SUPPORTING INFORMATION

Jumping Tensegrity Robots based on Torsionally Pre-stretched SMA Springs

Yoon Seop Chung, Ji-Hyeong Lee, Jae Hyuck Jang, Hyouk Ryeol Choi and Hugo Rodrigue*

School of Mechanical Engineering, Sungkyunkwan University, Suwon, 16419, South Korea

Corresponding Author: Hugo Rodrigue (rodrigue@skku.edu)

Table of contents

1. Derivation of thermoconstitutive model .............................................................. 2
2. Material properties .................................................................................................. 3
3. Manufacturing and testing jigs .............................................................................. 4
4. Explanation of two-state model ............................................................................. 5
5. Contraction speed of the TPSMA spring ............................................................... 6
6. Durability test of TPSMA springs versus SMA Springs ......................................... 7
7. Actuation results for springs with 6 mm coil radius ............................................. 8
8. Additional results for tensegrity robot rolling motion ......................................... 9
9. Additional results for tensegrity robot jumping motion ....................................... 11
1. Derivation of thermoconstitutive model

The model proposed in this work is a modification of the one-dimensional thermoconstitutive model proposed by Liang and Rogers. This thermoconstitutive model for SMA springs without torsional pre-strain can be written as

\[ y = \frac{4R^3N}{Gr^4} F - \frac{2\Delta N\pi R^2}{\sqrt{3}Gr} \xi \]  \hspace{1cm} (1)

The shear strain \( \gamma \) of a TPSMA spring is the same as that of a regular SMA spring plus or minus the torsional pre-strain \( \gamma_0 \) applied to the SMA wire, depending on the direction in which it is applied, and can be represented as

\[ \gamma = \frac{\tau}{G} - \frac{N}{\sqrt{3G}} \xi \pm \gamma_0 \]  \hspace{1cm} (2)

The shear stress in the TPSMA spring can be expressed as

\[ \tau = \frac{2FR}{\pi r^3} \]  \hspace{1cm} (3)

The deflection \( y \) of the TPSMA spring corresponds to that of a helical spring and can then be calculated from equation (2) as

\[ y = R \int_0^{2\pi R N} \frac{\gamma}{r} \, dx = \frac{2\pi R^2 N}{r} \left( \frac{\tau}{G} - \frac{N}{\sqrt{3G}} \xi \pm \gamma_0 \right) \]  \hspace{1cm} (4)

Substituting Equation (3) into Equation (4) yields Equation (5) which is the same as Equation (1) within the text.

\[ y = \frac{4R^3N}{Gr^4} F - \frac{2\Delta N\pi R^2}{\sqrt{3}Gr} \xi \pm \frac{2\pi R^2 N}{r} \gamma_0 \]  \hspace{1cm} (5)
2. Material properties

The material properties or the SMA (Flexinol LT, Dynalley) used in the text are as follows.

