

Bayesian data analysis reveals no preference for cardinal Tafel slopes in CO₂ reduction electrocatalysis

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Abstract

In this paper, we develop a Bayesian data analysis approach to estimate the Tafel slope from experimentally-measured current-voltage data. Our approach obviates the human intervention required by current literature practice for Tafel estimation, and provides robust, distributional uncertainty estimates. Using synthetic data, we illustrate how data insufficiency can unknowingly influence current fitting approaches, and how our approach allays these concerns. We apply our approach to conduct a comprehensive re-analysis of data from the CO₂ reduction literature. This analysis reveals no systematic preference for Tafel slopes to cluster around certain “cardinal values” (e.g. 60 or 120 mV/decade). We hypothesize several plausible physical explanations for this observation, and discuss the implications of our finding for mechanistic analysis in electrochemical kinetic investigations.

13 Introduction

14 Modern tools in data science provide the capability to reduce or outright eliminate sources
15 of human bias in the analysis and interpretation of experimental measurements. Despite
16 the wide availability of these tools, many communities continue to rely on more primitive
17 and bias-prone methods of data analysis. The calculation of Tafel slopes through linear
18 least squares fitting is one prominent example. Here, we present a robust Bayesian approach
19 to analyzing electrochemical current-voltage measurements that (1) eliminates the need to
20 manually exclude points in limiting-current regimes, and (2) provides a well-defined measure
21 of uncertainty in the fitting parameters (e.g. the Tafel slope). By applying this approach
22 to a large set of literature data, we identify a systematic but unjustified tendency for the
23 assignment of Tafel slopes to specific “cardinal” values. This finding highlights the role that
24 modern data science can play in uncovering and eliminating hidden sources of bias that exist
25 within various scientific communities.

26 Current-voltage measurements are a fundamental characterization tool for electrochem-
27 ical systems, as they report on the propensity for system response when pushed out of
28 equilibrium by a thermodynamic driving force. In the context of electrochemical catalysis,
29 current-voltage behavior is often summarized by the Tafel slope, a parameter that quantifies
30 the amount of electrochemical driving force required to produce a logarithmic increase in the
31 observed current.¹ Nearly all studies that develop a novel electrochemical catalyst report a
32 Tafel slope, and it is considered an important figure of merit when comparing catalysts. In
33 principle, the Tafel slope contains information about the microscopic mechanism underlying
34 the operation of a catalyst. The elementary kinetic steps of idealized reaction mechanisms
35 imply a strong tendency for Tafel slopes to exhibit certain “cardinal values”,^{2,3} and these
36 cardinal values are frequently referenced in the kinetic analysis of catalytic materials. How-
37 ever, several notable studies have reported Tafel slopes that differ significantly from their
38 predicted cardinal values.^{4,5} Because the kinetic steps associated with these cardinal values
39 are ubiquitous, there is a tendency to interpret Tafel slopes with off-cardinal values as if

40 they represent a truly cardinal value that has been altered by sources of experimental error,
41 despite relatively incomplete error quantification.⁶⁻¹² The flaw in this interpretive strategy
42 is that it relies on the validity of idealizing assumptions about the underlying kinetic mech-
43 anism, and does not account for the numerous ways in which deviations from ideality can
44 influence the Tafel slope.¹³

45 Despite the scientific and engineering relevance of the Tafel slope, current literature ap-
46 proaches for estimating this parameter from measured current-voltage data require subjective
47 human intervention, and are susceptible to numerous sources of systematic error. Subjective
48 considerations in the fitting procedure (namely, the manual demarcation of a linear fit re-
49 gion) make it impossible to determine, in a truly unbiased manner, the intrinsic distribution
50 of Tafel slopes, and whether they cluster around cardinal values. Additionally, human inter-
51 ventions in the fitting procedure enable researcher bias, both inadvertent and intentional, to
52 influence a quantitative catalyst benchmark. Such biases are difficult to recognize without
53 re-examining primary source data.

54 To address these concerns, we advance an alternative Bayesian data analysis method
55 that enables unbiased Tafel slope estimation. This method provides robust, distributional
56 uncertainty quantification, elucidating the credible range of Tafel slope values consistent
57 with the measured data. Because our method eliminates subjectivity in the fitting process,
58 it enables us to fairly evaluate the prevalence of cardinal Tafel slopes within re-analyzed
59 literature data.

60 In this manuscript, we begin by describing common literature practices for assigning Tafel
61 slopes from experimental current-voltage data. Subsequently, we develop the mathematical
62 formalism behind our Bayesian approach to Tafel slope estimation, and discuss its associated
63 benefits compared to existing approaches. Using synthetic data, we illustrate the benefits of
64 our approach, and show how it can be combined with iterative data acquisition procedures
65 to systematically reduce uncertainty in Tafel slope estimates. Finally, we apply our approach
66 to a large set of CO₂ reduction catalyst data from the literature, and compare our Tafel slope

67 estimates to the reported values. We find that clustering of reported Tafel slopes around
68 cardinal values is unjustified, and likely reflects systemic bias across the field. We conclude
69 by hypothesizing several plausible sources of mechanistic nonideality and estimating their
70 ability to modify Tafel slopes from their cardinal values.

