

# Analysis of Bitcoin Returns Volatility using AR-GARCH Modelling

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## Abstract

Bitcoin is a virtual/cryptocurrency, serving as a decentralized medium of digital exchange and not tied to any financial institution. It gained in popularity in the aftermath of the global financial crisis with the failure of many prominent banks and financial institutions, as it provided investors with direct control over their money.

The supply of Bitcoins is constrained due to its geometrically decreasing growth rate, with a limiting maximum supply of twenty-one million Bitcoins. Because of the limited number of Bitcoins in circulation and their increasing demand, Bitcoin prices tend to be highly volatile and increase/decrease at a very fast pace. Many noted economists have characterised Bitcoin prices as a speculative bubble. However, it is expected that, with wider acceptance and adoption of Bitcoins, Bitcoin prices would settle down and its volatility would stabilise.

The study examines the stability of Bitcoin price/returns volatility using an AR-GARCH model. The data for the study were the daily closing Bitcoin prices obtained from the bitcoin.com website<sup>1</sup> for the study period 01/01/2013 - 31/12/2017.

**Keywords:** Bitcons, crypto-currency, volatility, speculative bubble, AR-GARCH model, stability.

## Introduction

Bitcoin is the world's first digital cryptocurrency. It was first released as an open source by Satoshi Nakamoto in 2008. Bitcoins serve as a decentralised medium of digital exchange and are not tied to any financial institution. When large banking federations lost control over their management policies, investors shifted to cryptocurrencies to retain control over their money.

Bitcoins are not physically formed at all. They are mined through a process of transaction verification and solving a computational puzzle. The miner, a special user authorised to verify and secure the transactions, who solves the puzzle first gets the opportunity to claim Bitcoins as a reward, which becomes the newly released currency.

Bitcoin transactions are made through a public key as the address (similar to a bank account number). All the transaction details of any Bitcoin address are verified and recorded in a public distributed ledger (the blockchain) without the need for a trusted recordkeeping authority or central intermediary. Transaction blocks contain an SHA-256 cryptographic hash of previous transaction blocks, and are chained together, serving as an immutable record of all transactions that have ever occurred.

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<sup>1</sup> <https://charts.bitcoin.com/chart/price>

## **Problem Identification**

At the end of the last quarter of 2017, there were 16,794,762 Bitcoins in circulation, and 876,989 Bitcoin users (unique addresses); since Bitcoin's inception in 2009, there had been a total of 291,342,443 Bitcoin transactions, representing about 66.8% of the total estimated crypto-currency capitalisation. The number of bitcoins generated per block is set to decrease geometrically, with a 50% reduction every 210,000 blocks, or approximately four years. As a result, the limiting maximum supply of Bitcoins is twenty-one million.

Because of the limited number of Bitcoins in circulation, and the increased demand with each passing day, Bitcoin is highly volatile and increases or decreases at a very fast pace. Chu et al (2017) suggested that cryptocurrencies are highly volatile compared to traditional currencies. Cheah and Fry (2015) suggested that the Bitcoin market is highly speculative as compared to other currencies.

Speculators wish to take advantage of the volatility, but genuine investors think of it as risky; so, many investors do not invest in Bitcoins. Many noted economists, including Nobel laureates Paul Krugman, Robert J. Shiller, Joseph Stiglitz, Richard Thaler, James Heckman, Thomas Sargent, Angus Deaton, and Oliver Hart, have characterised Bitcoin prices as a speculative bubble. However, it is expected that, with wider acceptance and adoption, Bitcoin prices would settle down and its volatility would stabilise.

## **Literature Review**

Several studies have used econometric modelling, specifically GARCH-type models, to analyse Bitcoin prices (Zheng, 2018). Bouoiyour and Selmi (2016) examined Bitcoin volatility using the CMT-GARCH model, which they found to be optimal in terms of AIC, BIC, and HQ criteria. They suggested that Bitcoin volatility had reduced considerably from Jan'15. They also found evidence of asymmetric reaction to negative shocks as against positive shocks. Stavroyiannis and Babalos (2017) examined the dynamic properties of Bitcoin in the US market using univariate and multivariate GARCH models and Vector Auto-Regressive models. They found no evidence to suggest that Bitcoins provide investors with the benefits of hedge, diversifier, or safe-haven with respect to the movement of the US stock markets, in contrast to gold, which is used by investors in order to limit their losses, especially during adverse market conditions. Katsiampa (2017) compared different GARCH models for Bitcoin volatility estimation. He found that the optimal conditional heteroskedasticity model (in the sense of AIC, BIC, and Log-Likelihood criteria) is the AR-CGARCH model, highlighting the significance of including both a short-run and a long-run component of the conditional variance. Chu et al (2017) compared GARCH-type models for the seven most popular cryptocurrencies, viz. Bitcoin, Dash, Dogecoin, Litecoin, Maidsafecoin, Monero, and Ripple. They found that the IGARCH (1, 1) model gives the best fit for Bitcoin, Dash, Litecoin, Maidsafecoin and Monero; the GJRGARCH (1, 1) model gives the best fit for Dogecoin; and the GARCH (1, 1) model gives the best fit for Ripple.

