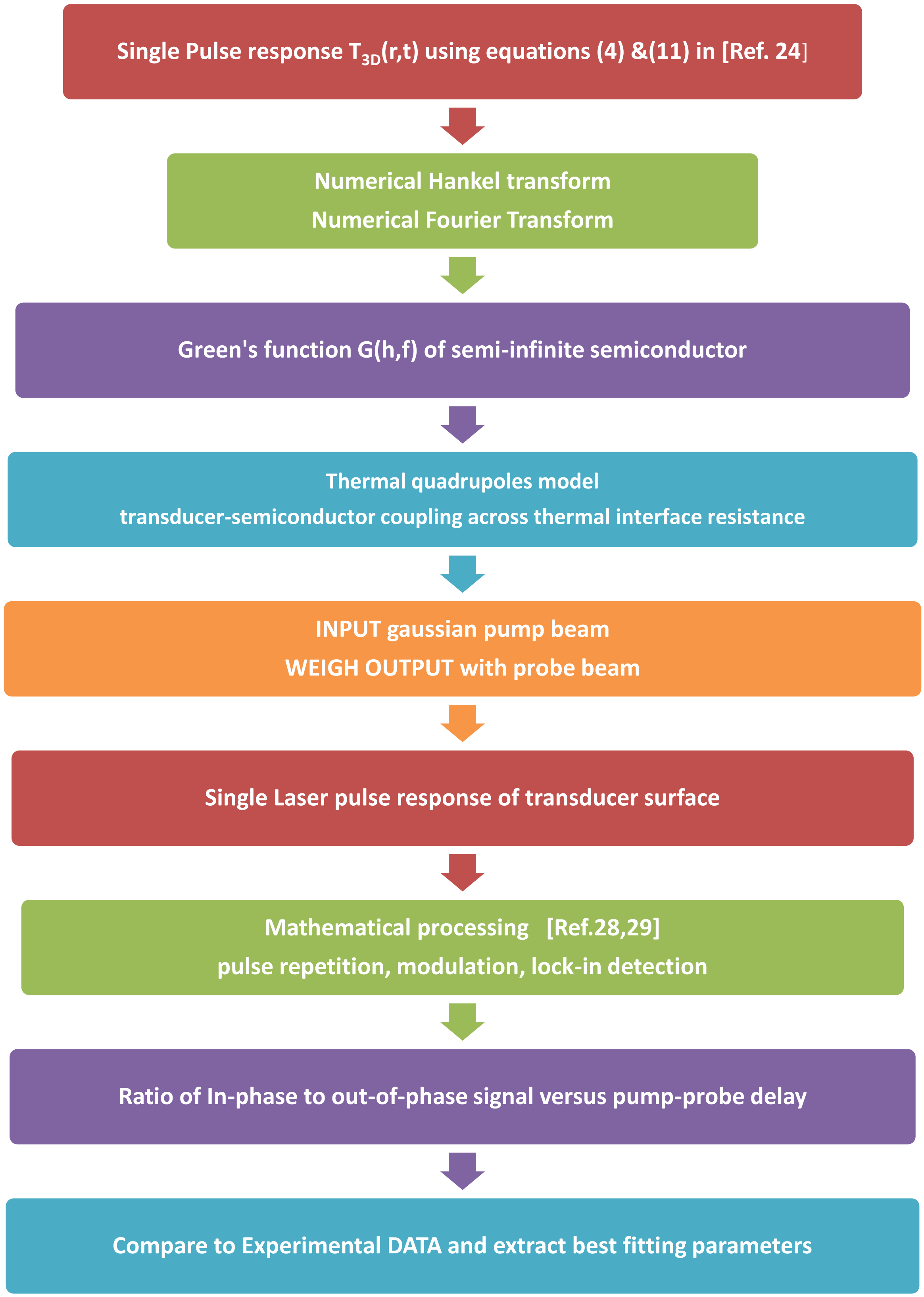
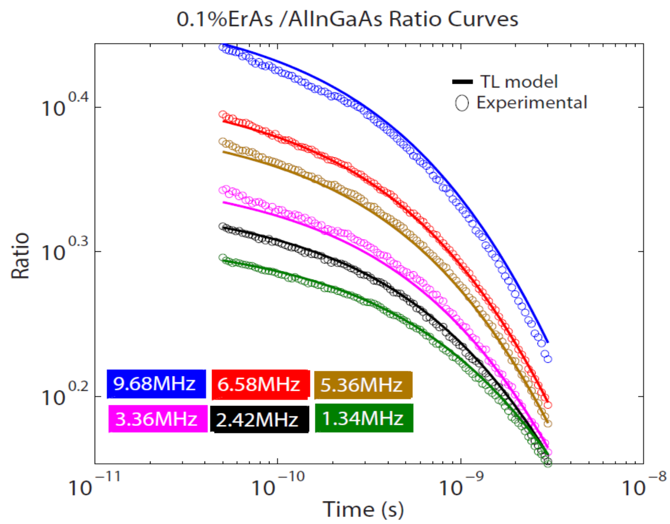
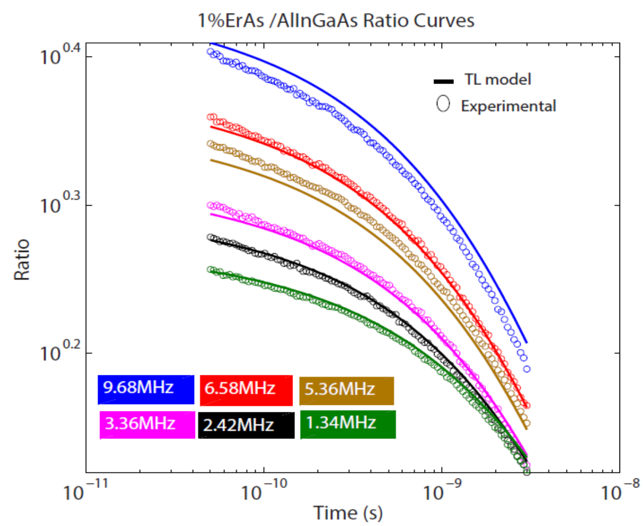
Supplemental materials