71 **Tafel Slopes in Electrocatalysis**

72 Electrochemical systems operate by converting the energy stored in chemical bonds into
73 electrical work (or vice-versa) by means of electron transport through an external circuit.
74 The electron transport process originates at an electrochemical interface, where a portion
75 of the external circuit (an electrode) is contacted with a chemical system (a reactive elec-
76 trolyte solution) in the presence of a catalyst. Catalyzed interfacial electron transfer serves
77 to inextricably link the electrical dynamics of the external circuit to the chemical dynamics
78 of the electrolyte. Electrochemical characterization techniques exploit this linkage, using the
79 voltage and current measured in the external circuit to report on the thermodynamic driving
80 forces and nonequilibrium currents in the chemical system. The simplest possible electro-
81 chemical experiment involves setting the applied potential at an electrode and measuring
82 the resultant electrical current (or vice-versa), generating a current-voltage trace.

83 Several phenomena can influence the shape of current-voltage traces. For example, the
84 electronic properties of the electrode, the chemical identity of the catalyst material, the
85 transport characteristics of reactive species in the electrolyte, and myriad other factors all
86 play an important role in structuring current-voltage behavior.¹⁴ When characterizing the
87 performance of a catalyst material, we are most interested in the kinetic control regime of a
88 current-voltage trace, where the measured current reports directly on the intrinsic rate of a
89 chemical reaction at the interface. Under kinetic control, the current is generally expected
90 to follow an exponential asymptotic dependence on the overpotential η , which quantifies
91 the difference between the applied electrode potential and the equilibrium potential for the

92 chemical reaction.¹ In the high $|\eta|$ limit (specifically, $|\eta| \gg k_{\text{B}}T/e$, the thermal voltage), the
93 logarithm of the kinetic current, i_{kin} , should depend linearly on the applied potential. The
94 (inverse) slope of this relationship is termed the Tafel slope,

$$\text{Tafel Slope} \equiv \left. \frac{d\eta}{d \log_{10} i_{\text{kin}}} \right|_{|\eta| \gg k_{\text{B}}T/e}, \quad (1)$$

95 and is generally reported in units of mV/decade.

96 The Tafel slope is an important parameter to judge the performance of a catalyst because
97 it quantifies the amount of additional applied potential required to observe a logarithmic in-
98 crease in the measured current. Hence, studies that develop a novel electrocatalytic material
99 often measure current-voltage data in the kinetic control regime, and then use this data to
100 estimate a Tafel slope for their catalytic system. Experimental limitations impose a number
101 of practical constraints on the Tafel slope estimation procedure. First, for several electro-
102 chemical reactions (CO_2 reduction, N_2 reduction, organic electrosynthesis, etc.), accurately
103 determining a kinetic current for a specific reaction requires product quantification after a
104 constant potential/current hold. Due to throughput limitations for quantification techniques,
105 the Tafel slope often must be estimated from just a handful of data points (roughly, 3–10).
106 Another important practical constraint arises from limiting current phenomena observed in
107 many electrocatalytic systems. At sufficiently high overpotentials, the reaction rate exceeds
108 the rate of another physical process required for the reaction to proceed. In this regime, the
109 system is no longer under kinetic control, and the measured current plateaus to a limiting
110 current, becoming independent of the applied voltage.¹⁵ Most commonly, limiting currents in
111 CO_2 reduction systems arise from diffusive transport limitations in the electrolyte, although
112 several other physical reasons for plateau currents have been hypothesized and investigated
113 in the literature.^{16–20} Consequently, experimentally measured Tafel data usually starts out
114 linear, but curves sub-linearly at sufficiently high overpotentials.

