cell along Cartesian axis α at a given time t can be written as²:

$$u^\alpha(jl, t) = \left(\frac{\hbar}{2Nm_j}\right)^{1/2} \sum_{\mathbf{q}, \nu} [\omega_\nu(\mathbf{q})^{-1/2} [\hat{a}_\nu(\mathbf{q})e^{-i\omega_\nu(\mathbf{q})t} + \hat{a}_\nu^\dagger(-\mathbf{q})e^{i\omega_\nu(\mathbf{q})t}] e^{i\mathbf{q}\cdot\mathbf{r}(jl)} e_\nu^\alpha(j, \mathbf{q})]$$

where m is the atomic mass, N is the number of unit cells, \mathbf{q} is the wave vector, ν is the branch index, $e_\nu^\alpha(j, \mathbf{q})$ is the polarization vector of the j^{th} atom in the l^{th} unit cell in mode ν , $\mathbf{r}(jl)$ is the atomic position and $\omega_\nu(\mathbf{q})$ is the phonon frequency. \hat{a}_ν and \hat{a}_ν^\dagger are phonon creation and annihilation operators. The expectation value of the squared atomic displacement is calculated as:

$$\langle |u^\alpha(jl, t)|^2 \rangle = \frac{\hbar}{2Nm_j} \sum_{\mathbf{q}, \nu} \omega_\nu(\mathbf{q})^{-1} (1 + 2n_\nu(\mathbf{q}, T)) |e_\nu^\alpha(j, \mathbf{q})|^2$$

where the phonon population $n_\nu(\mathbf{q}, T)$ is:

$$n_\nu(\mathbf{q}, T) = \frac{1}{e^{\left(\frac{\hbar\omega_\nu(\mathbf{q})}{k_B T}\right)} - 1}$$

The mean square displacements (MSD) were projected along the (111) direction as implemented in Phonopy³:

$$MSD = \frac{\hbar}{2Nm_j} \sum_{\mathbf{q}, \nu} \omega_\nu(\mathbf{q})^{-1} (1 + 2n_\nu(\mathbf{q}, T)) |n \cdot e_\nu^\alpha(j, \mathbf{q})|^2$$

We projected the MSDs along (111)-direction (Fig. S2) using a $31 \times 31 \times 31$ mesh-points, which were chosen based on rigorous convergence tests.

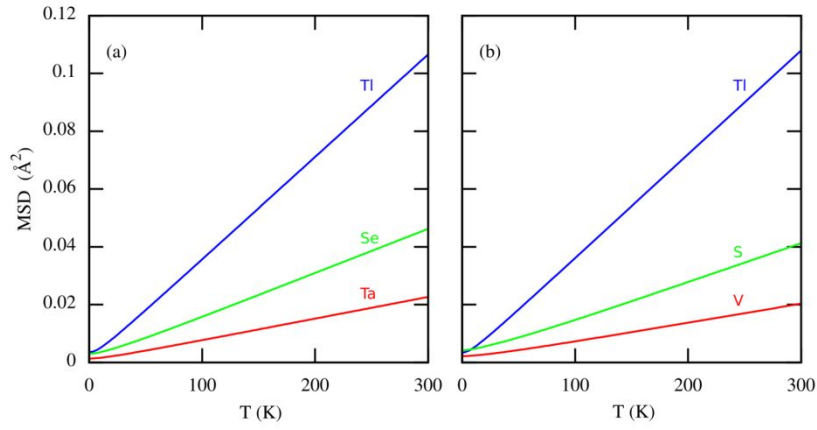


Figure S2: Mean square displacements of individual atoms for (a) Tl_3TaSe_4 and (b) Tl_3VS_4 at 300K.