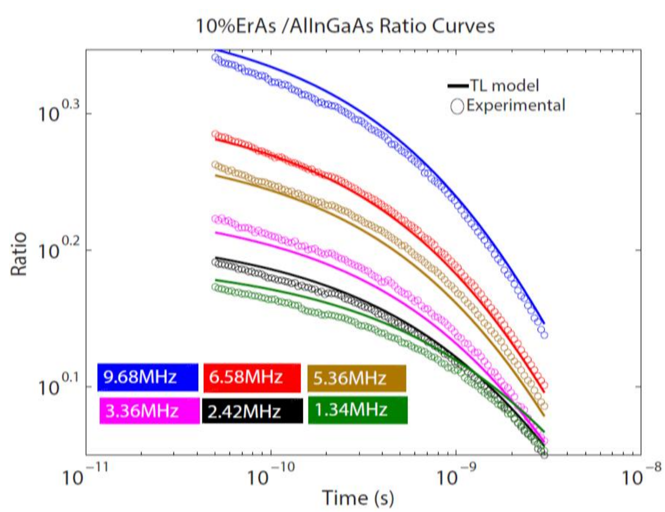
A. Truncated Levy model for TDTR Experiment



B. Raw Data Fittings using Truncate Levy model







C. Error Analysis

A MATLAB function lsqnonlin has been used to Extract TL model paramters by minimizing the error [***F(X)***] between experimental ratio curves and simulated one using TL model.

