Supporting Information for
Efficient Generation of Propagating Plasmons by Electron Beams

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Analytical Formalism for Plasmon Creation in Thin Films and Buried Planar Cavities

We provide here details on an analytical formalism to calculate plasmon excitation yields for an electron crossing two parallel interfaces under normal incidence. This formalism is an extension of part of Ref. 1, which is supplemented to isolate the contribution of surface plasmons to the electromagnetic field produced by the electron.

We first consider the electric field set up by an electron (charge $-e$) crossing the interfaces with constant velocity v along the normal direction z,

$$\mathbf{E}(\mathbf{r},t)=\int\frac{d^2\mathbf{k}_\parallel d\omega}{(2\pi)^3}\,\mathrm{e}^{\mathrm{i}\mathbf{k}_\parallel\cdot\mathbf{R}-\mathrm{i}\omega t}\,\mathbf{E}(\mathbf{k}_\parallel,z,\omega),$$

where the field is expressed in terms of parallel wavevector and frequency components, $\mathbf{k}_\parallel$ and ω, respectively, and $\mathbf{R}=(x,y)$. The field can be in turn separated into contributions arising from each medium j, described as an external electron field in an infinite bulk material (with $j=1,2,$ and 3 representing the materials below, inside, and above the intermediate layer, respectively, as shown in Figure 1) plus the field reflected at the film boundaries (i.e., the contribution of induced boundary charges and currents),

$$\mathbf{E}=\mathbf{E}_j^{\rm ext}+\mathbf{E}_j^{\rm ref}.$$

The former admits the expression (in Gaussian units)

$$\mathbf{E}_j^{\rm ext}(\mathbf{k}_\parallel,z,\omega)=\frac{4\pi\mathrm{i}\,e\,(\mathbf{q}-\mathbf{v}k\epsilon_j/c)}{v\epsilon_j(q^2-k^2\epsilon_j)}\,\mathrm{e}^{\mathrm{i}\omega z/v},$$

where $\mathbf{q}=(\mathbf{k}_\parallel,\omega/v)$, $k=\omega/c$, and ϵ_j is the dielectric function of medium j. Similarly, the magnetic field in bulk medium j reduces to

$$\mathbf{H}_j^{\rm ext}(\mathbf{k}_\parallel,z,\omega)\;=\;\frac{4\pi\mathrm{i}\,e\,k_\parallel\hat{\mathbf{t}}}{c(q^2-k^2\epsilon_j)}\,\mathrm{e}^{\mathrm{i}\omega z/v},$$

where $\hat{\mathbf{t}}$ is a vector parallel to the interfaces and perpendicular to $\mathbf{k}_\parallel$, so that the set $\{\hat{\mathbf{k}}_\parallel,\hat{\mathbf{t}},\hat{\mathbf{z}}\}$ forms a positively oriented 3D reference frame. The bulk fields are evanescent away from the trajectory, unless ϵ_j is a positive real number and $v>c/\sqrt{\epsilon_j}$, in which case Cherenkov radiation can be produced.

The reflected component of the fields finds its sources in the interfaces, so it can be expressed in terms of boundary currents and charges. The charges can be in turn expressed in terms of the currents by using the continuity equation. We find

$$\mathbf{E}^{\rm ref}(\mathbf{k}_\parallel,z,\omega)=\frac{2\pi\mathrm{i}k}{\Gamma_j}\times\begin{cases}\mathrm{e}^{\Gamma_1 z}\left[\mathbf{h}_1-\frac{1}{k^2\epsilon_1}(\mathbf{k}_\parallel,-\mathrm{i}\Gamma_1)\;(\mathbf{k}_\parallel\cdot\mathbf{h}_1)\right], & j=1\\[2ex] \mathrm{e}^{-\Gamma_2 z}\left[\mathbf{h}_2-\frac{1}{k^2\epsilon_2}(\mathbf{k}_\parallel,\mathrm{i}\Gamma_2)\;(\mathbf{k}_\parallel\cdot\mathbf{h}_2)\right]+\\ \mathrm{e}^{\Gamma_2(z-a)}\left[\mathbf{h}_3-\frac{1}{k^2\epsilon_2}(\mathbf{k}_\parallel,-\mathrm{i}\Gamma_2)\;(\mathbf{k}_\parallel\cdot\mathbf{h}_3)\right], & j=2\\[2ex] \mathrm{e}^{-\Gamma_3(z-a)}\left[\mathbf{h}_4-\frac{1}{k^2\epsilon_3}(\mathbf{k}_\parallel,\mathrm{i}\Gamma_3)\;(\mathbf{k}_\parallel\cdot\mathbf{h}_4)\right], & j=3\end{cases}$$

