Descriptors for Electron and Hole Charge Carriers in Metal Oxides

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Abstract

Metal oxides can act as insulators, semiconductors or metals depending on their chemical composition and crystal structure. Metal oxide semiconductors, which support equilibrium populations of electron and hole charge carriers, have widespread applications including batteries, solar cells, and display technologies. It is often difficult to predict in advance whether these materials will exhibit localized or delocalized charge carriers upon oxidation or reduction. We combine data from first-principles calculations of the electronic structure and dielectric response of 214 metal oxides to predict the energetic driving force for carrier localization and transport. We assess descriptors based on the carrier effective mass, static polaron binding energy, and Fröhlich electron–phonon coupling. Numerical analysis allows us to assign p and n type transport of a metal oxide to three classes: (i) band transport with high mobility; (ii) small polaron transport with low mobility; and (iii) intermediate behaviour. The results of this classification agree with observations regarding carrier dynamics and lifetimes and are used to predict 10 candidate p-type oxides.

Graphical TOC Entry
Crystalline compounds consisting of metals and oxygen may be electrically insulating (e.g. MgO), semiconducting (e.g. SnO), or even metallic (e.g. TiO). The variation in the behaviour of metal oxides is largely explained by the electronic configuration of the constituent metals. In the case of rocksalt TiO, and assuming formal oxidation states, Ti(II) is a d^2 cation, which results in a partially occupied t_{2g} d-level in an octahedral crystal field. The crystal structure also plays an important role and it is the edge-sharing network of Ti–O octahedra that provides the low energy conduction pathway in the case of TiO.

The mechanisms of electrical conduction in metal oxides with finite band gaps have been the subject of significant research for over a century. Electrolytic conduction can occur from a population of mobile ions, often in the form of charged vacancy or interstitial point defects. The transport of electron and hole charge carriers is also possible, with some oxides exhibiting high electrical conductivity similar to metals (e.g. In$_2$O$_3$ doped n-type with Sn) and others exhibiting low thermally-activated conductivity similar to disordered semiconductors (e.g. undoped Fe$_2$O$_3$). Metal oxide semiconductors often exhibit a doping asymmetry with a preference for electron or hole conduction, and the discovery of oxides with robust p-type conductivity remains a major challenge in the field.

In this study, we develop an efficient workflow for assessing the nature of charge carriers in non-metallic crystalline solids. By extracting data from existing metal oxide databases, we first assess the distribution of carrier (electron and hole) effective masses. These are then combined with (high and low-frequency) dielectric tensors to estimate the strength of carrier-lattice coupling in a static polaron model. The resulting binding energies are used to classify metal oxides into three groups. The dynamic electron–phonon interaction for a number of representative compounds are then calculated within Fröhlich electron–phonon coupling theory and an upper limit to the carrier mobility is predicted within the variational Feynman formalism for large (dielectric) polaron transport.

**Descriptor 1: Carrier effective mass.** The most simple descriptor for conductivity, which has been widely employed in high-throughput searches for p-type and thermoelectric oxides,
is the carrier effective mass, \( m^* \). The effective mass approximation assumes that the response of an electron in a periodic potential is equivalent to that of a free electron with a renormalized mass. There are a number of ways to fit \( m^* \) from a band dispersion calculation, \( E(k) \). One standard definition is as the local quadratic curvature at the band extrema

\[
\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2}.
\]

The effective mass describes how easy it is to accelerate the charge carriers. In the Drude model of carrier mobility, electrons with charge \( e \) are a classical gas with a constant relaxation time, therefore the mobility is calculated as \( \mu = e\tau/m^* \).

This relaxation time \( \tau \) is the inverse of the sum of all scattering rates (therefore the strongest scattering pathway dominates). Electrical conductivity \( (\sigma = \mu n) \) additionally depends on the carrier concentration \( n \). Assuming the scattering rate remains constant, a small effective mass \( (m^* < 1) \) supports a high mobility and conductivity. Asymmetry in the electron and hole effective masses leads to mobility and conductivity differences in n-type \( (m^*_e \ll m^*_h) \) or p-type \( (m^*_h \ll m^*_e) \) semiconductors.