The Estimation Error is to be evaluated using the Variance –covariance matrix extracted as following:

$$Variance - Covariance\ matrix = Residual\ * (Jacobian * Jacobian^{T})^{-1}$$

The Jacobian is evaluated as following:

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_m}{\partial x_1} & \cdots & \frac{\partial F_m}{\partial x_n} \end{bmatrix}$$

Where **X** will be TL parameters and F_m is the error function at certain delay time.

Residual is the difference between the Experimental ratio curves and simulated one at the optimum extracted parameters.

$$Estimation\ Error = \sqrt{Diag(var - covar\ matrix)}$$

Nanoparticles concentration(%)	σ_{Alpha}	σ_{uBD}	σ_{k}
0(Control Sample)	0.015	0.637	0.075
0.1	0.022	0.110	0.049
0.5	0.068	0.180	0.114
1	0.046	0.107	0.087
5	0.089	0.016	0.062
10	0.113	0.029	0.066

Covariance between α and u_{BD}

The covariance for control sample is higher than all nanoparticles sample except 0.5% (almost equal). However, Error map for the 0.5% sample show the confidence area to be confined and kind of far from hyperbolic shape (sign for correlation). It worth saying that for our nonlinear problem, the covariance is not ultimate measure to the correlation but we can make comparative judgments through it. This implies higher correlation for the control sample.

Nanoparticles concentration (%)	$\sigma_{\alpha,uBD}$
0(Control Sample)	9.25E-03
0.1	2.36E-03
0.5	1.17E-02
1	4.68E-03
5	4.66E-04
10	-6.82E-05