The present study contributes to the literature by attempting to quantify the variation of Bitcoin returns using the AR-GARCH models.

## Methodology

The objective of the study is to understand and model the volatility of Bitcoin returns using univariate time series models.

The data for the study were the daily closing Bitcoin prices obtained from the bitcoin.com website<sup>2</sup> for the study period 01/01/2013 - 31/12/2017. Following Bouoiyour and Selmi (2016), the Bitcoin returns series was analysed in the subperiod 01/01/2015 - 31/12/2017 separately and compared with the overall period. The Bitcoin returns series were computed from the closing prices using the log-returns formula

$$r_t = \ln (B_t/B_{t-1})$$

In order to tests for stationarity of the Bitcoin returns series, the ADF unit root test was applied. The specification of the ADF unit root test used in the study is given by

$$\Delta r_t = \alpha + \beta t + \gamma r_{t-1} + \delta_1 \Delta r_{t-1} + \dots + \delta_k \Delta r_{t-k} + \epsilon_t$$

i.e. with trend and lagged first order differences of the log-returns. The lag structure was selected using the Akaike Information Criterion (AIC) and/or the Bayesian Information Criterion (BIC). The null hypothesis of a unit root is rejected if the coefficient  $\gamma$  is statistically significant and negative.

In order to understand the autocorrelation structure of the Bitcoin returns series, the ACF and PACF correlograms were examined. The Bitcoin returns series was further modelled using the ARMA model. The specification of the ARMA(p,q) model used in the study is given by

$$r_t = a + \phi_1 r_{t-1} + \dots + \phi_p r_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q}$$

The AR and MA lags p and q were selected to minimise the AIC and BIC. The model was estimated using the maximum likelihood method.

In order to understand the volatility dynamics of the Bitcoin returns series, the AR-GARCH model was used. The specification of the AR(p)-GARCH(1, 1) model used in the study is as follows. The conditional mean equation is given by

$$r_t = \mu + \varphi_1 r_{t-1} + \dots + \varphi_p r_{t-p} + \epsilon_t$$

with  $\epsilon_t = \sigma_t h_t$ , where  $\{h_t\}$  is a strong white noise process, and the conditional variance equation is given by

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

The AR lag length p was selected to minimise the AIC and BIC. The model was estimated using the maximum likelihood method.

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<sup>2</sup> <https://charts.bitcoin.com/chart/price>

## Findings

The results of the ADF tests are presented in the tables below.

### Panel A (01/01/2013 - 31/12/2017):

Augmented Dickey-Fuller test for Bitcoin\_returns  
including 19 lags of (1-L)Bitcoin\_returns  
(max was 25, criterion AIC)  
sample size 1806  
unit-root null hypothesis:  $a = 1$

test with constant  
model:  $(1-L)y = b_0 + (a-1)y(-1) + \dots + e$   
estimated value of  $(a - 1)$ : -0.570591  
test statistic:  $\tau_c(1) = -7.25543$   
asymptotic p-value 6.632e-011  
1st-order autocorrelation coeff. for e: -0.000  
lagged differences:  $F(19, 1785) = 4.831 [0.0000]$

with constant and trend  
model:  $(1-L)y = b_0 + b_1*t + (a-1)y(-1) + \dots + e$   
estimated value of  $(a - 1)$ : -0.570544  
test statistic:  $\tau_{ct}(1) = -7.2528$   
asymptotic p-value 4.469e-010  
1st-order autocorrelation coeff. for e: -0.000  
lagged differences:  $F(19, 1784) = 4.829 [0.0000]$

### Panel B (01/01/2015 - 31/12/2017):

Augmented Dickey-Fuller test for Bitcoin\_returns  
including one lag of (1-L)Bitcoin\_returns  
(max was 30, criterion AIC)  
sample size 1094  
unit-root null hypothesis:  $a = 1$

test with constant  
model:  $(1-L)y = b_0 + (a-1)y(-1) + \dots + e$   
estimated value of  $(a - 1)$ : -0.893518  
test statistic:  $\tau_c(1) = -22.5932$   
asymptotic p-value 5.723e-051  
1st-order autocorrelation coeff. for e: 0.002

with constant and trend  
model:  $(1-L)y = b_0 + b_1*t + (a-1)y(-1) + \dots + e$   
estimated value of  $(a - 1)$ : -0.909129  
test statistic:  $\tau_{ct}(1) = -22.8503$   
asymptotic p-value 2.699e-084  
1st-order autocorrelation coeff. for e: 0.001

The results of the ADF tests indicate that the null hypothesis of unit root was rejected for Bitcoin returns in both panels. This implies stationarity of the Bitcoin returns series.