115 In the face of these practical limitations, studies in the literature use a relatively standard

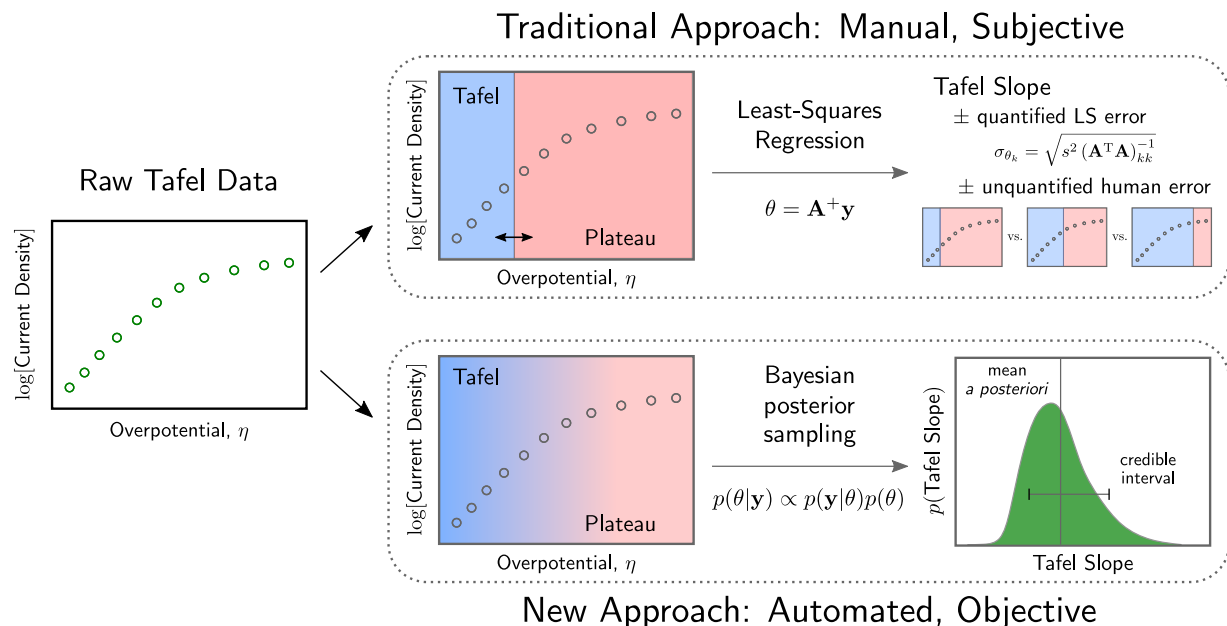


Figure 1: Schematic comparison of the traditional approach to Tafel fitting and the new approach we describe in this manuscript. Starting from raw current-voltage data on the left, the current literature approach begins with manual identification of a linear Tafel region on the plot. The rest of the data is discarded, and a linear fit to the Tafel region yields a Tafel slope, with an associated uncertainty corresponding to the standard error of the ordinary least squares (OLS) estimator. In addition to this quantified uncertainty, an additional unquantified source of uncertainty arises from manual selection of a Tafel region on the plot. The new approach described here considers all of the data in the context of a nonlinear model that smoothly interpolates between the traditional Tafel region and a plateaued region (e.g., due to mass transport limitations). Our approach uses a Monte Carlo method to sample from the Bayes posterior distribution over the parameters of the model, yielding a probabilistic distribution over Tafel slopes that are consistent with the measured data.

116 protocol for estimating the Tafel slope, depicted schematically in the upper half of Fig. 1.
117 First, a researcher must manually identify an ad hoc cutoff between a linear, kinetically-
118 controlled regime (the Tafel regime) and a limiting current (plateau) regime.²¹ All data points
119 in the plateau regime are subsequently discarded, and a Tafel slope is fitted by ordinary least
120 squares (OLS) linear regression to the data in the Tafel regime. The OLS procedure offers
121 a prescription for extracting the standard error of the Tafel slope; this standard error is
122 sometimes used to construct a confidence interval for the Tafel slope estimate.^{22,23}

123 We believe the current standard literature practice bears several drawbacks. First and
124 foremost, manual identification of a cutoff between the kinetic and limiting-current regimes
125 introduces subjectivity into Tafel slope estimation, potentially incorporating undesirable hu-
126 man influence into the quantification of an important metric for electrocatalyst performance.
127 Second, reporting the error associated with a linear fit to a manually selected set of data
128 points systematically under-estimates the actual error associated with estimating a Tafel
129 slope from a small number of data points. The OLS slope standard error quantifies the
130 uncertainty associated with the linear fit to a given set of data, but there is an additional
131 unquantified error associated with selection of the linear regime. Third and finally, ad hoc se-
132 lection of a regime cutoff can introduce a systematic bias in the Tafel slope, since the final few
133 points of the kinetic regime will suffer at least some effects from limiting current curvature,
134 causing the current at these points to deviate slightly from the true kinetic current.

135 **Bayesian Data Analysis Algorithm**

136 Our approach for Tafel slope estimation seeks to obviate manual demarcation between the
137 linear and plateau regions in current voltage data. To this end, we choose to fit *all* current-
138 voltage data measured in a Tafel experiment to a phenomenological model that smoothly

139 interpolates between the kinetic control and plateau regimes. The model reads,

$$\frac{1}{i(\eta)} = \frac{1}{i_{\text{lim}}} + \frac{1}{i_0 \exp [m_{\text{T}}^{-1} \cdot |\eta|]}, \quad (2)$$

140 where $i(\eta)$ is the measured current density as a function of overpotential. The unknown
141 parameters in the model are i_{lim} , the limiting current density, i_0 , the exchange current
142 density, and m_{T}^{-1} , the inverse Tafel slope. The mathematical structure of Eq. (2) can be
143 shown to arise, for example, when the surface concentration of a redox-active species changes
144 with applied overpotential due to diffusive transport effects. Alternatively, Eq. (2) could be
145 motivated by interpreting the presence of limiting-current phenomena as an additional series
146 resistance to current in the equivalent circuit for an electrochemical cell.¹ Most generally,
147 this model can describe any physical phenomenon that imposes a “speed limit” on the passed
148 current.²⁴