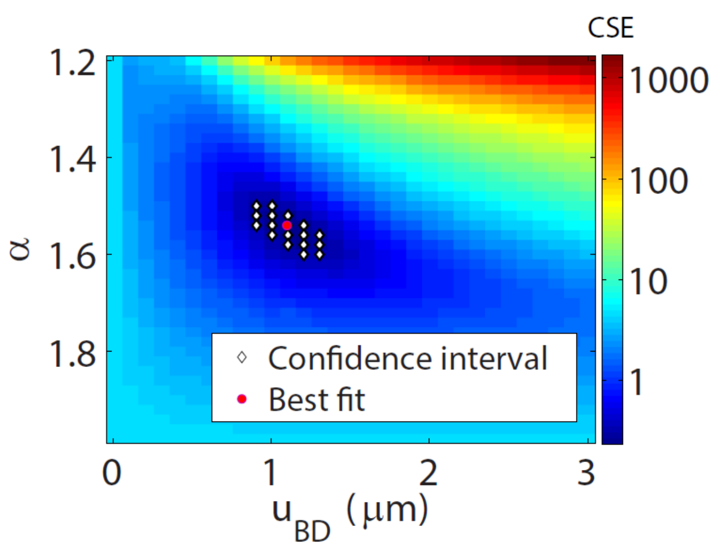
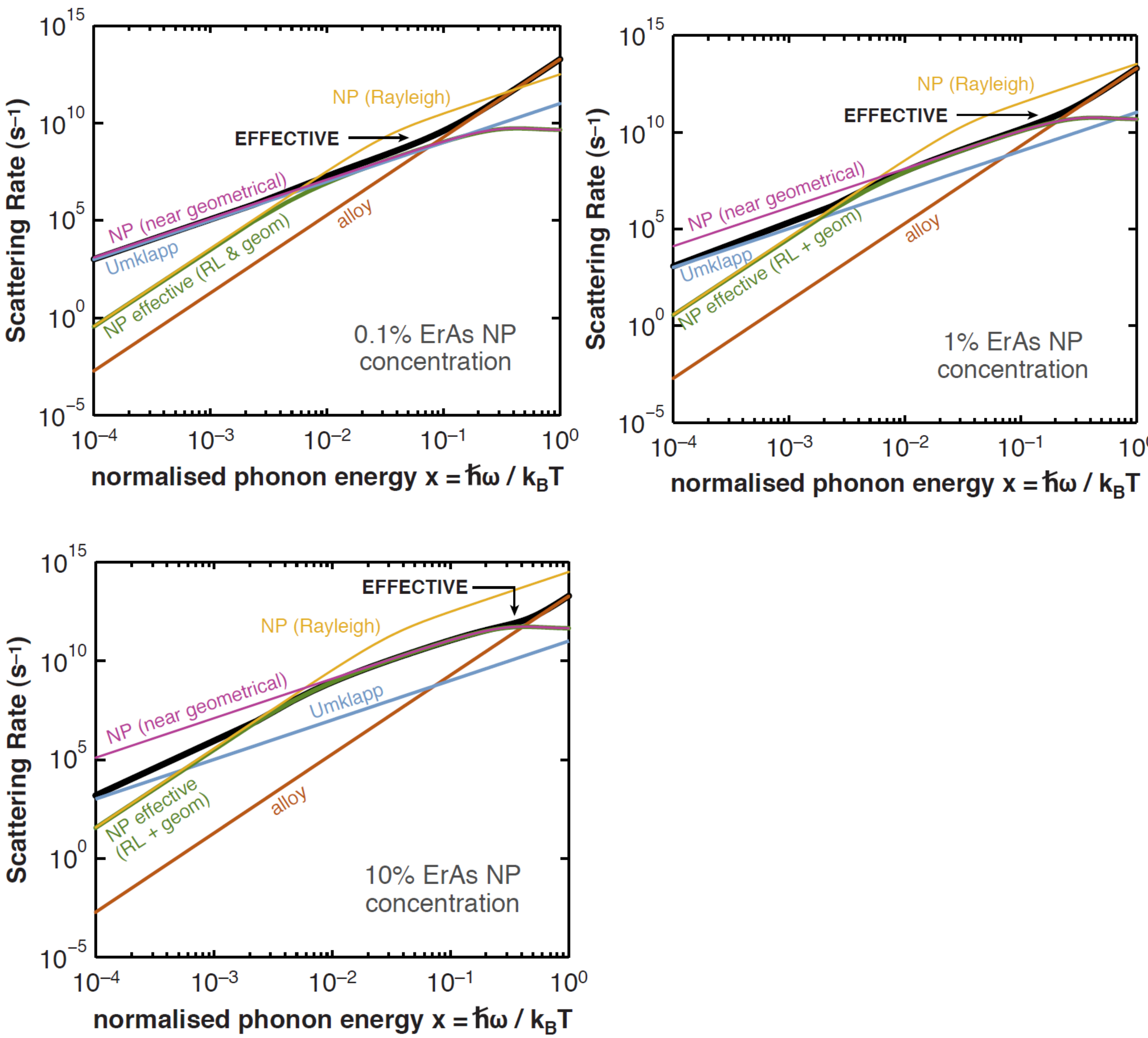


Figure C1: Error map (logarithm of the cumulative square error) with confidence intervals for the extracted TL parameters (α, u_{BD}). Error map for 0.5 % ErAs/AlInGaAs sample. The confidence interval shows that the correlation between α, u_{BD} is very minimal.