Table S1. Properties of the shape memory alloy used for all experiments

<table>
<thead>
<tr>
<th>Property</th>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austenite modulus value</td>
<td>$D_a$</td>
<td>$67 \times 10^3$</td>
<td>Mpa</td>
</tr>
<tr>
<td>Martensite modulus value</td>
<td>$D_m$</td>
<td>$26.3 \times 10^3$</td>
<td>Mpa</td>
</tr>
<tr>
<td>Thermal coefficient</td>
<td>$\Theta$</td>
<td>0.55</td>
<td>Mpa/°C</td>
</tr>
<tr>
<td>Martensite final temperature</td>
<td>$M_f$</td>
<td>42</td>
<td>°C</td>
</tr>
<tr>
<td>Martensite start temperature</td>
<td>$M_s$</td>
<td>52</td>
<td>°C</td>
</tr>
<tr>
<td>Austenite start temperature</td>
<td>$A_s$</td>
<td>68</td>
<td>°C</td>
</tr>
<tr>
<td>Austenite final temperature</td>
<td>$A_f$</td>
<td>78</td>
<td>°C</td>
</tr>
<tr>
<td>Stress influence coefficient</td>
<td>$C_M$</td>
<td>12</td>
<td>Mpa/°C</td>
</tr>
<tr>
<td>Stress influence coefficient</td>
<td>$C_A$</td>
<td>12</td>
<td>Mpa/°C</td>
</tr>
<tr>
<td>Critical stress at start</td>
<td>$\sigma_s^{cr}$</td>
<td>100</td>
<td>Mpa</td>
</tr>
<tr>
<td>Critical stress at final</td>
<td>$\sigma_f^{cr}$</td>
<td>140</td>
<td>Mpa</td>
</tr>
<tr>
<td>Maximum residual strain</td>
<td>$\epsilon_L$</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>
3. Manufacturing and testing jigs
The manufacturing jig is shown in Figure S1a. It consists of a motor at the top where one end of the SMA wire is attached and a guide where the other end of the wire is attached. The guide is free to move up and down but cannot rotate. A force/torque sensor or torsional load cell can be attached between the guide and the SMA wire to measure the torque applied to the SMA wire but is not necessary for the fabrication of TPSMA springs. A weight is attached to the bottom of the guide to maintain tension during the application of a torsional force by the motor to twist the SMA wire. The SMA wire used for the TPSMA springs in this paper has a diameter of 305 mm and a weight of 700 g was attached to the end of the guide throughout the process. An SMA wire with a length of 30 cm was used and 30, 50 and 70 rotations were applied to produce a torsional pre-strain of 36, 60 and 84 °/mm, respectively.
The universal testing machine with the environmental chamber used to produce the curves in Figure 2a-c and Figure S4a-c is shown in Figure S1b.

Figure S1. (a) Jig used for applying the torsional pre-strain to the SMA wire with a force-torque sensor to measure the torsional force applied to the SMA wire, and (b) the tensile testing machine with an environmental chamber.
4. Explanation of two-state model

A two-state model is used to predict the performance of SMA coils by assuming static conditions with a given load by assuming that the actuator will stop moving at equilibrium on the predicted curve of the fully martensite curve at low temperature and on the fully austenite curve at high temperature.\cite{27} The difference in deflection of the spring between these two curves at a given force will give the displacement of the spring from being heated until its fully martensite state and allowed to settle (Fig. S2). This was used for both the SMA and TPSMA springs to predict the contraction ratio for different payloads.

Figure S2. Displacement predicted using the two-state model
5. Contraction speed of the TPSMA spring

The displacement of the TPSMA spring with different current conditions was used to measure the contraction speed of the springs by calculating the contraction ratio divided by the time taken to reach the peak contraction using the data obtained as shown in Figure S3.

The current of 1.5 A produced an optimal steady-state contraction without overshoot that can be sustained, so it was used as the value for the basic actuation of the tensegrity robot. Currents of 3 A and higher produced overshoot and burned if the current was maintained for too long. A current of 5 A was used for high-speed rolling and jumping motions as it produced the most significant overshoot in a short time.

The thermoconstitutive model shows that an increase in temperature over the austenite finish transition temperature further increases the performance of the actuator. But this is only valid up to approximately 140–160 °C, above which thermal fatigue occurs due to overheating and suppresses the shape memory effect necessary for actuation. Lengthening of TPSMA springs after contraction under a sustained current above 2.0 A is due to such overheating of the spring.

![Figure S3. Displacement of the TPSMA spring at different applied currents.](image-url)
6. Durability test of TPSMA springs versus SMA Springs

An SMA spring and a TPSMA spring were manufactured with the same dimensions as those within tested previously, and a periodic current of 5 A applied for 1 s with a resting period of 2 seconds between cycles was applied to the springs with a constant weight of 100 g applied at one end. This cycle was repeated 63 times to test the deviation of the springs with regards to their initial position and to verify their durability as shown in Figure S4. Both TPSMA and SMA exhibit similar behaviour whereas their stretched and contracted positions drift from their initial point and the increase in performance of the TPSMA over the SMA is maintained throughout this change in contraction range. This shows that the durability of the spring is not significantly affected by the addition of the torsional pre-strain.

