APPENDIX

Calculation of Meniscus, Pressures, and Frequency Shift Due to Partial Wetting of a Sinusoidally Corrugated Surface

The surface is assumed to vary sinusoidally in one direction with periodicity $\Lambda$ and amplitude $A$:

$$y = A \cos(kx) \quad (A1)$$

where $k = 2\pi/\Lambda$. At equilibrium, a cylindrical liquid/air meniscus is formed. For minimum surface area, the meniscus is assumed to be circular in cross section, with radius $r$ and centered at $(x_c, y_c)$, as shown in Fig. A1 (a and b). The meniscus makes contact with the surface at $(x_l, y_l)$ at an angle $\theta$ determined by the surface condition. We solve for the meniscus parameters—surface intersection point $(x_l, y_l)$, radius $r$, and center $(x_c, y_c)$—in terms of $x_l$. We next express the capillary pressure and compression pressure of the trapped air bubble in terms of $x_l$. We then find $x_l$ such that the contact angle and pressure constraints are satisfied.

Liquid/Air Meniscus Calculation

The meniscus calculation is divided into two subcases: $\theta \leq \pi/2$ and $\pi/2 < \theta < (\pi - \phi)$, where $\theta$ and $\phi$ are defined in Fig. A1 (a and b).

(a) $\theta \leq \pi/2$. Referring to Fig. A1 (a), the meniscus contacts the surface at point $(x_l, y_l)$ at an angle $\theta$. The tangent line to the surface at this point, $T$, has a slope $m$ given by:

$$m = \frac{dy}{dx} = -Ak \sin(kx_l). \quad (A2)$$

Noting that $\tan(\phi) = -\frac{dy}{dx}$, we can solve for $\phi$ in terms of $x_l$:

$$\phi = \tan^{-1}[Ak \sin(kx_l)]. \quad (A3)$$

Since $\theta + \phi + \psi = \pi/2$, we can solve for $\psi$:

$$\psi = \frac{\pi}{2} - \theta - \tan^{-1}[Ak \sin(kx_l)]. \quad (A4)$$

Then the meniscus is an arc of a circle with radius $r$, given by
\[ r = \frac{x_c - x_1}{\cos(\psi)} \]  \hspace{1cm} (A5)

centered at \((x_c, y_c)\), where \(x_c = \pi/k = \Lambda/2\) and \(y_c = y_1 + (x_c - x_1) \tan(\psi)\).

(b) \(\pi/2 < \theta < (\pi - \phi)\): The angle \(\phi\) remains the same as in case (a). We introduce an angle \(\alpha\) between the tangent line \(T\) and the radius \(r\), as shown in Fig. A1 (b).

\[ \alpha = (\theta - \frac{\pi}{2}) \]  \hspace{1cm} so that \hspace{1cm} (A6)

\[ r = \frac{x_c - x_1}{\cos(\phi + \alpha)} \]  \hspace{1cm} and \hspace{1cm} (A7)

\[ y_c = y_1 - (x_c - x_1) \tan(\phi + \alpha) \]  \hspace{1cm} and \hspace{1cm} \(x_c = \pi/k = \Lambda/2\) as before. \hspace{1cm} (A8)

**Gas Bubble Pressure**

**Capillary pressure:** The capillary pressure of the gas under the meniscus is:

\[ P_1 = \frac{1}{r_1} + \frac{1}{r_2} = \frac{\gamma}{r} \]  \hspace{1cm} (A9)

where \(\gamma\) is the surface tension of the liquid; \(r_1\) and \(r_2\) are the meniscus radii on the \(xy\) and \(yz\) planes, respectively. Since the bubble is assumed cylindrical (unvarying in the \(z\) direction), \(r_2 \rightarrow \infty\) and the equation simplifies to:

\[ P_1 = \frac{\gamma}{r} \]  \hspace{1cm} \hspace{1cm} (A10)

where \(r\) is given by Eqs. A5 and A7.

**Compression pressure:** Assuming air initially lying below the corrugation peaks (cross-sectional area per period \(A_1\)) is compressed to the region below the meniscus (cross-sectional area per period \(A_2\)), then assuming conservation of gas molecules (constant \(PV\)) gives:

\[ P_2 = P_0 \left( \frac{A_1}{A_2} - 1 \right) \]  \hspace{1cm} (A11)

where \(P_0\) is the ambient air pressure and the cross-sectional areas are calculated as follows.

The initial cross-section of the gas pocket is:
\[ A_i = 2A \int_0^{\pi/k} [1 - \cos(kx)] dx = AA. \] (A12)

The area (per period) between the circular meniscus and sinusoidal surface can be divided into two contributions: \( A_2 = A_{2A} + A_{2B} \), where \( A_{2A} \) is the area under the meniscus but above the horizontal line \( H \) and \( \theta' \) is defined in fig A1 (a):

\[ A_{2A} = \frac{r^2}{2} \left[ \theta' - \sin(\theta') \right] \] (A13)

with

\[ \theta' = 2 \tan^{-1} \left( \frac{x - x_i}{y - y_c} \right) \quad \text{for} \; y > y_c \quad \text{or} \quad (A14a) \]

\[ \theta' = \pi \quad \text{for} \; y = y_c \quad \text{or} \quad (A14b) \]

\[ \theta' = 2\pi - 2 \tan^{-1} \left( \frac{x - x_i}{y - y_c} \right) \quad \text{for} \; y < y_c; \quad (A14c) \]

and \( A_{2B} \) is the area between line \( H \) and the sinusoidal surface:

\[ A_{2B} = 2 \int_{x_i}^{x_c} \left[ y_1 - A \cos(kx) \right] dx = 2A \left[ (x_c - x_i) \cos(kx_i) + \frac{1}{k} \sin(kx_i) \right]. \] (A15)

To determine the meniscus parameters, we iteratively find \( x_1 \) \( (0 < x_i < x_c) \) such that \( P_1 = P_2 \), noting that both \( P_1 \) and \( P_2 \) are dependent upon \( x_1 \). With a sinusoidal surface, this uniquely determines a meniscus.

**Frequency shift due to partial wetting**

The frequency shift caused by trapped and entrained fluid is:

\[ \Delta f = -\frac{2f_s^2 \rho}{N(\rho_q \mu_q)^{1/2}} \left( d + \frac{\delta}{2} \right) \] (A16)

where \( d \) is the effective thickness of trapped fluid and \( \delta \) is the decay length in the fluid (Eq. 2).

For partial wetting of a sinusoidal surface, the effective thickness of trapped fluid is:

\[ d = \left( \frac{A_i - A_x}{\Lambda} \right) \] (A17)
where $A_1$ and $A_2$ are the cross-sectional areas calculated above.

Substituting Eq. A17 into Eq. A16 gives the frequency shift due to partial wetting of the sinusoidal surface:

$$-\frac{\Delta f}{\rho} = \frac{2f^2}{N}\left[\frac{A_1 - A_2}{\Lambda} + \left(\frac{\delta}{2}\right)\right]$$  \hspace{1cm} (A18)

**Appendix Fig. 1A**

**Appendix Fig. 1A:** Illustration of geometric variables for the liquid/air meniscus calculation; a) $\theta \leq \pi/2$ case and b) $\pi/2 < \theta < (\pi - \phi)$ case.