D. Scattering mechanisms vs normalized frequency

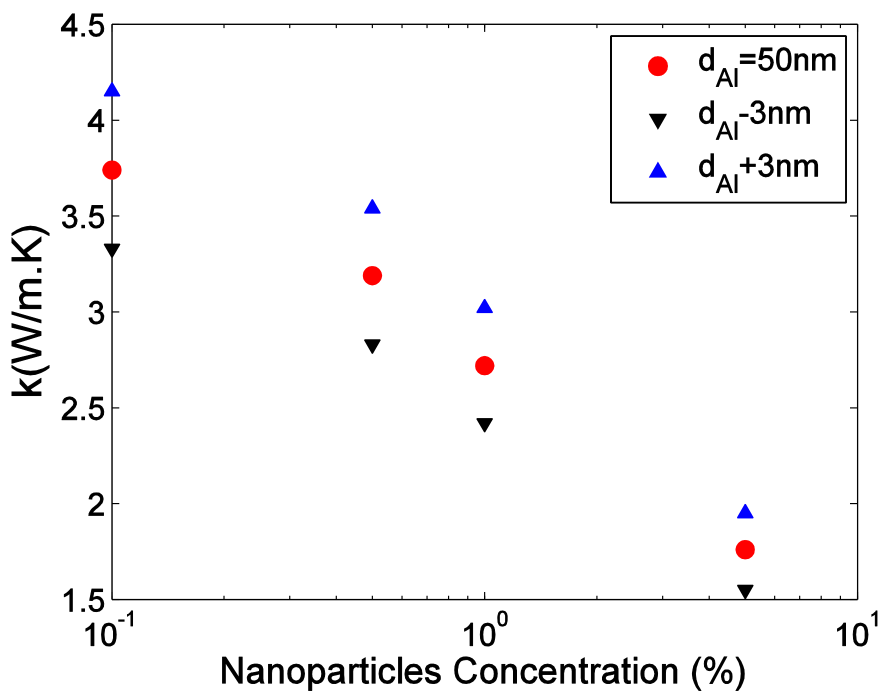
In this section, we present the frequency dependence of scattering rate for different mechanisms. Also, the effective scattering rate using Mathieson's rule is shown. For three nanoparticles concentrations (0.1%,1%,10%), Umklapp mostly dominates at low frequency (relevant range for thermal conductivity at low temperature); while the nanoparticles starts to affect the middle region of the frequency range. Then, alloy scattering dominates at high frequency range.



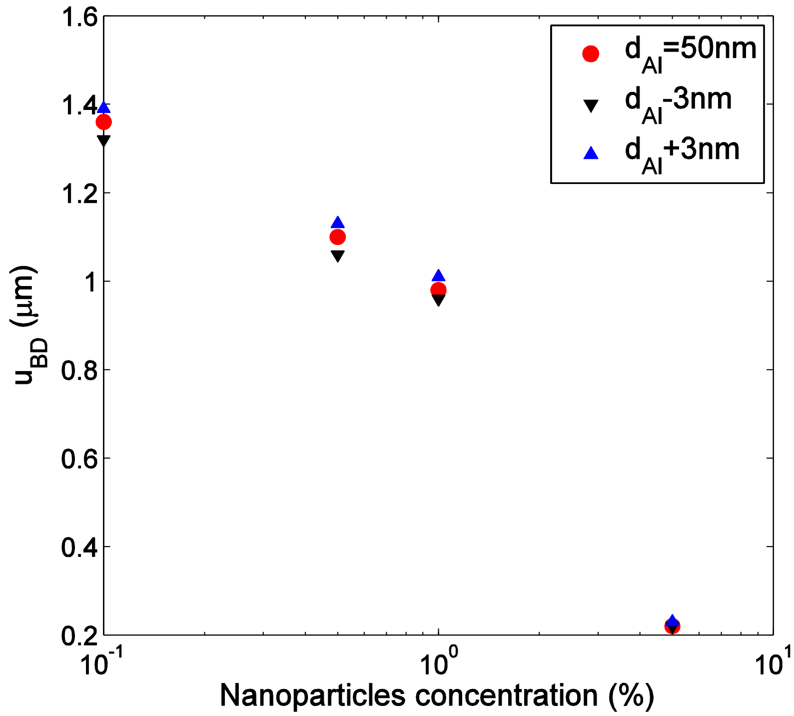
E. The Impact of Al transducer thickness on the Extracted Parameters