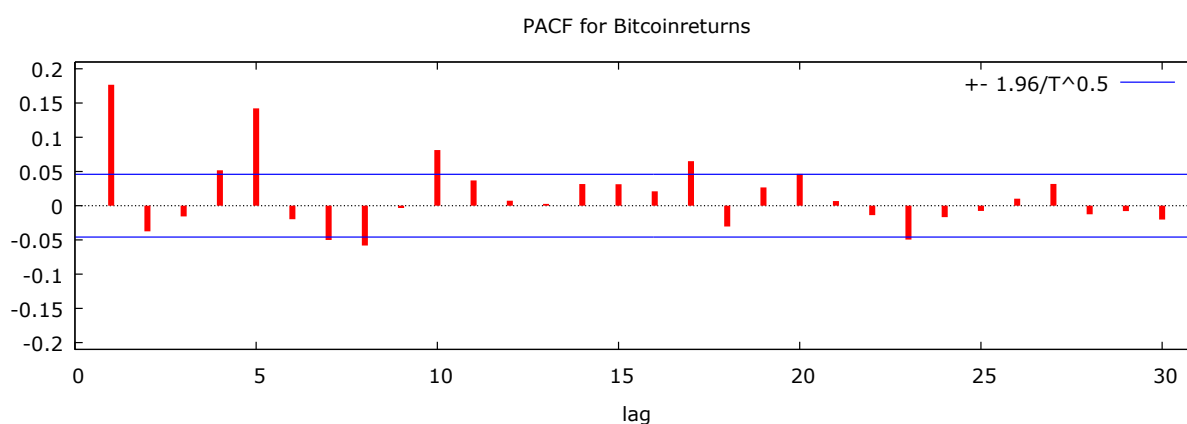
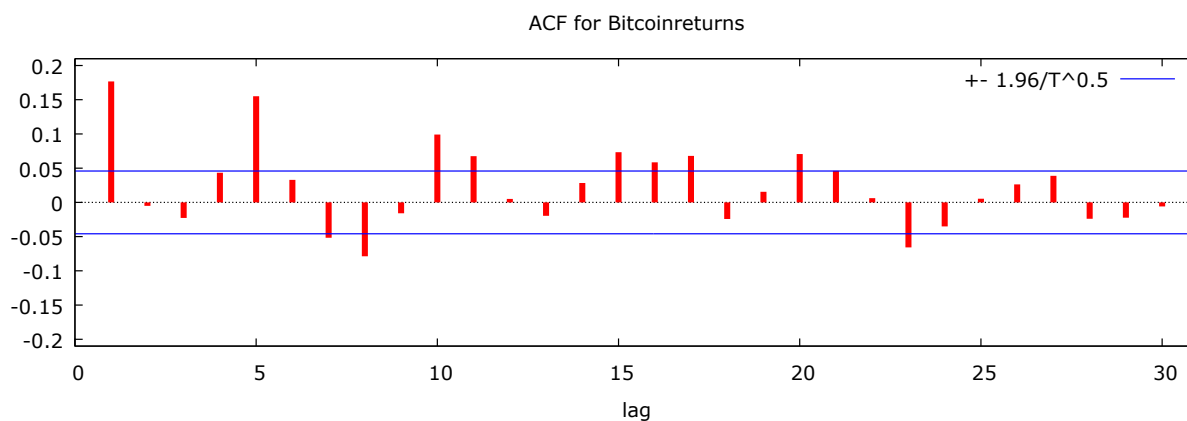
The ACF and PACF correlograms for Bitcoin returns are presented below.

**Panel A (01/01/2013 - 31/12/2017):**

Autocorrelation function for Bitcoin\_returns

\*\*\*, \*\*, \* indicate significance at the 1%, 5%, 10% levels  
using standard error  $1/T^{0.5}$

