From Discrete Measurements to Bounded Gradient Estimates: A Look at Some Regularizing Structures (Supporting Information)

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Results of Study 1

The following are the results of Study 1 for the test functions $f_2(\mathbf{u})$, $f_3(\mathbf{u})$, $f_5(\mathbf{u})$, $f_6(\mathbf{u})$.

Table S 1: Results of Study 1 for $f_2(\mathbf{u})$.

	Linear	Quadratic	Convex/Concave	DBC
$\sigma = 0.1$	$\alpha_a = 0.9931 \pm 0.0119$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 0.9879 \pm 0.0138$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9477 \pm 0.0327$	$\alpha_p = 0.9833 \pm 0.0050$	$\alpha_p = 0.7865 \pm 0.1599$	$\alpha_p = 0.0000 \pm 0.0000$
$\sigma = 0.4$	$\alpha_a = 0.9843 \pm 0.0214$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 0.9715 \pm 0.0241$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9435 \pm 0.0346$	$\alpha_p = 0.9390 \pm 0.0181$	$\alpha_p = 0.7832 \pm 0.1562$	$\alpha_p = 0.0000 \pm 0.0000$
$\sigma = 0.7$	$\alpha_a = 0.9755 \pm 0.0301$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 0.9608 \pm 0.0303$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9450 \pm 0.0295$	$\alpha_p = 0.8898 \pm 0.0321$	$\alpha_p = 0.7505 \pm 0.1441$	$\alpha_p = 0.0000 \pm 0.0000$
$\sigma = 1.0$	$\alpha_a = 0.9567 \pm 0.0424$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 0.9449 \pm 0.0318$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9520 \pm 0.0219$	$\alpha_p = 0.8271 \pm 0.0514$	$\alpha_p = 0.7393 \pm 0.1460$	$\alpha_p = 0.0000 \pm 0.0000$

Table S 2: Results of Study 1 for $f_3(\mathbf{u})$.

	Linear	Quadratic	Convex/Concave	DBC
$\sigma = 0.1$	$\alpha_a = 0.9781 \pm 0.0175$	$\alpha_a = 0.9965 \pm 0.0028$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9320 \pm 0.1485$	$\alpha_p = 0.9549 \pm 0.1020$	$\alpha_p = 0.8433 \pm 0.0852$	$\alpha_p = 0.0418 \pm 0.0498$
$\sigma = 0.4$	$\alpha_a = 0.9880 \pm 0.0123$	$\alpha_a = 0.9937 \pm 0.0068$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9399 \pm 0.1010$	$\alpha_p = 0.9770 \pm 0.0378$	$\alpha_p = 0.7745 \pm 0.0968$	$\alpha_p = 0.0032 \pm 0.0097$
$\sigma = 0.7$	$\alpha_a = 0.9892 \pm 0.0155$	$\alpha_a = 0.9902 \pm 0.0083$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9552 \pm 0.0581$	$\alpha_p = 0.9809 \pm 0.0149$	$\alpha_p = 0.7459 \pm 0.1038$	$\alpha_p = 0.0003 \pm 0.0023$
$\sigma = 1.0$	$\alpha_a = 0.9866 \pm 0.0199$	$\alpha_a = 0.9878 \pm 0.0127$	$\alpha_a = 1.0000 \pm 0.0000$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9633 \pm 0.0280$	$\alpha_p = 0.9783 \pm 0.0163$	$\alpha_p = 0.7089 \pm 0.1109$	$\alpha_p = 0.0000 \pm 0.0000$

Table S 3: Results of Study 1 for $f_5(\mathbf{u})$.

	Linear	Quadratic	Convex/Concave	DBC
$\sigma = 0.1$	$\alpha_a = 0.9941 \pm 0.0123$	$\alpha_a = 0.9935 \pm 0.0134$	$\alpha_a = 0.9917 \pm 0.0348$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9718 \pm 0.0196$	$\alpha_p = 0.9544 \pm 0.0412$	$\alpha_p = 0.8378 \pm 0.1500$	$\alpha_p = 0.0000 \pm 0.0000$
$\sigma = 0.4$	$\alpha_a = 0.9819 \pm 0.0213$	$\alpha_a = 0.9743 \pm 0.0304$	$\alpha_a = 0.9939 \pm 0.0169$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9662 \pm 0.0180$	$\alpha_p = 0.9387 \pm 0.0387$	$\alpha_p = 0.7288 \pm 0.1443$	$\alpha_p = 0.0000 \pm 0.0000$
$\sigma = 0.7$	$\alpha_a = 0.9705 \pm 0.0259$	$\alpha_a = 0.9627 \pm 0.0357$	$\alpha_a = 0.9984 \pm 0.0080$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9671 \pm 0.0148$	$\alpha_p = 0.9271 \pm 0.0373$	$\alpha_p = 0.5195 \pm 0.1222$	$\alpha_p = 0.0000 \pm 0.0000$
$\sigma = 1.0$	$\alpha_a = 0.9675 \pm 0.0305$	$\alpha_a = 0.9886 \pm 0.0166$	$\alpha_a = 0.9992 \pm 0.0047$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9260 \pm 0.0098$	$\alpha_p = 0.7980 \pm 0.0653$	$\alpha_p = 0.3705 \pm 0.0950$	$\alpha_p = 0.0000 \pm 0.0000$

Table S 4: Results of Study 1 for $f_6(\mathbf{u})$.

	Linear	Quadratic	Convex/Concave	DBC
$\sigma = 0.1$	$\alpha_a = 0.9963 \pm 0.0050$	$\alpha_a = 0.9993 \pm 0.0023$	$\alpha_a = 0.9994 \pm 0.0022$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9656 \pm 0.0201$	$\alpha_p = 0.8453 \pm 0.2119$	$\alpha_p = 0.8791 \pm 0.1061$	$\alpha_p = 0.0639 \pm 0.0723$
$\sigma = 0.4$	$\alpha_a = 0.9938 \pm 0.0110$	$\alpha_a = 0.9960 \pm 0.0110$	$\alpha_a = 0.9992 \pm 0.0030$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9505 \pm 0.0381$	$\alpha_p = 0.8924 \pm 0.1546$	$\alpha_p = 0.8504 \pm 0.1058$	$\alpha_p = 0.0411 \pm 0.0528$
$\sigma = 0.7$	$\alpha_a = 0.9933 \pm 0.0149$	$\alpha_a = 0.9924 \pm 0.0144$	$\alpha_a = 0.9981 \pm 0.0076$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.9441 \pm 0.0485$	$\alpha_p = 0.9274 \pm 0.0776$	$\alpha_p = 0.8326 \pm 0.1103$	$\alpha_p = 0.0300 \pm 0.0428$
$\sigma = 1.0$	$\alpha_a = 0.9835 \pm 0.0301$	$\alpha_a = 0.9963 \pm 0.0088$	$\alpha_a = 0.9997 \pm 0.0024$	$\alpha_a = 1.0000 \pm 0.0000$
	$\alpha_p = 0.8735 \pm 0.0136$	$\alpha_p = 0.7053 \pm 0.0929$	$\alpha_p = 0.3161 \pm 0.0867$	$\alpha_p = 0.0000 \pm 0.0000$