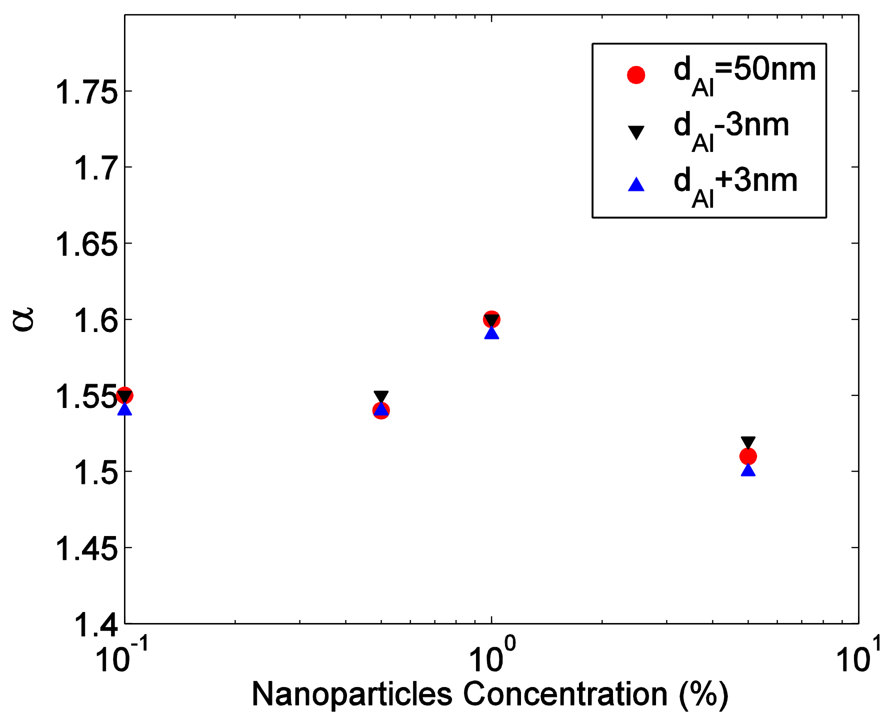
From the sensitivity analysis, Aluminum transducer thickness is known to be the largest source of uncertainty. Even though Al thickness is measured independently using acoustic echoes in the TDTR signal[1], a small error of ~1ps in identifying the location of the acoustic echo will correspond to variation in extracted thermal conductivity by ~10% .

The Al transducer for all ErAs:InGaAlAs samples was deposited at the same time such that any deviation in the desired thickness will be the same for all samples. However, it is still very useful to study how the variation of the Al thickness will impact the extracted parameters. The nominal thickness of the Al transducer layer was 50nm based on electron beam evaporator setting. Acoustic echo in the TDTR signal was at the same location for all ErAs samples and it showed an Aluminum thickness of 50.1nm. In the figures below, we have run the fitting algorithm for Al thicknesses larger and smaller than the nominal thickness by 3nm (6%). For the extracted thermal conductivity, the variations by 11-12% are almost twice as the variation in the Al thickness. This is similar to the results obtained in Fourier parameter identification. For the ballistic-diffusive transition length (u_{BD}), the estimated number is within 4%. This can be understood from its relation with the effective mean-free-path that should vary in response to the thermal conductivity variation. More importantly, the fractal Levy exponent (α) is robust against the variations in the Al thickness (<0.6%).



(1) Koh, Y. K.; Singer, S. L.; Kim, W.; Zide, J. M. O.; Lu, H.; Cahill, D. G.; Majumdar, A.; Gossard, A. C. *J. Appl. Phys.* **2009**, *105*, 054303.





*Note: The 10% ErAs nanoparticle results are not shown above. It turns out that varying Al layer by 3nm, will introduce large errors in extracted Levy transport parameters for this particular sample. We have observed that error bounds for samples with short range superdiffusive transport are relatively large (see Error maps & Error bars in the main manuscript). In cases where Lévy transport happens over very small length scales (short u_{BD}), it is hard to extract non-diffusive characteristics of the transport with narrow confidence interval.