Supporting Material

The method we describe builds on work by Cahn [9] aimed at producing three-dimensional structures formed by spinodal decomposition and extended by Berk [10,11] and others [12-14,16-18] to analyze scattering data from bicontinuous structures. Here a Gaussian random function \( \Psi(r) \) is generated by the superposition of many sinusoidal waves with random directions and phases. This function is then clipped to produce a representative three-dimensional morphology: all points \( r \) with \( \Psi(r) > \alpha \) are assigned to one phase, and all other points are in the other phase. The clipping parameter \( \alpha \) is determined by the porosity. The wavevectors for the sinusoidal waves are obtained from a spectral function \( f(k) \), which gives the characteristic wavelengths of the physical morphology and thus describes the essential morphological features. Often [9-12] there is an assumed form for the spectral function. Rather than using this assumed form (which failed to describe our data), we have directly obtained the spectral function from the scattering data using the method from Ref. [13]. This insures that the generated morphology yields X-ray scattering consistent with the experimental data and that the length scales of the morphological features of the two-phase material are faithfully reproduced.

The analysis procedure is similar to that in Refs. [16] and [17] and is illustrated in Fig. 4 for 40% porogen loading. First, the SAXS data \( I(q) \) are extrapolated to high \( q \) as \( q^{-4} \). Although this is not necessary, it prevents ringing in subsequent steps. The \( I(q) \) is then Fourier transformed to produce the Debye correlation function \( \Gamma^a(r) \),

\[
\Gamma^a(r) = \int_0^\infty 4\pi q^2 I(q)[\sin(qr)/(qr)]dq. \tag{1}
\]
Here \( q = (4\pi/\lambda)\sin(\theta/2) \), where \( \lambda \) and \( \theta \) are the X-ray wavelength and the scattering angle, respectively. This function is normalized such that \( \Gamma^a(\infty) = (\Gamma^a(0))^2 \). It is related to the correlation function, \( g(r) \), of \( \Psi(r) \) by [29]

\[
\Gamma^a(r) = \Gamma^a(0) - \frac{(1)}{2\pi \varrho(r)} \int \frac{\exp[-\alpha^2/(1+t)]}{\sqrt{1-t^2}} dt, \tag{2}
\]

where \( \Gamma^a(0) \) is the volume fraction of one of the phases (e.g., the porosity) and hence is known. It is related to the clipping parameter \( \alpha \) by

\[
\Gamma^a(0) = \frac{1}{\sqrt{2\pi \alpha}} \int \exp[-x^2/2] dx. \tag{3}
\]

Equations (1)-(3) permit determination of both \( \alpha \) and \( g(r) \). The spectral function, \( f(k) \), of the Gaussian random function \( \Psi(r) \) is then obtained by the inverse Fourier transform of \( g(r) \):

\[
f(k) = \int_0^\infty 4\pi r^2 g(r)[\sin(kr)/(kr)] dr. \tag{4}
\]

Hence, \( f(k) \) is obtained from the experimental data with no \textit{a priori} assumptions about its functional form. The Gaussian random function \( \Psi(r) \) is generated by summing many (\( N=10000 \)) sinusoidal waves

\[
\Psi(r) = \sqrt{\frac{2}{N}} \sum_{i=1}^N \cos(k_i \cdot r + \varphi_i), \tag{5}
\]

where the directions of wavevectors \( k_i \) and phases \( \varphi_i \) are random, but the magnitudes of wavevectors \( k_i \) are obtained from the spectral function given by Eq. (4). To obtain the two-phase morphologies, the field is then clipped to the corresponding \( \alpha \) calculated in Eq. (3). All points \( r \) with \( \Psi(r) > \alpha \) are assigned to one phase and all other points are assigned to the other phase.
The illustrative SAXS data set in figure 4 is for 40% porogen loading and has the MSSQ contribution subtracted; this contribution dominates the scattering above about 0.3 Å⁻¹, but is negligible (<1%) below about 0.1 Å⁻¹. For the weakest scattering films (5% loading), the MSSQ scattering is still less than <10% of the SAXS from the pores.