LAG	ACF		PACF		Q-stat.	[p-value]
1	0.1767	***	0.1767	***	57.1230	[0.000]
2	-0.0051		-0.0375		57.1700	[0.000]
3	-0.0227		-0.0156		58.1110	[0.000]
4	0.0433	*	0.0517	**	61.5491	[0.000]
5	0.1551	***	0.1421	***	105.6306	[0.000]
6	0.0329		-0.0197		107.6104	[0.000]
7	-0.0516	**	-0.0501	**	112.5026	[0.000]
8	-0.0788	***	-0.0581	**	123.8899	[0.000]
9	-0.0159		-0.0034		124.3539	[0.000]
10	0.0991	***	0.0814	***	142.3988	[0.000]
11	0.0675	***	0.0370		150.7857	[0.000]
12	0.0051		0.0073		150.8333	[0.000]
13	-0.0196		0.0026		151.5421	[0.000]
14	0.0284		0.0317		153.0234	[0.000]
15	0.0734	***	0.0314		162.9456	[0.000]
16	0.0585	**	0.0210		169.2615	[0.000]
17	0.0680	***	0.0651	***	177.7829	[0.000]
18	-0.0242		-0.0304		178.8675	[0.000]
19	0.0155		0.0267		179.3127	[0.000]
20	0.0706	***	0.0465	**	188.5265	[0.000]
21	0.0460	**	0.0068		192.4460	[0.000]
22	0.0062		-0.0138		192.5171	[0.000]
23	-0.0658	***	-0.0495	**	200.5267	[0.000]
24	-0.0350		-0.0167		202.8017	[0.000]
25	0.0054		-0.0077		202.8550	[0.000]
26	0.0263		0.0102		204.1367	[0.000]
27	0.0388	*	0.0318		206.9315	[0.000]
28	-0.0240		-0.0126		208.0002	[0.000]
29	-0.0224		-0.0078		208.9317	[0.000]
30	-0.0059		-0.0203		208.9967	[0.000]



**Panel B (01/01/2015 - 31/12/2017):**

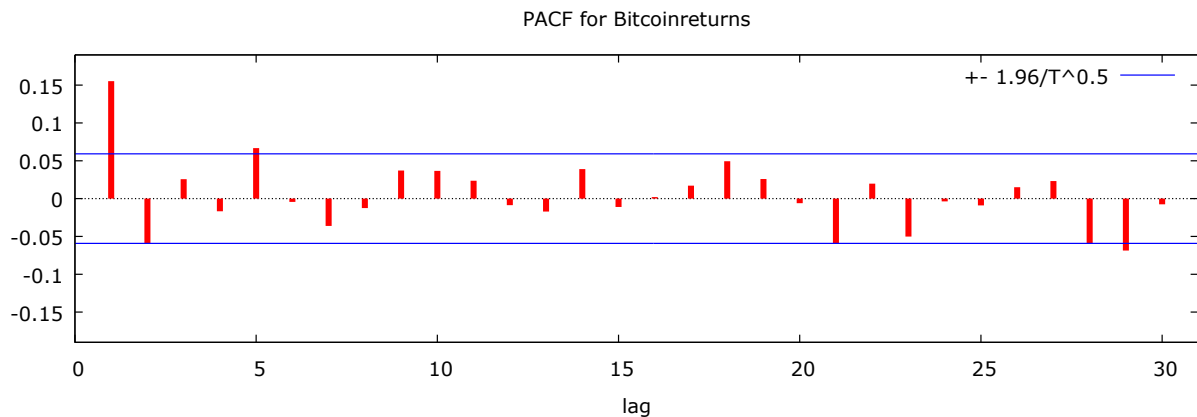
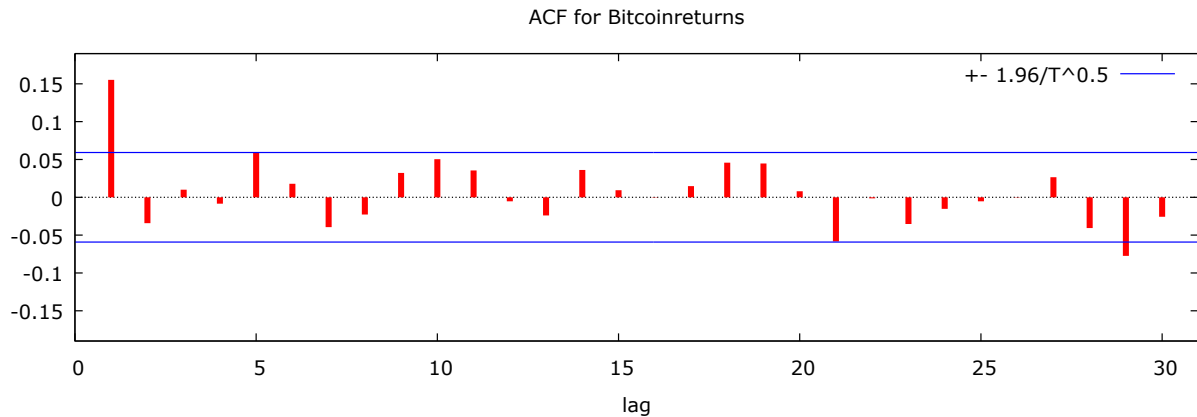
Autocorrelation function for Bitcoin\_returns