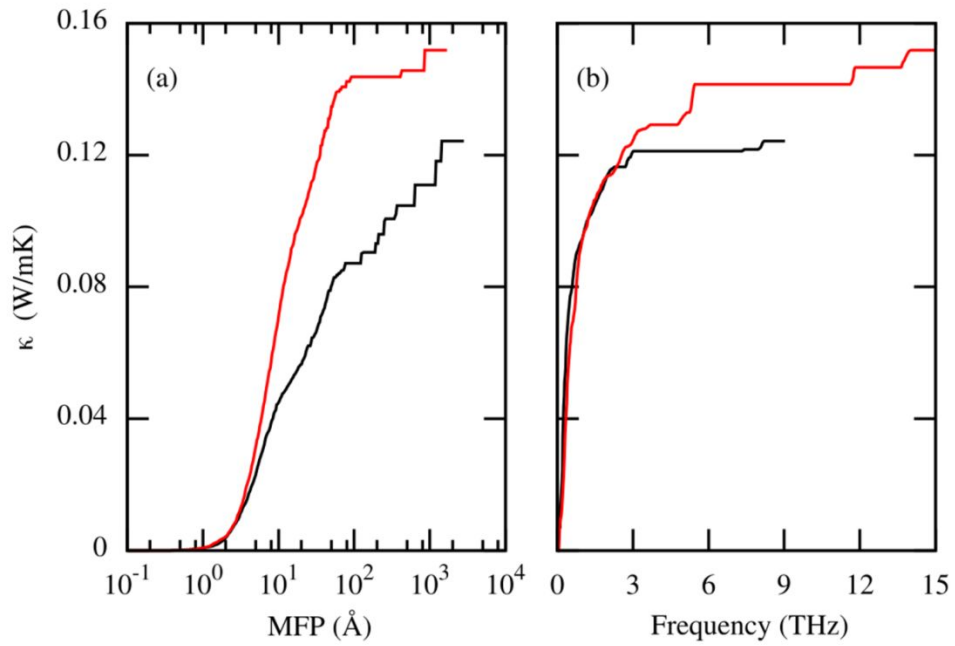


Figure S3: Accumulated κ for Tl_3TaSe_4 (black) and Tl_3VS_4 (red) at 300K as a function of (a) mean free path (MFP) and (b) phonon frequency.

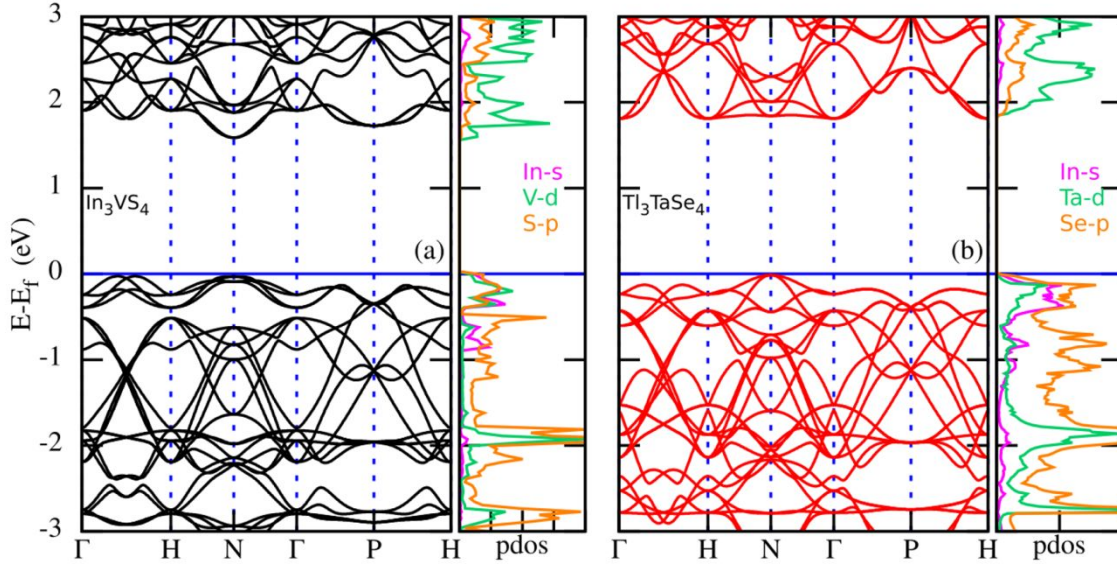


Figure S4: Electronic band structure and density of states projected on orbitals for (a) In_3VS_4 and (b) In_3TaSe_4 . The blue horizontal lines are the Fermi levels of the corresponding compounds.