The availability of density functional theory (DFT) materials databases including \( m^* \) makes it attractive as a low-cost descriptor for high-throughput computational studies. The distribution of electron and hole masses for 5,548 metal oxides calculated by Ricci et al. and available from the Materials Project database is shown in Figure 1. The upper valence band of most metal oxides is formed from the weak overlap of relatively localized O 2p orbitals, while the lower conduction band is usually formed of metal s (e.g. ZnO), metal d (e.g. TiO\(_2\)) or metal p (e.g. SnO) orbitals with higher principal quantum number. As a result, the hole masses are generally larger and show a wider distribution reflecting the range of crystal environments of oxygen in metal oxide compounds. From the data shown in Figure 1, the electron masses have a mean of 2.5 while the hole masses have a mean of 5.1 \( m_e \). Note the data has been truncated to omit very large values \( (m_e > 20) \) due to a breakdown in the
effective mass approximation for bands with low dispersion.

The smallest value of electron (hole) effective mass is found for BaSnO₃ (Rb₃AuO), respectively.

![Graph: Distribution of electron and hole carrier effective masses](image)

**Figure 1:** Distribution of electron and hole carrier effective masses (semi-local density functional theory) for 5,548 metal oxides from a public dataset.¹⁷ The values are in units of free electron mass and $m^* < 1$ is typical of conventional band semiconductors, which represents 78% and 11% of the dataset for electrons and holes, respectively.

**Descriptor 2: Static polaron binding.** Scattering of charge carriers in non-metallic crystals is often dominated by the dielectric polarization of the host.¹⁹ For a static point charge, the long-range polarization is the static (low-frequency) dielectric constant, $\epsilon_s$, which includes both ionic and electronic response. The Pekar factor²⁰ separates out the slow ionic response
(\epsilon_{\text{ion}}) which can only respond beyond the localised charge carrier, from the fast high-frequency response (\epsilon_{\infty}) that acts everywhere. The result is an effective dielectric response driving localisation of the charge carrier,

\[ \frac{1}{\epsilon_{\text{eff}}} = \left( \frac{1}{\epsilon_{\infty}} - \frac{1}{\epsilon_{s}} \right). \]  

(2)

Self-consistently minimizing the total energy of an additional, stationary, effective mass \((m^*)\) charge added to a dielectric crystal with the Pekar dielectric response yields a simple expression for the polaron binding energy of\(^{21}\)

\[ E_{\text{polaron}} = \frac{1}{8\pi^2} \frac{m^*}{\hbar^2} e^4 \epsilon_{\text{eff}}. \]  

(3)

Intuitively, electron localization is favored by large \(m^*\) (reduced kinetic energy penalty) and a large difference between \(\epsilon_{\infty}\) and \(\epsilon_{s}\) (strong dielectric ionic response, and therefore large dielectric electron-phonon coupling). Significant extensions of polaron theory have been since developed to take account of the dynamic properties of both the electron and the lattice including phonon creation and annihilation, as well as other sources of electron-phonon coupling, including the short-range forces that lead to small polaron formation.\(^{22}\)

The availability of datasets for both effective masses\(^{17}\) and dielectric constants\(^{23}\) make Eqn. 3 attractive as an alternative descriptor for charge carrier behaviour. It describes the result of the dielectric electron-phonon coupling on a static effective-mass charge carrier. Its coarseness has to be recognised as it assumes isotropic effective masses and dielectric response, and ignores band degeneracy and the angular momentum of the orbitals involved. However, as part of a screening procedure, the \(m^*/\epsilon_{\text{eff}}^2\) ratio can differentiate between systems with similar effective masses. The associated energies have been estimated for electrons and holes in each of the 214 metal oxides that are common to both databases.

The distribution of polaron behaviour is shown in Figure 2. The estimated binding energies range from 0.03\,meV (an electron in PtO\(_2\)) to 1980\,meV (a hole in K\(_3\)TaO\(_8\)). The
general trends follows those of the effective masses and standard expectations, e.g. ZnO with delocalized Zn 4s conduction band has a weak electron binding energy (4 meV), while \( \text{K}_2\text{TiO}_3 \) has a high electron binding energy arising from the localized Ti 3d conduction band (497 meV). In addition to ZnO, other known n-type conductors have weak electron binding energies including \( \text{SnO}_2 \) and \( \text{BaSnO}_3 \) (10 and 11 meV, respectively). A number of compounds with similar effective masses are estimated to have very different polaron binding energies. For example, \( \text{LiRhO}_2 \) and \( \text{K}_2\text{SrTa}_2\text{O}_7 \) exhibit hole \( E_{\text{polaron}} \) values of 129 meV and 730 meV, respectively, despite having \( m^*_e \) within 0.02 \( m_e \) of each other.