\*\*\*, \*\*, \* indicate significance at the 1%, 5%, 10% levels

using standard error  $1/T^{0.5}$

LAG	ACF		PACF		Q-stat.	[p-value]
1	0.1553	***	0.1553	***	26.4963	[0.000]
2	-0.0342		-0.0598	**	27.7831	[0.000]
3	0.0100		0.0256		27.8939	[0.000]
4	-0.0084		-0.0167		27.9721	[0.000]
5	0.0592	**	0.0668	**	31.8410	[0.000]
6	0.0179		-0.0044		32.1945	[0.000]
7	-0.0395		-0.0361		33.9158	[0.000]
8	-0.0228		-0.0125		34.4899	[0.000]
9	0.0322		0.0371		35.6374	[0.000]
10	0.0504	*	0.0366		38.4476	[0.000]
11	0.0355		0.0236		39.8418	[0.000]
12	-0.0053		-0.0086		39.8730	[0.000]
13	-0.0240		-0.0171		40.5120	[0.000]
14	0.0361		0.0390		41.9654	[0.000]
15	0.0093		-0.0110		42.0622	[0.000]
16	-0.0004		0.0021		42.0623	[0.000]
17	0.0148		0.0171		42.3060	[0.001]
18	0.0458		0.0494		44.6440	[0.000]
19	0.0447		0.0259		46.8777	[0.000]
20	0.0079		-0.0061		46.9482	[0.001]
21	-0.0584	*	-0.0594	**	50.7633	[0.000]

22	-0.0014	0.0197	50.7654	[0.000]
23	-0.0353	-0.0502 *	52.1598	[0.000]
24	-0.0152	-0.0037	52.4195	[0.001]
25	-0.0053	-0.0090	52.4509	[0.001]
26	0.0004	0.0151	52.4510	[0.002]
27	0.0266	0.0232	53.2452	[0.002]
28	-0.0406	-0.0587 *	55.1052	[0.002]
29	-0.0775 **	-0.0687 **	61.8709	[0.000]
30	-0.0257	-0.0076	62.6158	[0.000]



The ACF and PACF of Bitcoin returns were found to have similar patterns in both panels. However, there were differences in the pattern of ACF and PACF between the panels. There were significant autocorrelations and partial autocorrelations at the 1<sup>st</sup> and 5<sup>th</sup> lags in both panels. Further, there were significant autocorrelations and partial autocorrelations at the 8<sup>th</sup>, 10<sup>th</sup>, 17<sup>th</sup>, 20<sup>th</sup>, and 23<sup>rd</sup> lags in panel A, while for panel B the autocorrelations and partial autocorrelations at the 21<sup>st</sup> and 29<sup>th</sup> lags were significant. This suggests that an ARMA model would be appropriate to model the series.

The ARMA model estimates for Bitcoin returns are presented in the tables below.

**Panel A (01/01/2013 - 31/12/2017):**

Model 1: ARMA, using observations 2013-01-01:2017-12-31 (T = 1826)

Estimated using Kalman filter (exact ML)

Dependent variable: Bitcoin\_returns

Standard errors based on Hessian

	coefficient	std. error	z	p-value	
const	0.00372382	0.00125158	2.975	0.0029	***
phi_1	1.32096	0.0885326	14.92	2.42e-050	***
phi_2	-1.32699	0.0686121	-19.34	2.46e-083	***
phi_3	0.563438	0.0837538	6.727	1.73e-011	***
theta_1	-1.14980	0.0972912	-11.82	3.15e-032	***
theta_2	1.11662	0.0768071	14.54	6.96e-048	***
theta_3	-0.374737	0.0899452	-4.166	3.10e-05	***
Mean dependent var	0.003750	S.D. dependent var	0.041341		
Mean of innovations	1.06e-06	S.D. of innovations	0.039993		
Log-likelihood	3286.861	Akaike criterion	-6557.721		
Schwarz criterion	-6513.642	Hannan-Quinn	-6541.462		

Model 2: ARMA, using observations 2013-01-01:2017-12-31 (T = 1826)

Estimated using Kalman filter (exact ML)

Dependent variable: Bitcoin\_returns

Standard errors based on Hessian

	coefficient	std. error	z	p-value	
const	0.00372272	0.00125816	2.959	0.0031	***
phi_1	0.376448	0.0853184	4.412	1.02e-05	***
phi_2	-0.0480315	0.0375122	-1.280	0.2004	
phi_3	-0.715583	0.0426224	-16.79	2.94e-063	***
phi_4	0.566908	0.0773092	7.333	2.25e-013	***
theta_1	-0.195605	0.0936096	-2.090	0.0367	**
theta_2	-0.0205830	0.0456054	-0.4513	0.6518	
theta_3	0.724689	0.0494022	14.67	1.02e-048	***
theta_4	-0.401966	0.0865966	-4.642	3.45e-06	***
Mean dependent var	0.003750	S.D. dependent var	0.041341		
Mean of innovations	1.75e-06	S.D. of innovations	0.039870		
Log-likelihood	3292.382	Akaike criterion	-6564.763		
Schwarz criterion	-6509.664	Hannan-Quinn	-6544.439		