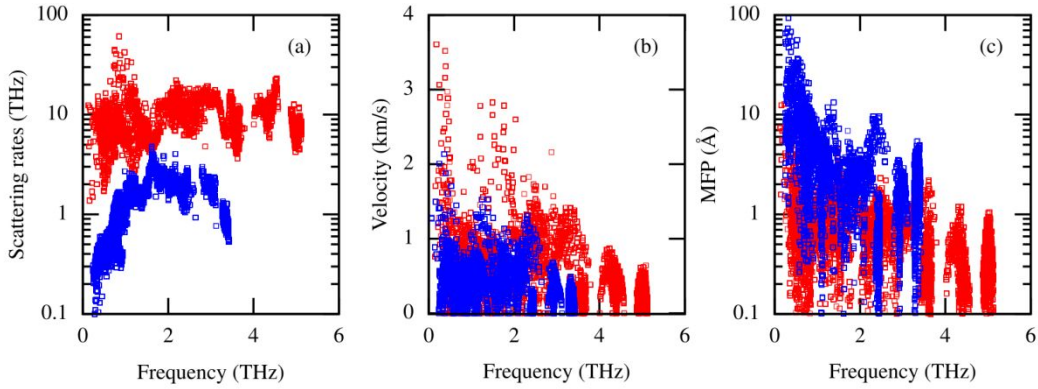


Figure S5: Comparison of lattice dynamical properties of In_3VS_4 (red) and Tl_3VSe_4 (blue): (a) scattering rates, (b) group velocity and (c) mean free path.

Thermal conductivity: Higher scattering rates in In_3XY_4 than those in Tl_3XY_4 (as shown in Fig. S4) give lower κ (<0.1 W/mK) In_3XY_4 . This was verified by calculating κ from almaBTE⁴ and Phono3py⁵ which give similar κ .

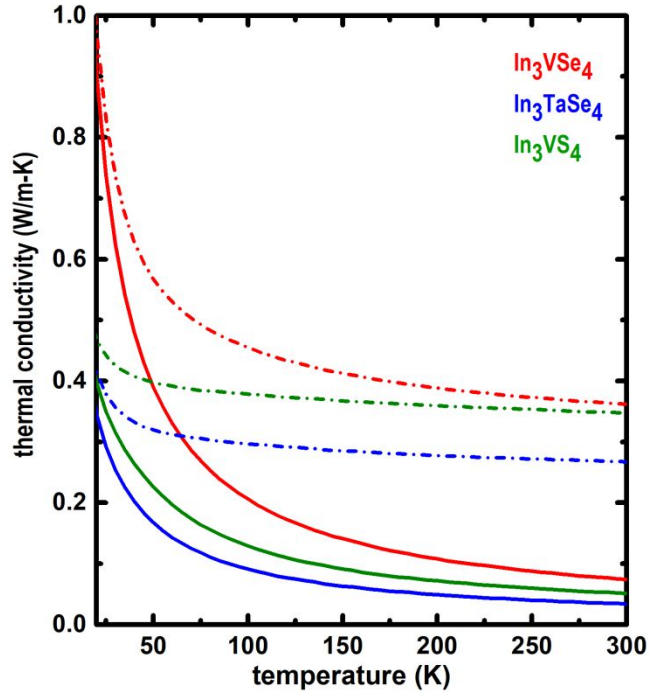


Figure S6: Thermal conductivity for In_3VSe_4 (red), In_3TaSe_4 (blue) and In_3VS_4 (green). The solid lines are κ - contributions from the phonon Boltzmann equation^{4,5} and the dashed are κ from the two channel model⁶.