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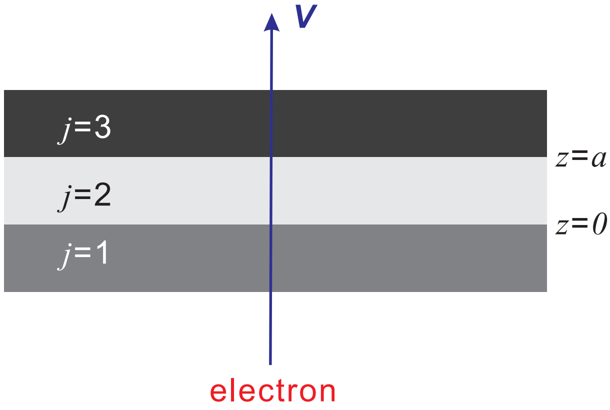

FIG. 1: Schematic representation of an electron moving along the positive z direction with velocity v and crossing two parallel interfaces situated in the planes $z = 0$ and $z = a$, respectively. The homogeneous materials that these interfaces separate are labeled $j = 1, 2$, and 3, as shown by text insets.

and

$$\mathbf{H}_j^{\rm ref}(\mathbf{k}_\parallel, z, \omega) = \frac{2\pi{\rm i}}{\Gamma_j} \times \begin{cases} {\rm e}^{\Gamma_1 z}(\mathbf{k}_\parallel, -{\rm i}\Gamma_1) \times \mathbf{h}_1, & j = 1 \\ {\rm e}^{-\Gamma_2 z}(\mathbf{k}_\parallel, {\rm i}\Gamma_2) \times \mathbf{h}_2 + {\rm e}^{\Gamma_2(z-a)}(\mathbf{k}_\parallel, -{\rm i}\Gamma_2) \times \mathbf{h}_3, & j = 2 \\ {\rm e}^{-\Gamma_3(z-a)}(\mathbf{k}_\parallel, {\rm i}\Gamma_3) \times \mathbf{h}_4, & j = 3 \end{cases}$$

where $\Gamma_j = \sqrt{k_\parallel^2 - k^2\epsilon_j}$, the square root is chosen to yield positive real parts, $\mathbf{h}_i \perp \hat{\mathbf{z}}$, and a is the thickness of the intermediate layer (see Figure 1). Here, $\mathbf{h}_1$ and $\mathbf{h}_3$ are boundary currents defined on the bottom side of the lower and upper interfaces of Figure 1, respectively, whereas $\mathbf{h}_2$ and $\mathbf{h}_4$ are defined on the top side of these interfaces. The continuity of the parallel components of the electric and magnetic fields allows us to determine the boundary currents, which turn out to only have components along $\mathbf{k}_\parallel$, such that $\mathbf{h}_i = h_i\,\hat{\mathbf{k}}_\parallel$, with the coefficients h_i satisfying

$$\begin{pmatrix} -\Gamma_1\epsilon_2 & \Gamma_2\epsilon_1 & \Gamma_2\epsilon_1{\rm e}^{-\Gamma_2 a} & 0 \\ 1 & 1 & -{\rm e}^{-\Gamma_2 a} & 0 \\ 0 & \Gamma_2\epsilon_3{\rm e}^{-\Gamma_2 a} & \Gamma_2\epsilon_3 & -\Gamma_3\epsilon_2 \\ 0 & {\rm e}^{-\Gamma_2 a} & -1 & -1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix} = \frac{-2ek_\parallel}{v} \times \begin{pmatrix} k\left(\frac{\epsilon_2}{q^2-k^2\epsilon_1} - \frac{\epsilon_1}{q^2-k^2\epsilon_2}\right) \\ \frac{{\rm i}v}{c}\left(\frac{1}{q^2-k^2\epsilon_1} - \frac{1}{q^2-k^2\epsilon_2}\right) \\ k{\rm e}^{{\rm i}\omega a/v}\left(\frac{\epsilon_2}{q^2-k^2\epsilon_3} - \frac{\epsilon_3}{q^2-k^2\epsilon_2}\right) \\ \frac{{\rm i}v}{c}{\rm e}^{{\rm i}\omega a/v}\left(\frac{1}{q^2-k^2\epsilon_3} - \frac{1}{q^2-k^2\epsilon_2}\right) \end{pmatrix}.$$