149 Typically, the parameters in a model like Eq. (2) would be adjusted to achieve optimal
150 agreement between the model and experimental data. Despite the nonlinearity inherent to
151 the model, numerical optimization schemes for determining the optimal set of parameters are
152 mature and well-studied. Here, we employ a different approach based on Bayesian sampling
153 that can quantify not only an optimal set of parameters, but also a *distribution* over the likely
154 values of the model parameters given the available experimental data.²⁵ Before carrying out
155 any current-voltage measurements, we generally have at least *some* idea of the reasonable
156 values of parameters in Eq. (2). For example, one would be very leery of a Tafel slope
157 $m_{\text{T}} \notin [10^1, 10^3]$ mV/decade, and one can use tabulated values of a species diffusion coefficient
158 and an estimate of the cell boundary layer thickness to compute a ballpark estimate of a
159 limiting current, i_{lim} , arising from diffusive transport limitations.^{1,24} For a general set of
160 parameters θ , this knowledge is encoded in a *prior* distribution over the parameters $p(\theta)$.
161 Upon observing some data \mathbf{y} , we can compute an updated *posterior* distribution $p(\theta|\mathbf{y})$ (the

162 probability of the parameters given the observed data) using Bayes' rule,²⁶

$$p(\theta|\mathbf{y}) = \frac{p(\mathbf{y}|\theta) \times p(\theta)}{p(\mathbf{y})}. \quad (3)$$

163 The likelihood $p(\mathbf{y}|\theta)$ is supplied by a model like Eq. (2), in conjunction with an assumption
164 about the distribution over the error at each data point. Here, we assume the error at each
165 data point is independently normally distributed with a standard deviation $\sigma = 0.10 \log$
166 units, though our approach is flexible in this respect (see SI for sensitivity analysis with
167 respect to the σ parameter).

168 Despite the fact that the partition function $p(\mathbf{y})$ is an unknown constant, Eq. (3) yields a
169 prescription for sampling from the posterior distribution over parameters $p(\theta|\mathbf{y})$. Briefly, one
170 can use a Markov chain Monte Carlo sampling scheme²⁷ to glean several parameter samples
171 from the posterior distribution, which are expected to concentrate in density around the
172 optimal values of the parameters (additional detail on implementation can be found in the
173 SI). As depicted in the lower panel of Fig. 1, this Bayesian posterior sampling technique
174 reveals both the optimal values of the model parameters and a distributional uncertainty
175 estimate over their values, accounting for all sources of uncertainty in the model. If we truly
176 want to collapse this distributional information to a single value of the parameter, say for
177 quoting a Tafel slope associated with a catalyst, we can compute,

$$\langle \theta \rangle \equiv \int d\theta \cdot \theta \cdot p(\theta|\mathbf{y}), \quad (4)$$

178 where $\langle \theta \rangle$ is termed the mean *a posteriori* (MAP) parameter estimate. When the poste-
179 rior distribution $p(\theta|\mathbf{y})$ is strongly peaked around an optimal set of parameters, the MAP
180 estimate will line up with the parameters gleaned from a nonlinear optimization technique.
181 When the posterior distribution is broad or multimodal, the MAP estimate and the opti-
182 mal parameter values may differ, signaling a high degree of uncertainty associated with the
183 optimal parameter estimate.

184 The Bayesian posterior sampling approach offers several key advantages over the tradi-
185 tional approach to Tafel slope estimation. First, it removes subjectivity from the analysis of
186 Tafel data: users of our algorithm need only select a model such as Eq. (2) to interpret the
187 observed data, which can be justified on the basis of rigorous physical arguments, unlike a
188 subjective delineation between linear and plateau regimes. Second, our approach yields ac-
189 curate quantification the uncertainty associated with a Tafel slope estimate. Specifically, we
190 believe the *distributional* uncertainty quantification afforded by our algorithm will be useful
191 when assessing and discriminating between disparate sets of experimental data. Finally, be-
192 cause the model in Eq. (2) analytically extrapolates away curvature-related attenuation of
193 the kinetic current, our approach is free of the systematic bias present in current literature
194 practice.

195 As a caveat, we stress that while the model in Eq. (2) is appropriate for fitting some
196 current-voltage data in the CO₂ reduction literature, experimental and theoretical stud-
197 ies have noted the possibility of multiple kinetic control regimes with distinct Tafel slopes
198 before a limiting current plateau regime.^{3,28,29} This phenomenon can arise due to potential-
199 dependent surface coverage effects or a potential-dependent switch in the microscopic reac-
200 tion mechanism. When multiple distinct Tafel regimes are present, the experimental data
201 must be interpreted under a model that allows for this possibility; once a suitable model is se-
202 lected, the Bayesian posterior sampling approach can still be employed (see SI for additional
203 discussion on model flexibility).