We can separate the compounds into three types based on the available thermal energy \( (k_B T \sim 25 \text{ meV}) \) at room temperature: (I) \( E_{\text{polaron}} < 25 \text{ meV} \); (II) \( 25 \text{ meV} < E_{\text{polaron}} < 250 \text{ meV} \); (III) \( E_{\text{polaron}} > 250 \text{ meV} \). From the 214 entries, for electrons, there are 66 type I, 141 type II, and 7 type III compounds. For holes, there are 19 type I, 106 type II, and 89 type III compounds. This classification fits with the trends in \( m^* \) (Figure 1) and observations that metal oxides are more often n-type (majority electron conductors) than p-type (majority hole conductors).\(^{24-26}\) The three types are colored in Figure 2 and a more detailed breakdown is provided in the Supplementary Information.

**Descriptor 3: Fröhlich electron–phonon coupling.** To assess the limitation of the first two descriptors and to apply these approaches to identify metal oxides with potential for hole delocalization and p-type conduction, we chose a subset of 10 compounds with the smallest \( E_{\text{polaron}}^{\text{hole}} \) and performed hybrid-DFT (HSE06)\(^{27,28}\) calculations to assess the electronic, vibrational, and dielectric properties. The use of hybrid functionals is necessary to calculate key quantities such as carrier effective masses and Born effective charges accurately, as they both suffer from band gap errors. As a control, we included two materials with large calculated polaron binding energies, namely \( \text{K}_2\text{TiO}_3 \) (electrons) and \( \text{NaSbO}_3 \) (holes).

Our calculated values for the two descriptors are compared to the public datasets in Table 1 (and Figure S2). The main difference is that the databases used semi-local exchange-correlation functionals (PBE\(^{29}\) for \( m^* \) and PBEsol\(^{30}\) for \( \epsilon \)), which results in a band gap
Figure 2: Distribution of electron and hole large polaron binding energies calculated using the effective masses and dielectric constants (semi-local density functional theory)\textsuperscript{17,23} for 214 metal oxides. The values are categorized into type I (orange), II (yellow), and III (blue).

underestimation. For example, the HSE06 band gap of LaZnAsO (1.36 eV) is much closer to the experimentally measured value (1.5 eV)\textsuperscript{31} than the PBE band gap from the database (0.58 eV). In general, from perturbation theory, a smaller band gap is associated with larger estimates of $\varepsilon_\infty$ (enhanced electronic polarization) and smaller values of $m^*$ (stronger band coupling).\textsuperscript{32} Hybrid functionals have also been shown directly to improve the description of static and dynamic screening properties compared to semi-local functionals.\textsuperscript{33} A particularly bad case is PtO$_2$, which exhibits the largest discrepancy in $\varepsilon_{\text{eff}}$. This arises as the dataset value of $\varepsilon_\infty = 18.8$ compared to $\varepsilon_\infty = 7.1$ from our calculations. Calculations at the Hybrid
DFT level also predict $m_e^* < m_h^*$ for PtO$_2$ and YZnPO, the opposite trend to that predicted at the GGA level. The qualitative trends are maintained across the series, but the absolute values also show significant variation. This highlights a risk in relying on large materials datasets generated with less computationally expensive methods.

Table 1: Comparison of polaron energies calculated using database values (PBE and PBEsol)$^{17,23}$ and our calculated values (HSE06) for twelve metal oxides. Consideration of the effective optical phonon frequency $\omega_{\text{optic}}$ (HSE06) also allows the prediction of large polaron mobility ($\mu$) presented at $T = 300$ K. Values are presented as electron / hole.

