Supporting Information for

**Polarization Tunable Plasmonic Structured Illumination**

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1. Unequal coupling efficiency of the fishbone grating for orthogonal linear polarization and the corresponding optimization.

![Figure S1](image)

**Figure S1.** Simulated average intensity (a) and phase (b) of the SPI for different $D$ values. The phase shift of SPI is not strictly equal to $2\theta$ because of the unequal coupling efficiency between x and y polarized incident. And it can be eliminated by optimizing structural parameters. Figure S1 (a) shows the average intensities of SPI with the different polarized incident. When the $D=260\text{nm}$, the coupling efficiencies are almost equal. The conversion efficiencies reach 2.5% for TM and TE polarized incident beam. And the phase shift of SPI varies linearly with the polarization (Figure S1 (b)).

2. Image reconstruction algorithm utilizing polarization tunable structured illumination.
Figure S2. The OTF of conventional fluorescence microscope (a), SIM (b) and PSIM (d).

The red circles correspond to the spatial information within the pass band of conventional microscopy. The blue circles represent the sideband high spatial frequency information components accessible by SIM and PSIM, respectively. The dashed circles represent the maximum of Fourier space region detectable by SIM and PSIM, respectively. (c) SPPs shows a large wave vector $k_{spp}$ than the light $k_{photon}$ at the same frequency $\omega_0$.

Different from conventional fluorescence microscope can only transform spatial frequencies that reside within the passband of the optical transfer function (OTF) of the objective (Figure S2 (a)), the plasmonic SIM (PSIM) utilizes illuminations with varying intensity to couple the high spatial frequency information of the object to spatial frequency information with the pass band of microscope objective. Owing to SPPs provide higher spatial frequency than light at the same frequency (Figure S2 (c)), the PSIM utilizing SPI can achieve greater than 2-fold resolution improvement.

In the imaging performance simulations, surface plasmon interference (SPI) patterns serve as the structured illumination patterns to illuminate a fluorescent object. The
numerical algorithm of reconstructing high-resolution image from intermediate images is the same as that of SIM \(^1,^2\). The sinusoidal SPI patterns under different polarization angle \(\theta\) excitation can be expressed as:

\[
I_\theta(x) = I_0[1 + m \cos(k_{SPP}x + 2\theta)] \tag{S-1}
\]

where \(I_0\) and \(m\) is the average intensity and modulation depth of SPI, respectively. \(k_{SPP}\) is the magnitude of SPPs wave vector. And the frequency content of SPI can be expressed as:

\[
I_\theta(k) = I_0[\delta(k) + \frac{m}{2} \delta(k - k_{SPP})e^{2i\theta} + \frac{m}{2} \delta(k + k_{SPP})e^{-2i\theta}] \tag{S-2}
\]

where \(k\) is the wave vector of fluorescence. By using convolution theorem\(^3\), the Fourier transform of observed image under the structured illumination can be expressed as:

\[
D_\theta(k) = [C(k) \otimes I_\theta(k)] \cdot OTF(k) \tag{S-3}
\]

where \(C(k)\) represents frequency content of fluorophore density distribution within specimen. \(OTF(k)\) represents the optical transfer function (OTF) of the optical system. Thus, the observed image \(D_\theta(k)\) is a linear combination of frequency content within three circular regions of specimen \(C(k)\), as shown in Figure S1 (d).

To obtain a faithful high-resolution image, the three components in Fourier space need to be solved and moved back to their original positions. Consequently, three different SIM images \((D_{\theta_1}(r), D_{\theta_2}(r)\) and \(D_{\theta_3}(r))\) of the specimen are acquired corresponding to three
different polarizations \((\theta_1, \theta_2 \text{ and } \theta_3)\) of incident light. Subsequently, the contents of \(C(k)\), \(C(k-k_{\text{app}})\) and \(C(k+k_{\text{app}})\) can be obtained by Wiener Filter \(^4\), as shown following:

\[
\begin{bmatrix}
C(k) \cdot \text{OTF}(k) \\
C(k+k_{\text{app}}) \cdot \text{OTF}(k) \\
C(k-k_{\text{app}}) \cdot \text{OTF}(k)
\end{bmatrix} = \frac{1}{I_0 \cdot (e^{-2i(\theta_1-\theta_3)} - e^{2i(\theta_1-\theta_3)} + e^{-2i(\theta_2-\theta_3)} - e^{2i(\theta_2-\theta_3)} + e^{-2i(\theta_3-\theta_1)} - e^{2i(\theta_3-\theta_1)})}
\]

\[
\begin{bmatrix}
D_{\theta_1}(k)[e^{2i(\theta_1-\theta_3)} + e^{-2i(\theta_1-\theta_3)}] + D_{\theta_2}(k)[e^{2i(\theta_2-\theta_3)} - e^{-2i(\theta_2-\theta_3)}] + D_{\theta_3}(k)[e^{-2i(\theta_3-\theta_1)} - e^{2i(\theta_3-\theta_1)}] \\
\frac{2}{m} \left[D_{\theta_1}(k)[e^{2i\theta_1} - e^{-2i\theta_1}] + D_{\theta_2}(k)[-e^{2i\theta_2} + e^{2i\theta_2}] + D_{\theta_3}(k)[e^{-2i\theta_3} - e^{-2i\theta_3}]\right] \\
\frac{2}{m} \left[D_{\theta_1}(k)[-e^{-2i\theta_1} + e^{-2i\theta_1}] + D_{\theta_2}(k)[e^{-2i\theta_2} - e^{2i\theta_2}] + D_{\theta_3}(k)[-e^{-2i\theta_3} + e^{-2i\theta_3}]\right]
\end{bmatrix}
\]  
(S-4)

By employing Fourier shift theorem\(^5\), frequency contents \(C(k \pm k_{\text{app}})\) can be sub-pixelly shifted to their correct locations in the reciprocal space. Thus, the frequency content of resolution-enhanced images can be calculated. Its resolution improvement can be expressed as:

\[
\rho_{\text{app}} = \frac{k+k_{\text{app}}}{k} 
\]  
(S-5)

Here, three polarization angles \((\theta_1=30^\circ, \theta_2=90^\circ \text{ and } \theta_3=150^\circ)\) are selected to reconstructed image, and the phases of cosinoidal structured illumination are about \(\pi/3, \pi\) and \(5\pi/3\), respectively. The radiation wavelength of fluorescent beads (such as Fluorescent dye: CY5) is chosen as 670nm under the excitation of SPPs. The fluorescent signal is detected by an objective with NA = 1. By manipulating the polarization angle of incidence, SPI laterally shifts along \(x\) axis. Three images with illumination phases \(\pi\) and \(\pi\pm2\pi/3\) (Figure S3 (b-d)) are obtained and used to reconstruct the final high-resolution image through the numerical algorithm above. The reconstructed image (Figure S3 (g)) and
calculated OTF (Figure S3 (h)) of the PSIM indicate a greater than 2-fold resolution improvement can be achieved comparing with the conventional fluorescence microscopy (Figure S3 (e-f)).

Figure S3. (a) The fluorescence emission distribution of specimen for imaging. (b-d) Images of specimen with illumination phases $-2\pi/3$, 0, $2\pi/3$. (e, f) The image and OTF of conventional fluorescence microscopy. (g, h) The reconstructed image (Figure S3 (g)) and calculated OTF (Figure S3 (h)) of the PSIM.

Reference