![Figure S4](image)

**Figure S4.** Repeatability test of TPSMA springs versus SMA springs.
7. Actuation results for springs with 6 mm coil radius
The tests were repeated for TPSMA springs with torsional pre-strains of 60 and 84°/mm and a SMA spring with a coil diameter of 6 mm. Similar differences can be seen for the force versus displacement curves of the martensite, partially austenite and fully austenite phases of the springs (Fig. S4a-c). The contraction ratio versus load was measured and an increase in the contraction ratio was measured throughout the range of tested loads (Fig. S4d).

The displacement through cyclic motions at a periodic current of 2 A with a period of 4 s and a duty ratio of 10 % with a constant load of 40 g and a periodic current of 2.5 A with a period of 3 s and a duty ratio of 10 % with a constant load of 40 g were recorded (Fig. S4e,f). From these results, it can be seen that the effect of adding torsional pre-strain in SMA springs with a larger coil diameter also has a similarly significant impact on the performance of the actuator.

Figure S5. Force versus displacement curves at (a) 25, (b) 60 and (c) 120 °C. The (d) contraction ratio versus load curves, and the (e) contraction speed, (f) contraction ratio and (g) displacement for different applied currents. The cyclic displacement for (h) period of 4 s and (i) period of 3 s.
8. Additional results for tensegrity robot rolling motion

The actuation of a single SMA spring with a spring diameter of 30 mm, a wire diameter of 305 mm and an actuation current of 1.5 A is insufficient on its own to shift the center of mass of the actuator beyond any of the edges of the base triangle due to the insufficient contraction (Fig. S5a,b). It is possible to use a high current to produce overshoot that will make the same spring produce a rolling motion (Fig. S5c), but this requires the application of high currents. A thicker SMA wire with a diameter of 0.381 mm and other dimensions maintained is able to produce more force that results in a larger contraction that makes a successful rolling motion (Fig. S5d), but this requires more power to produce the same rolling motion. Two SMA springs where one is located on the top triangle and the other on the bottom triangle can produce a rolling motion while requiring significantly less contraction (Fig. S5e), but this requires double the power input of that using a single TPSMA spring. Using four TPSMA springs located such that their location is always at the bottom of the current triangle, it is possible to produce a rolling motion that will reposition the tensegrity robot back to its original orientation (Fig. S5f). A current of 3.0 A for 0.4 s was applied to each TPSMA spring with a resting time of 3.6 s to recover the initial shape, and the average speed of the robot was measured to be 2.88 cm/s or 0.14 BL/s for the sequence of 4 rolls.
Figure S6  (a) Tensegrity robot, (b) unsuccessful rolling motion with single SMA spring, successful rolling motion using (c) higher current, (d) thicker SMA spring, (e) two SMA springs, and (f) successive rolling motion of the tensegrity robot.
9. Additional results for tensegrity robot jumping motion

In addition to the jumping method shown in the text using six SMA springs along the circumference of the tensegrity robot, two other methods to induce a tubular shape to the robot were tested. The first consists of using three SMA springs across the cross-section (Fig. S6a), and the second consists of using six SMA springs where three are attached along the circumference of the bottom triangle of the robot and the other three along the circumference of the top triangle of the robot (Fig. S6b). Both methods were tested using SMA and TPSMA springs using actuation currents of 5.2 A, and the first method resulted in jumping heights of 0.54 BL and 0.58 BL for SMA and TPSMA springs, respectively (Fig. S6c,d). The second method results in jumping heights of 0.17 BL and 0.25 BL for SMA and TPSMA springs, respectively (Fig. S6e,f). A comparison of all methods is shown Fig. S7. TPSMA springs were able to increase the jumping height irrespective of the method due to the faster movement and larger contraction, and the method using six SMA springs around the circumference of the tensegrity robot was shown to be better than the other two methods.
Figure S7 SMA spring configuration for (a) jumping motion using three transversal springs and (b) six SMA springs positioned along the circumference of the top and bottom triangles. Results for jumping height using three transversal springs with (c) SMA springs and (d) TPSMA springs. Results for jumping height using six SMA springs positioned along the circumference of the top and bottom triangles for (e) SMA springs and (f) TPSMA springs.
Figure S8 Comparison of the three jumping methods with SMA and TPSMA springs.