<table>
<thead>
<tr>
<th>Formula</th>
<th>$E_{\text{polaron}}$ (meV)</th>
<th>$\omega_{\text{optic}}$ (cm$^{-1}$)</th>
<th>$\mu$ (cm$^2$V$^{-1}$s$^{-1}$)</th>
<th>$m^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>database</td>
<td>HSE06</td>
<td>database</td>
<td>HSE06</td>
</tr>
<tr>
<td>PtO$_2$</td>
<td>0.04 / 0.03</td>
<td>0.52 / 1.48</td>
<td>713</td>
<td>1.55 / 1.18</td>
</tr>
<tr>
<td>CuRhO$_2$</td>
<td>0.14 / 0.54</td>
<td>0.51 / 2.20</td>
<td>590</td>
<td>0.58 / 2.31</td>
</tr>
<tr>
<td>LiAg$_3$O$_2$</td>
<td>3.50 / 2.49</td>
<td>17.51 / 11.35</td>
<td>429</td>
<td>1.10 / 0.78</td>
</tr>
<tr>
<td>NaNbO$_2$</td>
<td>4.86 / 2.64</td>
<td>5.89 / 2.88</td>
<td>464</td>
<td>1.39 / 0.75</td>
</tr>
<tr>
<td>Ca$_4$Bi$_2$O</td>
<td>3.34 / 4.84</td>
<td>12.46 / 13.68</td>
<td>105</td>
<td>0.33 / 0.47</td>
</tr>
<tr>
<td>YZnAsO</td>
<td>5.93 / 5.13</td>
<td>17.13 / 29.97</td>
<td>307</td>
<td>0.47 / 0.41</td>
</tr>
<tr>
<td>NaAg$_3$O$_2$</td>
<td>5.21 / 5.99</td>
<td>13.62 / 16.98</td>
<td>439</td>
<td>1.04 / 1.20</td>
</tr>
<tr>
<td>LiNbO$_2$</td>
<td>24.43 / 6.45</td>
<td>36.28 / 6.62</td>
<td>420</td>
<td>2.88 / 0.76</td>
</tr>
<tr>
<td>K$_2$TiO$_3$</td>
<td>496.6 / 522.8</td>
<td>284.6 / 371.1</td>
<td>370</td>
<td>2.44 / 2.57</td>
</tr>
<tr>
<td>NaSbO$_3$</td>
<td>41.47 / 1182.0</td>
<td>60.67 / 1500</td>
<td>453</td>
<td>0.41 / 11.7</td>
</tr>
</tbody>
</table>

Finally, we can directly calculate polaron charge carrier mobility. For polar semiconductors, optic mode (Fröhlich) scattering often limits the relaxation time $\tau$ at room temperature and above.$^{34}$ In addition to the effective mass and dielectric constants, calculation of the carrier mobility also requires knowledge of the optical phonon ($\omega_{\text{optic}}$) spectrum and dielectric activity.$^{21}$ These data are used in a calculation of the static dielectric constant, but are often not directly exposed. By computing the phonon modes and energies, along with the Born effective charges, we calculate an effective $\omega_{\text{optic}}$ following the averaging procedure of Hellwarth.$^{35}$ The polaron mobility calculation is implemented in the POLARONMOBILITY.JL package,$^{36,37}$ and solves the Feynman variational solution for a finite temperature extension of Fröhlich’s polaron Hamiltonian, and then directly integrates the polaron response function.
to produce a mobility. The method is finite temperature, requires no empirical parameters, includes all electron phonon scattering processes. Only considering one scattering process and one source of electron-phonon coupling, the result is always an upper limit to the mobility in a perfect crystal.

The calculated electron and hole mobility values are listed in Table 1. The control $\text{K}_2\text{TiO}_3$ was chosen due to its large electron polaron binding energy, and indeed the calculated upper limit to the room temperature mobility of $1\,\text{cm}^2\text{V}^{-2}\text{s}^{-1}$. Similarly, $\text{NaSbO}_3$ which has an even larger hole polaron binding energy has an associated hole mobility of just $0.1\,\text{cm}^2\text{V}^{-2}\text{s}^{-1}$. In terms of the 10 candidate p-type oxides, all have a predicted hole mobility greater than $10\,\text{cm}^2\text{V}^{-2}\text{s}^{-1}$ showing the power of the simpler descriptors. Higher mobilities are predicted for compounds with a higher frequency of $\omega^\text{optic}$. At $T = 300\,\text{K}$ these have both lower occupancy and are more energetically inaccessible to scatter into, and thus scatter less. For example, the highest hole mobility is predicted for $\text{PtO}_2$ which also has the largest $\omega^\text{optic}$. Looking in the experimental literature, there is evidence that several of these candidates are indeed p-type semiconductors, including $\text{CuRhO}_2$, $\text{Li}_{1-x}\text{NbO}_2$, and the family of zinc oxypnictides.