204 **Identifying and Addressing Data Insufficiency**

205 As mentioned previously, practical throughput considerations imposed by product quantifi-
206 cation and other experimental requirements limit the amount of data generally used in a
207 Tafel analysis to 3–10 points. In a survey of Tafel data reported in CO₂ reduction studies
208 in the literature, we found that a significant number of papers conduct Tafel analyses on a

209 set of 3–5 data points within a narrow overpotential window. With such few data points,
 210 trends often appear linear, and so many studies simply extract the Tafel slope from a linear
 211 fit to all the data. While seemingly benign, estimating a Tafel slope without accounting
 212 for the possibility of limiting-current nonlinearity can lead to systematic error arising from
 213 data insufficiency. This phenomenon is elegantly identified and addressed by our Bayesian
 214 posterior sampling approach. For the sake of clarity, we illustrate how this systematic error
 215 can emerge by analyzing a set of synthetic data. The parable narrated by this data is easily
 216 relatable to a specific set of experimental data, and it emphasizes another distinct advantage
 217 of the Bayesian posterior sampling approach.

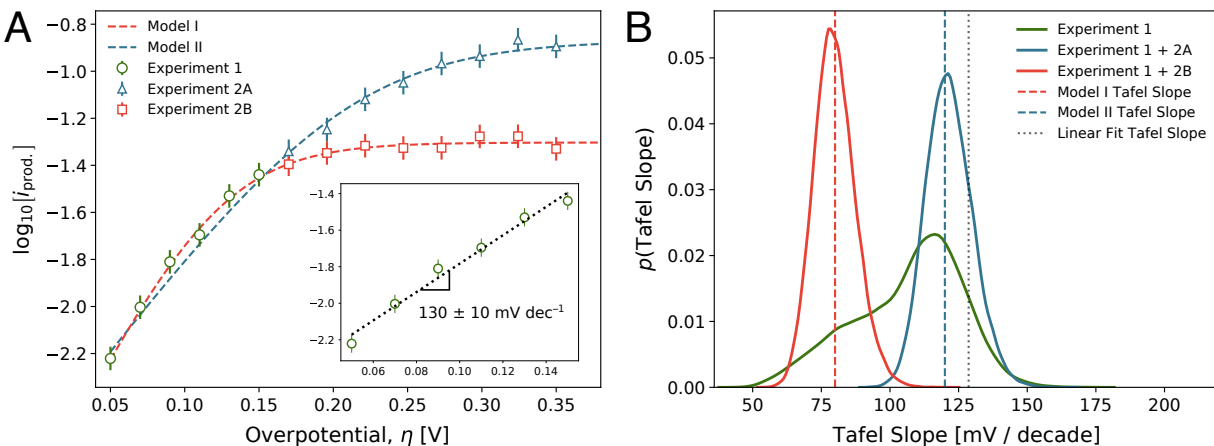


Figure 2: Bimodal posterior distributions signify Tafel slope ambiguity that cannot be clarified by the available data. (A, inset) Synthetic current-voltage data collected in hypothetical Experiment 1 appears linear over a 100 mV overpotential region, with a Tafel slope of 130 ± 10 mV/decade. (A) Synthetic current-voltage data over a broader range of overpotentials. The dashed lines show two models (I, II) with different Tafel slopes and plateau currents that could both reasonably fit the Experiment 1 data. Synthetic error bars represent one standard error of the mean of uncertainty in the synthetic data. Experiment 2A (blue triangles) and 2B (red squares) represent two possible outcomes of experiments that probe a broader range of overpotentials, which can clearly distinguish between Model I and Model II. (B) Bayes posterior distributions over the Tafel slope determined by our algorithm given various sets of observed data. If the algorithm is fed just the Experiment 1 data, the posterior distribution over the Tafel slope is broad and weakly bimodal, indicating that the Experiment 1 data is insufficient to discriminate between Model I and Model II. When fit to the Experiment 2A or 2B data in addition to the Experiment 1 data, this bimodality splits cleanly into two separate modes centered at the Model I and Model II Tafel slopes. Note that the linear fit to just the Experiment 1 data is distinct from both the Model I and Model II Tafel slopes.

218 The inset of Fig. 2A depicts a set of (synthetic) Tafel data measured over a 100 mV
219 overpotential window. To the eye, the data looks entirely linear, and fitting a Tafel slope
220 to the entire dataset using OLS regression yields a Tafel slope of 130 mV/decade, with a
221 standard error of 10 mV/decade. However, Fig. 2A illustrates the issue with this traditional
222 Tafel analysis. Indeed, the original set of green data in the narrow overpotential regime is
223 essentially entirely consistent, within experimental error, with two models possessing very
224 different Tafel slopes. Model I has a Tafel slope of 80 mV/decade, while Model II has a Tafel
225 slope of 120 mV/decade. Despite this wide berth in Tafel slope, the models align in the
226 initial overpotential window because of their distinct limiting currents i_{lim} , which differ by
227 a modest half order of magnitude. Clearly, the set of data measured in the inset of Fig. 2A
228 is insufficient to distinguish between the two models, but this data insufficiency is entirely
229 hidden by the traditional analysis approach.