Model 3: ARMA, using observations 2013-01-01:2017-12-31 (T = 1826)

Estimated using Kalman filter (exact ML)

Dependent variable: Bitcoinreturns

Standard errors based on Hessian

	coefficient	std. error	z	p-value	
const	0.00372515	0.00124636	2.989	0.0028	***
phi_1	-0.336384	0.114855	-2.929	0.0034	***
phi_2	-0.0847084	0.145945	-0.5804	0.5616	
phi_3	-0.431283	0.0966263	-4.463	8.07e-06	***
phi_4	-0.282620	0.103454	-2.732	0.0063	***
phi_5	0.477338	0.101105	4.721	2.34e-06	***



theta_1	0.512645	0.122279	4.192	2.76e-05	***
theta_2	0.153897	0.155498	0.9897	0.3223	
theta_3	0.449332	0.102771	4.372	1.23e-05	***
theta_4	0.383680	0.109875	3.492	0.0005	***
theta_5	-0.284181	0.112596	-2.524	0.0116	**
Mean dependent var	0.003750	S.D. dependent var	0.041341		
Mean of innovations	1.01e-06	S.D. of innovations	0.039866		
Log-likelihood	3292.672	Akaike criterion	-6561.344		
Schwarz criterion	-6495.226	Hannan-Qu inn	-6536.955		

**Panel B (01/01/2015 - 31/12/2017):**

Model 1: ARMA, using observations 2015-01-01:2017-12-31 (T = 1096)

Estimated using Kalman filter (exact ML)

Dependent variable: Bitcoin\_returns

Standard errors based on Hessian

	coefficient	std. error	z	p-value	
const	0.00336879	0.00110242	3.056	0.0022	***
phi_1	-0.328622	0.172547	-1.905	0.0568	*
theta_1	0.495631	0.159695	3.104	0.0019	***
Mean dependent var	0.003382	S.D. dependent var	0.032933		
Mean of innovations	-2.65e-06	S.D. of innovations	0.032424		
Log-likelihood	2202.865	Akaike criterion	-4397.729		
Schwarz criterion	-4377.732	Hannan-Quinn	-4390.163		

The results of the ARMA model estimation for panel A indicate that the best models with respect to the AIC and BIC criteria were ARMA(4, 4) and ARMA(3, 3), respectively. On the other hand, the ARMA(5, 5) model showed significant  $\varphi_5$  and  $\vartheta_5$  coefficients and had higher log-likelihood, but had higher AIC and BIC than the former models. All three of these models can be considered as appropriate models for Bitcoin returns.

On the other hand, the results of the ARMA model estimation for panel B indicate that the best model with respect to all three criteria was ARMA(1, 1).

The results of the AR-GARCH models are presented in the tables below.

**Panel A (01/01/2013 - 31/12/2017):**

Model: GARCH(1,1) [Bollerslev] (Normal)\*

Dependent variable: Bitcoin\_returns

Sample: 2013-01-02 -- 2017-12-31 (T = 1825), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.00121570	0.000476328	2.552	0.0107	**
AR1	0.225169	0.0295263	7.626	2.42e-014	***

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.62198e-05	6.31971e-06	2.567	0.0103	**
alpha	0.268974	0.0438955	6.128	8.92e-010	***

beta 0.773459 0.0291573 26.53 4.71e-155 \*\*\*

Llik: 3833.41627 AIC: -7656.83254  
BIC: -7629.28586 HQC: -7646.67107

Model: GARCH(1,1) [Bollerslev] (Normal)\*  
Dependent variable: Bitcoin\_returns  
Sample: 2013-01-06 -- 2017-12-31 (T = 1821), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.00114861	0.000463102	2.480	0.0131	**
AR1	0.233063	0.0309269	7.536	4.85e-014	***
AR2	-0.0427224	0.0308597	-1.384	0.1662	
AR3	0.0382540	0.0307154	1.245	0.2130	
AR4	-0.0290588	0.0291487	-0.9969	0.3188	
AR5	0.0646978	0.0280800	2.304	0.0212	**

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.60188e-05	6.27284e-06	2.554	0.0107	**
alpha	0.267058	0.0441643	6.047	1.48e-09	***
beta	0.774968	0.0293130	26.44	5.05e-154	***