In conclusion, we have considered descriptors for assessing the behaviour of charge carriers in metal oxides, which can be extended to other semiconductors. Our study has shown the utility of modern materials datasets to facilitate rapid screening, but also their limitations both in terms of data quality (level of theory for calculated quantities) and availability (not all values and properties are available, even if they must have been calculated at some point to produce the presented values). Based on the carrier binding energies, we separated the metal oxides into three classes: I (weak coupling) where band transport is expected; II (intermediate behaviour) where the balance between short and long-range forces may be subtle; III (strong coupling) where small polaron transport is expected. One application to identifying p-type metal oxides yielded 10 candidates, which are all predicted to have room temperature hole mobility values greater than $10\,\text{cm}^2\text{V}^{-2}\text{s}^{-1}$. 

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Going beyond this approach, explicit consideration of defect energies is necessary to predict equilibrium charge carrier concentration and electrical conductivity, but these end up being dependant on the growth environment (chemical potentials) and require the development of appropriate high-throughput procedures and associated databases. Our calculated mobilities represent an upper limit for perfect crystals with no impurity or defect scattering, and with only dielectric electron phonon coupling. The same defect calculations could be used to extend the mobility metric.

**Methods and Data Access**

Periodic DFT calculations were performed within the Vienna *Ab Initio* Simulation Package (VASP)\(^{42-45}\) using scalar relativistic pseudopotentials within the projector-augmented wave (PAW) method.\(^{46}\) For geometry optimizations, electronic structure and optical property calculations, the functional of Heyd, Scuseria and Ernzerhof (HSE06) was used, with a screening parameter of 0.207 Å\(^{-1}\).\(^{27,47}\) All calculations utilised a plane wave energy cutoff of 600 eV and a consistent \(k\)-point density, with a spacing between \(k\)-points of no more than 0.25 Å\(^{-1}\), to ensure equivalence between multiple structure types. Convergence criteria of \(1 \times 10^{-5}\) eV and 0.01 eV Å\(^{-1}\) were enforced on the total energy and forces per atom respectively. The high-frequency dielectric function was calculated using the method of Gajdoš *et al.*,\(^{48}\) while the ionic response was calculated using Density Functional Perturbation Theory with the related functional of Perdew, Burke and Ernzerhof (PBE).\(^{29}\) The scripts necessary to reproduce the results presented here from raw data, along with the raw data itself, are available from https://doi.org/10.5281/zenodo.3520679. This analysis was facilitated by several open-source Python packages, including **Pymatgen**,\(^{49}\) **Matminer**,\(^{50}\) **SMACT**\(^{51,52}\) and **Phonopy-Spectroscopy**,\(^{53}\) as well as the open-source Julia package **PolaronMobility.jl**.\(^{36,37}\)
Acknowledgement

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Supporting Information Available

The Supporting Information contains data and analysis on trends in polaron binding energies, a summary of the sources of discrepancy between $E_{\text{polaron}}$ values calculated using DFT and hybrid-DFT, and a visualization of the variation of $\epsilon_{\text{eff}}$ with $\epsilon_s$ and $\epsilon_\infty$. This material is available free of charge via the Internet at http://pubs.acs.org/.
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Supplementary Information
Figure S1: Electron and hole polaron binding energies calculated using the effective masses and dielectric constants from public data-sets. Some compositions have multiple entries due to polymorphism.
Figure S2: a) Polaron energy ($E_{\text{polaron}}$) values as calculated using hybrid-DFT (solid bars) and database (hatched bars) for electrons (orange bars) and holes (blue bars). b) Absolute difference (database derived minus Hybrid-DFT derived) in $\epsilon_{\text{eff}}$ (green bars) and effective mass ($m^*$) for electrons (orange bars) and holes (blue bars).
Figure S3: Visualization of the variation of $\epsilon_{eff}$ (y-axis) with $\epsilon_s$ (x-axis) at different fixed values of $\epsilon_{\infty}$. For $\epsilon_s > 20$ the system is in a strong screening regime where $\epsilon_{eff}$ shows little variation.