1 Dye chemical structures

2 pH influence

![Graph showing the influence of pH on Carmine Indigo removal.](image)

Fig. 2. Influence of pH on Carmine Indigo removal

3 Linearization of Langmuir and Freundlich models

The Langmuir equation may be linearized in the following way:

$$q_c = \frac{k_{l1} C_l}{1 + k_{l2} C_l} \Rightarrow \frac{1}{q_c} = \frac{1}{k_{l1} C_l} = \frac{C_l}{q_c} = \frac{1}{k_{l1}} + \frac{k_{l2}}{k_{l1}} \cdot C_l$$

On the other hand, the Freundlich equation may be linearized in the following way:

$$q_c = k_f \cdot C_l^{n_f} \Rightarrow \ln q_c = \ln k_f + n_f \cdot \ln C_l$$
5 Design of experiments

In order to determine if there exist a relationship between the factors and the response variables investigated, the data collected must be analyzed in a statistically manner using regression. In developing the regression equation, the test factors were coded according to equation 1:

\[
\chi_i = \frac{X_i - X_i^x}{\Delta X_i}
\] (1)

where \(\chi_i\) is the coded value of the \(i\)th independent variable, \(X_i\) the natural value of the \(i\)th independent variable, \(X_i^x\) the natural value of the \(i\)th independent variable at the center point and \(\Delta X_i\) is the value of the step change.

Each response \(Y\) can be represented by a mathematical equation that correlates the response surface (equation 2):

\[
Y = b_0 + \sum_{j=1}^{k} b_j \chi_j + \sum_{i,j=1}^{k} b_{ij} \chi_i \chi_j + \sum_{j=1}^{k} b_{jj} \chi_j^2
\] (2)

where \(Y\) is the predicted response, \(b_0\) the offset term, \(b_j\) the linear effect, \(b_{ij}\) the first-order interaction effect, \(b_{jj}\) the squared effect and \(k\) is the number of independent variables.
We have selected a Central Composite Design (CCD) which is one of the most popular class of second-order design. It involves the use of a two-level factorial design with $2^k$ points combined with $2k$ axial points and $n$ center runs, $k$ being the number of factors. The total number of experiments, $N$, with $k$ factors is:

$$N = 2^k + 2 \cdot k + n$$

(3)

$n$ is considered to be 8 and the axial distance is $\sqrt{2}$ in order to guarantee an orthogonal and rotatable design.

5.1 Graphical analysis for the design of experiments

5.1.1 Interaction of factors

Fig. 4. Interaction graphic for both variables: temperature and initial dye concentration

* Initial Dye Concentration

Fig. 4. Interaction graphic for both variables: temperature and initial dye concentration
5.1.2 Main effects

Fig. 5. Main effects graphic for both variables: temperature and initial dye concentration

* Initial Dye Concentration