Llik: 3826.61805 AIC: -7635.23610  
BIC: -7585.67183 HQC: -7616.95071

Model: GARCH(1,1) [Bollerslev] (Normal)\*  
Dependent variable: Bitcoin\_returns  
Sample: 2013-01-07 -- 2017-12-31 (T = 1820), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.00110432	0.000461019	2.395	0.0166	**
AR1	0.230633	0.0306677	7.520	5.46e-014	***
AR2	-0.0426079	0.0310242	-1.373	0.1696	
AR3	0.0364607	0.0305167	1.195	0.2322	
AR4	-0.0286436	0.0292709	-0.9786	0.3278	
AR5	0.0557621	0.0280686	1.987	0.0470	**
AR6	0.0462937	0.0268039	1.727	0.0841	*

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.61777e-05	6.25866e-06	2.585	0.0097	***
alpha	0.268739	0.0441863	6.082	1.19e-09	***
beta	0.773638	0.0291916	26.50	9.16e-155	***

Llik: 3825.59347 AIC: -7631.18694  
BIC: -7576.12102 HQC: -7610.87130

Model: GARCH(1,1) [Bollerslev] (Normal)\*

Dependent variable: Bitcoin\_returns  
 Sample: 2013-01-08 -- 2017-12-31 (T = 1819), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.00114563	0.000454421	2.521	0.0117	**
AR1	0.233691	0.0302732	7.719	1.17e-014	***
AR2	-0.0413357	0.0306436	-1.349	0.1774	
AR3	0.0342451	0.0303183	1.130	0.2587	
AR4	-0.0242986	0.0287539	-0.8451	0.3981	
AR5	0.0523958	0.0276314	1.896	0.0579	*
AR6	0.0576161	0.0260215	2.214	0.0268	**
AR7	-0.0525383	0.0269303	-1.951	0.0511	*

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.58601e-05	6.10038e-06	2.600	0.0093	***
alpha	0.275346	0.0454330	6.060	1.36e-09	***
beta	0.769257	0.0290645	26.47	2.31e-154	***

Llik: 3825.23759      AIC: -7628.47518  
 BIC: -7567.90871      HQC: -7606.12959

Model: GARCH(1,1) [Bollerslev] (Normal)\*  
 Dependent variable: Bitcoin\_returns  
 Sample: 2013-01-23 -- 2017-12-31 (T = 1804), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.000950645	0.000446144	2.131	0.0331	**
AR1	0.233012	0.0300499	7.754	8.89e-015	***
AR2	-0.0373272	0.0301070	-1.240	0.2150	
AR3	0.0325942	0.0292789	1.113	0.2656	
AR4	-0.0341181	0.0287797	-1.185	0.2358	
AR5	0.0452607	0.0275861	1.641	0.1009	
AR6	0.0597387	0.0256034	2.333	0.0196	**
AR7	-0.0470114	0.0262174	-1.793	0.0730	*
AR8	-0.0213515	0.0256324	-0.8330	0.4049	
AR9	0.00500548	0.0240879	0.2078	0.8354	
AR10	0.0255735	0.0255341	1.002	0.3166	
AR11	0.00477708	0.0247719	0.1928	0.8471	
AR12	-0.0271160	0.0245905	-1.103	0.2702	
AR13	-0.00326454	0.0255385	-0.1278	0.8983	
AR14	0.0427626	0.0248259	1.723	0.0850	*
AR15	0.0171878	0.0269989	0.6366	0.5244	
AR16	0.00972889	0.0241233	0.4033	0.6867	
AR17	0.0129611	0.0228686	0.5668	0.5709	
AR18	-0.0125906	0.0222389	-0.5662	0.5713	
AR19	0.0194808	0.0229565	0.8486	0.3961	
AR20	0.0129829	0.0235798	0.5506	0.5819	
AR21	-0.0282592	0.0217867	-1.297	0.1946	
AR22	0.0395271	0.0225362	1.754	0.0794	*

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.54035e-05	5.85565e-06	2.631	0.0085	***
alpha	0.281951	0.0494783	5.698	1.21e-08	***
beta	0.764704	0.0313845	24.37	3.96e-131	***
Llik:	3800.19093	AIC:	-7548.38185		
BIC:	-7405.44005	HQC:	-7495.62242		

**Panel B (01/01/2015 - 31/12/2017):**

Model: GARCH(1,1) [Bollerslev] (Normal)\*

Dependent variable: Bitcoin\_returns

Sample: 2015-01-02 -- 2017-12-31 (T = 1095), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.00137055	0.000536462	2.555	0.0106	**
AR1	0.213178	0.0365509	5.832	5.46e-09	***

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.08146e-05	4.71648e-06	2.293	0.0219	**
alpha	0.218956	0.0416002	5.263	1.41e-07	***
beta	0.804679	0.0310790	25.89	8.32e-148	***
Llik:	2495.66518	AIC:	-4981.33037		
BIC:	-4956.33782	HQC:	-4971.87340		