This implies that the electric field is entirely made of TM waves.

Finally, we isolate the contribution of surface plasmons from the above solution by separating the plasmon poles in the $\mathbf{k}_\parallel$ integral. This is the so-called plasmon-pole approximation [2, 3]. More precisely, the determinant of the above linear system of equations can have two plasmonic solutions $k_\parallel = k_{\parallel,n}$ (with $n = 1, 2$) for either a thin film (if $j = 2$ is a metal and $j = 1, 3$ are dielectrics) or a metal waveguide (if $j = 2$ is a dielectric and $j = 1, 3$ are metals). This allows us to approximate the coefficients h_i as

$$h_i \approx \sum_{n=1,2} \frac{A_{in}}{k_\parallel - k_{\parallel,n}}, \tag{1}$$

where A_{in} is independent of $k_\parallel$ and the roots $k_{\parallel,n}$ are obtained upon numerical solution of the above equations. The dispersion relations of the surface plasmons in these systems (e.g., those shown in Figures 1-3 of the main paper) are obtained by representing $k_{\parallel,n}$ as a function of free-space wavelength $\lambda = 2\pi/k$. Plasmon excitation yields are separately obtained for each of the modes by (1) inserting the plasmon-pole expression back into the boundary components of the fields; (2) performing the azimuthal integral of $\mathbf{k}_\parallel$ to yield Bessel functions with $k_\parallel R$ argument; (3) approximating the integral over $k_\parallel$ by using the $k_\parallel = k_{\parallel,n}$ pole of Eq. (1), so that the Bessel functions become Hankel functions; (4) retaining the large $|k_{\parallel,n}R|$ limit, which yields an overall $\exp({\rm i}k_{\parallel,n}R)/\sqrt{R}$ factor in the fields; and (5) using the remaining coefficients that multiply that factor to calculate the flux of electromagnetic energy propagating along the surface, away from the electron trajectory. We find:

$$
\begin{aligned}
\Gamma_n^{SP}(\omega) \;=\; & \frac{1}{\hbar k^2}\left(\frac{|A_{1n}|^2}{2\mathrm{Re}\{\Gamma_1\}}\mathrm{Re}\left\{\frac{k_{\parallel,n}|k_{\parallel,n}|}{\epsilon_1}\right\}\right. \\
& +\frac{|A_{4n}|^2}{2\mathrm{Re}\{\Gamma_3\}}\mathrm{Re}\left\{\frac{k_{\parallel,n}|k_{\parallel,n}|}{\epsilon_3}\right\} \\
& \left.+\left[\left(1-\mathrm{e}^{-2\mathrm{Re}\{\Gamma_2\}a}\right)\,\frac{\left(|A_{3n}|^2+|A_{2n}|^2\right)}{2\mathrm{Re}\{\Gamma_2\}}+\frac{\mathrm{Im}\left\{\left(1-\mathrm{e}^{2\mathrm{i}\mathrm{Im}\{\Gamma_2\}a}\right)\,\mathrm{e}^{-\Gamma_2 a}A_{3n}A_{2n}^*\right\}}{\mathrm{Im}\{\Gamma_2\}}\right]\,\mathrm{Re}\left\{\frac{k_{\parallel,n}|k_{\parallel,n}|}{\epsilon_2}\right\}\right).
\end{aligned}
\tag{2}
$$

It should be mentioned that the approximation of Eq. (1) is fully justified in the large R limit regime that we have examined to derive the plasmon energy flux far away from the electron trajectory [3].

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