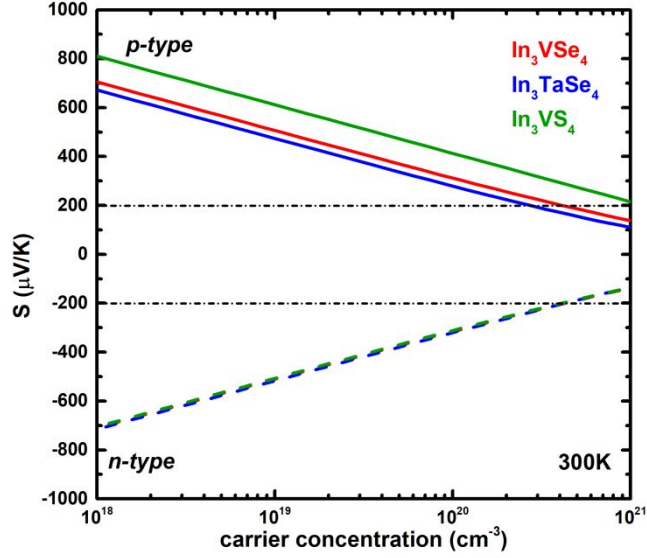


Figure S7: Calculated Seebeck coefficient of In_3VSe_4 , In_3TaSe_4 and In_3VS_4 as a function of carrier concentration at 300K.

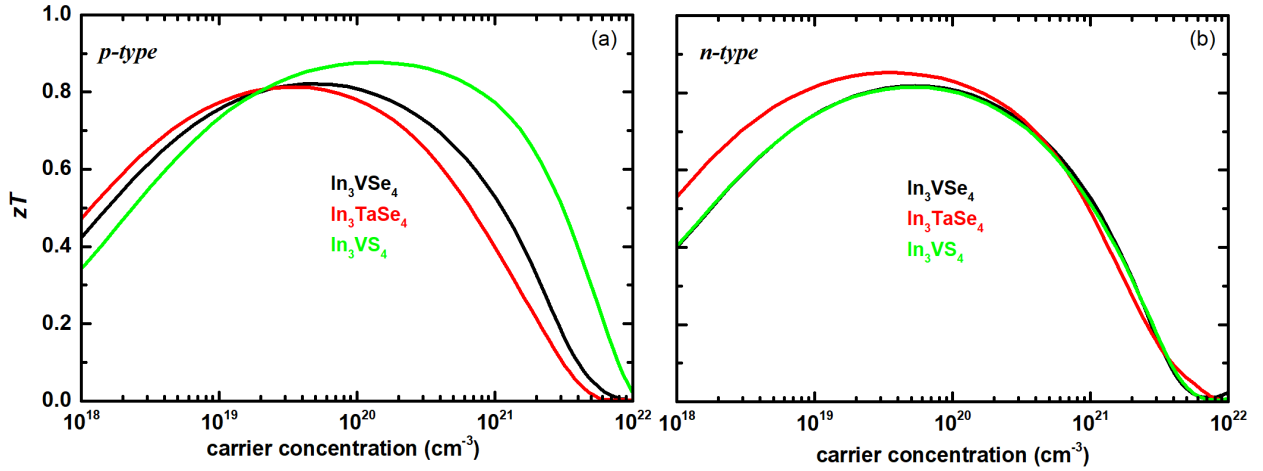


Figure S8: Thermoelectric properties of In_3VSe_4 , In_3TaSe_4 and In_3VS_4 as a function of carrier concentration at 300K when doped (a) p-type and (b) n-type. A relaxation time; $\tau = 9 \times 10^{-14}\text{s}$ was used for all the cases to be consistent with Tl_3XY_4 discussed in the main text.