230 Unlike the traditional analysis approach, Bayesian posterior sampling correctly identifies
231 the Tafel slope ambiguity present in the original data. The green trace in Fig. 2B depicts
232 the posterior distribution over the Tafel slope using solely the green data. This distribution
233 is broad and markedly multimodal, with concentrations of probability density around the
234 Model I and II Tafel slopes. In this manner, Bayesian posterior sampling correctly surmises
235 that the original data is insufficient to pin down a value of the Tafel slope with high cer-
236 tainty. One possible solution to this issue is to measure additional data over a wider range
237 of overpotentials. Depending on the true parameters of the electrochemical system under
238 measurement, this experiment could yield either the blue data or the red data in Fig. 2A.
239 When the Bayes posterior sampling algorithm is fed a combination of the green data and the
240 red data, it correctly predicts a posterior distribution of Tafel slopes concentrated around
241 the Model I Tafel slope. Conversely, when fed a combination of the green data and the blue
242 data, it correctly predicts a posterior distribution of Tafel slopes concentrated around the
243 Model II Tafel slope. In other words, multimodality in the posterior distribution predicted
244 by our algorithm is a hallmark of data insufficiency; when the underlying insufficiency is

245 addressed, the algorithm neatly splits the distributional modes according to the observed
246 data.

247 We highlight a couple of important conclusions from the synthetic data analysis presented
248 in Fig. 2. First, current-voltage data used for Tafel slope estimation should ideally be mea-
249 sured until clear curvature is observed. If such a measurement is unnecessarily inconvenient
250 or impossible, one should attempt to quantify the limiting current either through back-of-
251 the-envelope estimates or through direct experimental control over the limiting current (e.g.
252 with a rotating-disk electrode), and ensure that data used to estimate Tafel slopes is collected
253 well below the limiting current density. Without information that elucidates the magnitude
254 of the limiting current, it is impossible to ascertain the degree of limiting current-induced
255 attenuation suffered by the current measured in the Tafel regime. Consequently, Tafel slopes
256 estimated on the basis of a linear Tafel plot measured in a small overpotential window are
257 likely systematically unreliable, and can harbor significant unquantified uncertainty. Second,
258 the synthetic data analysis illustrates how the Bayesian posterior sampling approach can be
259 employed iteratively with data acquisition efforts. Since the posterior distributions accu-
260 rately quantify the uncertainty associated with a Tafel slope estimated given available data,
261 an experimentalist can use this uncertainty information to guide future data acquisition until
262 a desirable uncertainty threshold is achieved.

263 **Evaluating Cardinal Preferences in Literature Data**

264 In addition to being an important metric in assessing catalyst performance, the Tafel slope
265 can be valuable because it may yield insight into the mechanism of a catalyzed electro-
266 chemical reaction. The connection between the Tafel slope, a macroscopically-measurable
267 quantity, and the microscopic reaction mechanism is derived using microkinetic analysis in-
268 voking a whole host of ideality assumptions.^{2,3} For an electrochemical reaction that proceeds
269 through a number of elementary steps, one must assume that a single step determines the

270 rate, and that all steps prior to the rate-determining step (RDS) are in quasi-equilibrium.
271 Each of the quasi-equilibrated elementary steps carries an associated equilibrium constant
272 which is possibly dependent on the applied potential. For potential-dependent equilibrium
273 constants, one must additionally assume that the potential dependence goes exponentially in
274 the (strictly integer) number of electrons transferred in the elementary step. The RDS has
275 an associated forward rate constant, which is assumed to have a Butler-Volmer-like depen-
276 dence on the applied potential, with a symmetry coefficient $\alpha = 1/2$.¹ Under these restrictive
277 assumptions, one can derive (see SI) an equation for the Tafel slope of the entire chemical
278 reaction (at $T = 298$ K),

$$\text{Tafel Slope} = \frac{60 \text{ mV/decade}}{n + q/2}, \quad (5)$$

279 where n is the total number of electrons transferred in elementary steps prior to the RDS,
280 and q is the number of electrons transferred in the RDS.

281 Equation (5) gives rise to so-called “cardinal values” of the Tafel slope, which arise from
282 evaluating the Tafel slope for different values of (n, q) . Tafel slope values of 120, 60, and
283 40 mV/decade are familiar to most electrochemists, and arise from $(n, q) = (0, 1), (1, 0)$
284 and $(1, 1)$, respectively. Researchers routinely appeal to cardinal values to extract micro-
285 scopic insight from experimentally-measured Tafel slopes. A common argumentative thrust
286 goes: “my catalyst has a Tafel slope of 110 mV/decade, which is reasonably close to 120
287 mV/decade, indicating that the reaction proceeds through a rate-limiting first electron trans-
288 fer step.” This line of reasoning is only truly valid if the typical catalyst satisfies the ideality
289 assumptions involved in deriving Eq. (5), a point that has been emphasized several times in
290 the literature.³⁻⁵ Comprehensive analysis of literature Tafel data can shed light on whether a
291 typical catalyst satisfies these strict assumptions; if this is indeed true, then literature Tafel
292 slopes should tend to cluster around the cardinal Tafel values predicted by Eq. (5).