Model: GARCH(1,1) [Bollerslev] (Normal)\*

Dependent variable: Bitcoin\_returns

Sample: 2015-01-31 -- 2017-12-31 (T = 1066), VCV method: Robust

Conditional mean equation

	coefficient	std. error	z	p-value	
const	0.00155461	0.000483178	3.217	0.0013	***
AR1	0.215863	0.0356427	6.056	1.39e-09	***
AR2	-0.0469729	0.0342711	-1.371	0.1705	
AR3	0.00980338	0.0338962	0.2892	0.7724	
AR4	-0.0402046	0.0344394	-1.167	0.2430	
AR5	0.0301559	0.0337880	0.8925	0.3721	
AR6	0.0609462	0.0318737	1.912	0.0559	*
AR7	-0.0560488	0.0314022	-1.785	0.0743	*
AR8	-0.0549245	0.0311780	-1.762	0.0781	*
AR9	0.00208956	0.0305568	0.06838	0.9455	
AR10	0.0268740	0.0290504	0.9251	0.3549	
AR11	0.00951606	0.0302008	0.3151	0.7527	
AR12	-0.0240190	0.0305697	-0.7857	0.4320	
AR13	-0.00864688	0.0313523	-0.2758	0.7827	
AR14	0.0531280	0.0286303	1.856	0.0635	*
AR15	-0.00568987	0.0326670	-0.1742	0.8617	
AR16	-0.00470319	0.0300438	-0.1565	0.8756	
AR17	0.000130166	0.0254348	0.005118	0.9959	

AR18	0.0143437	0.0253718	0.5653	0.5718	
AR19	0.0128481	0.0267110	0.4810	0.6305	
AR20	-0.0390407	0.0278419	-1.402	0.1608	
AR21	-0.0427248	0.0257570	-1.659	0.0972	*
AR22	0.0600753	0.0278995	2.153	0.0313	**
AR23	0.00673543	0.0274744	0.2452	0.8063	
AR24	-0.000125798	0.0277502	-0.004533	0.9964	
AR25	-0.00571637	0.0277826	-0.2058	0.8370	
AR26	0.0300101	0.0285684	1.050	0.2935	
AR27	0.0196746	0.0333506	0.5899	0.5552	
AR28	0.000961953	0.0383573	0.02508	0.9800	
AR29	-0.0765444	0.0334690	-2.287	0.0222	**
AR30	-0.0237276	0.0346463	-0.6849	0.4934	

Conditional variance equation

	coefficient	std. error	z	p-value	
omega	1.14310e-05	4.85148e-06	2.356	0.0185	**
alpha	0.264934	0.0515745	5.137	2.79e-07	***
beta	0.769047	0.0346654	22.18	4.81e-109	***

Llik: 2484.22710      AIC: -4900.45419  
 BIC: -4731.41746      HQC: -4836.40808

The results of the AR-GARCH models show non-stationarity of the conditional variance (as all models have  $\alpha_1 + \beta_1 > 1$ ). The results also show significant AR terms at the 1<sup>st</sup>, 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup>, 13<sup>th</sup>, and 22<sup>nd</sup> lags. According to the AIC and BIC criteria, the best model was AR(1)-GARCH(1, 1).

## Discussion

The results of the study suggest that Bitcoin returns has non-stationary conditional variance. This may be interpreted as a speculative bubble, as some noted economists, including Nobel laureates Paul Krugman, Robert J. Shiller, Joseph Stiglitz, Richard Thaler, James Heckman, Thomas Sargent, Angus Deaton, and Oliver Hart, had observed. In fact, Bitcoin prices crashed drastically in late 2017, conforming this observation. The results are related with those presented by Katsiampa (2017).

Comparing the results of Panel A and Panel B, some similarities and differences are evident. The first and fifth ACFs and PACFs were significant for both panels; on the other hand, the tenth, fifteenth, and twentieth ACFs and PACFs were significant in Panel A, they were not significant in Panel B, while the twenty-ninth ACFs and PACFs were significant in Panel B, though they were not significant in Panel A. This is reflected in the ARMA results; for Panel A, ARMA(3, 3), ARMA(4, 4), and ARMA(5, 5) were the optimal models, while for Panel B, only ARMA (1, 1) was the best model. Both of these results suggest that that Bitcoin returns have a simpler lag structure in Panel B than in Panel A. This provides some evidence of change in the time series properties of Bitcoin returns post-2015.

The AR-GARCH model results, however, indicate that the non-stationarity of Bitcoin returns conditional variance persists. Thus, Bitcoin price volatility does not show evidence of stabilising, as suggested by Bouoiyour and Selmi (2016).

There are several limitations inherent in the study. The study has used data from only one Bitcoin exchange, and the study period was limited to five years. Also, the study did not consider macroeconomic factors affecting Bitcoin prices, including particularly exchange rates of some of the important currencies, interest rates, inflation, gold prices, and oil prices. Multivariate econometric models including such factors may better explain the instability of Bitcoin returns.

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