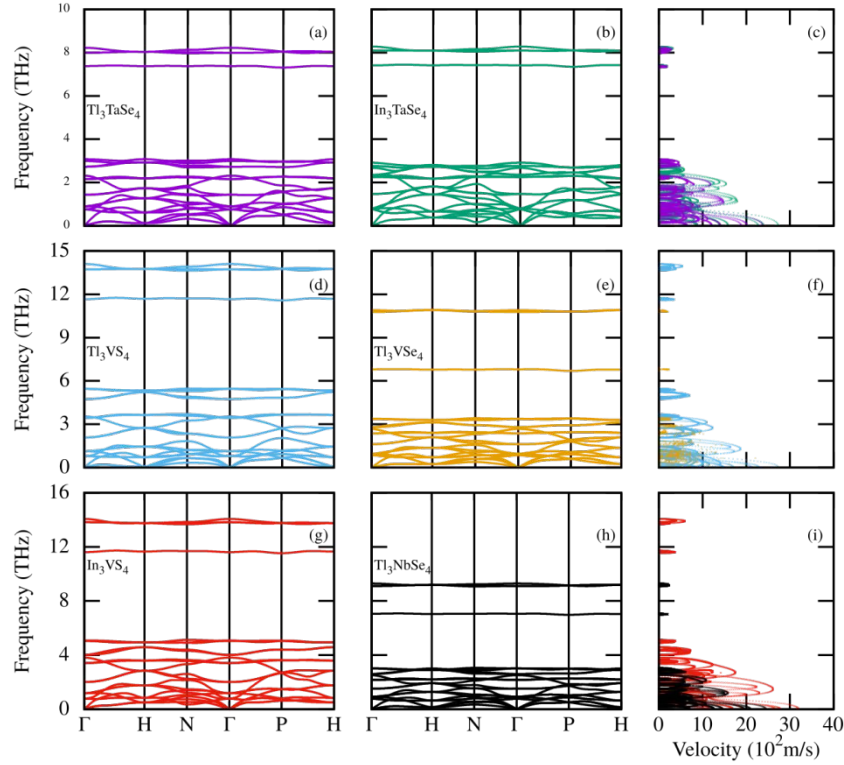


Figure S9: Comparative lattice dynamical properties in Z_3XY_4 ($X=In/Tl$, $X=Ta/V/Nb$ and $Y= S/Se$) as a result of variation of Z (a-c), Y (d-f) and X, Y, Z (g-i) atoms

References

- (1) Jain, A.; Ong, S. P.; Hautier, G.; Chen, W.; Richards, W. D.; Dacek, S.; Cholia, S.; Gunter, D.; Skinner, D.; Ceder, G.; Persson, K. A., Commentary: The Materials Project: A Materials Genome Approach to Accelerating Materials Innovation, *APL Materials*. **2013**, *1*, 011002 (1-11) .
- (2) Wallace, D. C. *Thermodynamics of Crystal*. (Dover Publications, 1998).
- (3) Togo, A.; Tanaka, I., First Principles Phonon Calculations in Materials Science, *Scr. Mater.* **2015**, *108*, (1-5).
- (4) Carrete, J.; Vermeersch, B.; Katre, A.; van Roekeghem, A.; Wang, T.; Madsen, G. K. H.;

Mingo, N., almaBTE : A Solver of the Space–Time Dependent Boltzmann Transport Equation for Phonons in Structured Materials, *Comput. Phys. Commun.* **2017**, *220*, (351-362)

(5) Togo, A., Chaput, L. & Tanaka, I. Distributions of Phonon Lifetimes in Brillouin Zones. *Phys. Rev. B.* **2015**, *91*, 094306 (1-31).

(6) Mukhopadhyay, S.; Parker, D. S.; Sales, B. C.; Poretzky, A. A.; McGuire, M. A.; Lindsay, L., Two-Channel Model for Ultralow Thermal Conductivity of Crystalline Ti_3VSe_4 , *Science.* **2018**, *360*, (1455-1458).