293 Our Bayesian posterior sampling algorithm for Tafel slope fitting allows us to carry out
294 an unbiased, automated survey of literature Tafel data to quantitatively test whether Tafel
295 slope values reported in the literature show any preference for cardinal values. In this study,

296 we choose to focus on re-analyzing Tafel data from the CO₂ reduction literature. We fo-
297 cus on this subsection of the literature because CO₂ reduction is a burgeoning field with
298 diverse catalyst materials and morphologies,³⁰ and because product quantification require-
299 ments place Tafel analysis in this field in the low-data regime, as discussed previously. To
300 carry out the literature survey, we digitized 344 distinct Tafel datasets from the CO₂ reduc-
301 tion literature and fed the resultant data to the Bayesian posterior sampling algorithm to
302 produce re-analyzed estimates of the Tafel slope. Further information on the data mining
303 and analysis procedure can be found in the Methods section.

304 Our re-analysis procedure uses Eq. (2) to interpret the literature Tafel datasets. As
305 mentioned earlier, this model is only truly appropriate in the case of one kinetic control
306 regime associated with a single Tafel slope; it cannot accurately capture current-voltage
307 behavior under multiple kinetic regimes, which may be operative in at least some of the
308 datasets we have analyzed. However, absent independent experimental confirmation of the
309 physical mechanism underlying the observed multiple kinetic regimes (e.g. from spectroscopy
310 or surface imaging), it is difficult to rigorously select a single model from the plethora that
311 arise from enumerating microkinetic possibilities for intermediates in CO₂ reduction. Since
312 the papers from the literature that we have re-analyzed fit a single Tafel slope to their data,
313 and because they lack the experimental evidence required to pin down a richer physical
314 model describing their data, we believe that uniform application of the model in Eq. (2)
315 is an appropriate choice for our literature survey study. Our usage of Eq. (2) to interpret
316 the data should *not* be construed as a blanket endorsement of this model in Tafel analysis.
317 Indeed, if there is solid experimental evidence motivating the usage of a different kinetic
318 model for a specific CO₂ reduction catalyst system, it can and should be employed under
319 our Bayesian framework.

320 Figure 3A depicts a correlation plot of the MAP Tafel slope estimated by the Bayes
321 posterior sampling approach versus the literature-reported Tafel slope. A significant fraction
322 of the datasets fall within the 20% parity line, a strong sign that our algorithm produces Tafel

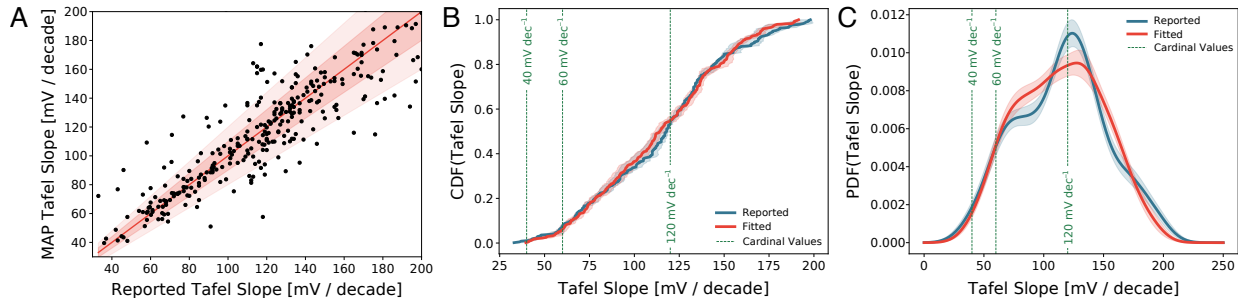


Figure 3: Unbiased refit of literature data using our Bayesian analysis approach reveals little preference for cardinal values of the Tafel slope for CO₂ reduction catalysts. (A) Correlation plot of reported Tafel slopes from the literature against MAP Tafel slopes fitted by our algorithm on identical data. The solid red line represents perfect agreement, while the red filled intervals are lines representing 10% and 20% relative error. (B) Cumulative distribution function of the Tafel slopes reported in literature data (blue), and those refitted by our algorithm (red). Error intervals correspond to one standard deviation of bootstrapped resamples. (C) Kernel density estimates (KDE) of the empirical probability distribution function of Tafel slopes reported in literature data (blue) and MAP Tafel slopes refitted by our algorithm (red). Error intervals correspond to one standard deviation of bootstrapped resamples. Green dashed lines in both (B, C) correspond to cardinal values of the Tafel slope predicted by Eq. (5).

323 slopes that are consistent with literature values when seeing identical data. Additionally,
 324 the MAP Tafel slope does not seem to systematically overestimate or underestimate the
 325 literature-reported value over a wide range of reported Tafel slopes. We note that complete
 326 parity between the MAP and literature Tafel slopes should not be expected; as explained
 327 previously, due to the possibility for subjectivity and systematic error with current literature
 328 practice for Tafel estimation, the MAP estimates derived by our algorithm are arguably *more*
 329 trustworthy than the literature-reported values.

330 Figures 3B and 3C depict estimates of the distributional tendencies of the MAP and
 331 literature-reported Tafel slopes. Figure 3B plots the empirical cumulative distribution func-
 332 tion (CDF) of the MAP Tafel slope (red trace) and the literature-reported Tafel slopes (blue
 333 trace). The low-opacity intervals in both Fig. 3B and Fig. 3C span one standard deviation
 334 of several bootstrapped resamples drawn with replacement (see Methods section for addi-
 335 tional detail on the bootstrapping procedure), and are useful for examining the sensitivity
 336 of our distributional results to the specific subsampling of literature data we have chosen to

337 analyze.³¹ The CDF value for a given Tafel slope value m_T tallies the running fraction of
338 datasets that have a Tafel slope value of at most m_T . If Tafel slopes truly cluster around
339 cardinal values, the running fraction should increase sharply around those preferred values,
340 and one would expect to see sigmoidal features in the CDF at the cardinal values. Figure 3
341 shows little evidence of such locally sigmoidal behavior; rather, we see something resembling
342 a straight line, corresponding to a roughly uniform distribution over the range of Tafel slopes
343 considered.

344 We can visualize the distributional data in a different way by examining the empirical
345 probability distribution function (PDF) of the Tafel slopes. Estimating the PDF of a distri-
346 bution given a set of samples is a notoriously difficult problem in statistics, because relatively
347 small amounts of sampling noise can result in the presence of spurious peaks in the PDF.
348 Several techniques exist for tackling this problem; here, we employ Gaussian kernel density
349 estimation, which constructs an estimate of the PDF by summing appropriately-normalized
350 Gaussian kernel functions centered at each of the observed data points. Additional details
351 on the kernel density estimation procedure are reported in the Methods section. Figure 3C
352 shows a kernel density estimate of the PDF of the MAP (red trace) and literature-reported
353 (blue trace) Tafel slopes. Based on the shape of the PDFs and their corresponding stan-
354 dard errors, we conclude that there is a slight preference to assign Tafel slope values around
355 70 and 125 mV/decade in the literature that essentially disappears when the same data is
356 re-analyzed with our approach. While a small peak persists around 125 mV/decade in the
357 MAP Tafel slopes, the height of the peak is roughly within the error bound. We also note
358 that any residual preference for cardinal values in the MAP Tafel slopes can possibly be
359 explained by data *acquisition* biases. While the Bayesian approach we develop removes sub-
360 jectivity from data analysis, we cannot remove biases introduced during data collection; it is
361 at least plausible that such biases exist given that the literature-reported values significantly
362 overestimate the concentration of Tafel slopes around 120 mV/decade compared to the MAP
363 values.

364 The results presented in Fig. 3 combine data from several studies using different catalyst
365 materials to assess whether or not a preference for cardinal Tafel slopes exists broadly across
366 all CO₂ reduction catalysts. In order to confirm that this apparent lack of cardinal preference
367 in the entire dataset is not simply an artifact introduced by pooling together separate cata-
368 lyst materials which each individually exhibit a cardinal preference, we broke out the PDF
369 analysis in Fig. 3 according to catalyst material identity. Upon examining the results from
370 the breakout analysis (see SI), we conclude that the apparent lack of cardinality we find in
371 Fig. 3 indeed persists when separately examining Tafel slopes from common materials used
372 in CO₂ reduction catalysts: Ag, Au, Cu, Sn, Zn. Curiously, it appears that Bi-based mate-
373 rials *do* show a preference for Tafel slope values around 120 mV/decade, which may inform
374 future mechanistic studies on these catalysts. As a caveat, we note our Bi results comprise
375 only 27 distinct Tafel datasets, and are hence subject to a high degree of variability arising
376 from the specific set of studies we chose to re-analyze; future work that attempts to weigh
377 in on this question should ideally perform new experiments on well-controlled Bi surfaces
378 and use the data acquisition and analysis recommendations identified in this work. Taken
379 together, the results presented in Fig. 3 and the material breakout analysis in the SI lead us
380 to conclude that, when analyzed in an unbiased fashion, experimental data in the literature
381 does not support a systematic preference for cardinal values of the Tafel slope among CO₂
382 reduction catalysts.

383 While we are not in a position to identify the cause for deviation from cardinal Tafel slope
384 values in each specific Tafel dataset comprising Fig. 3, we advance the hypothesis that these
385 deviations could originate from physical non-idealities that violate the assumptions involved
386 in deriving Eq. (5). To test this hypothesis, we attempt to ascertain the manner in which
387 a select few simple physical non-idealities can adjust the PDF of Tafel slopes that would
388 otherwise be concentrated around cardinal values. In this regard, we consider three possible
389 physical effects that violate the ideality assumptions used to arrive at Eq. (5). These three
390 effects comprise a very small subset of the menagerie of physical non-idealities that could

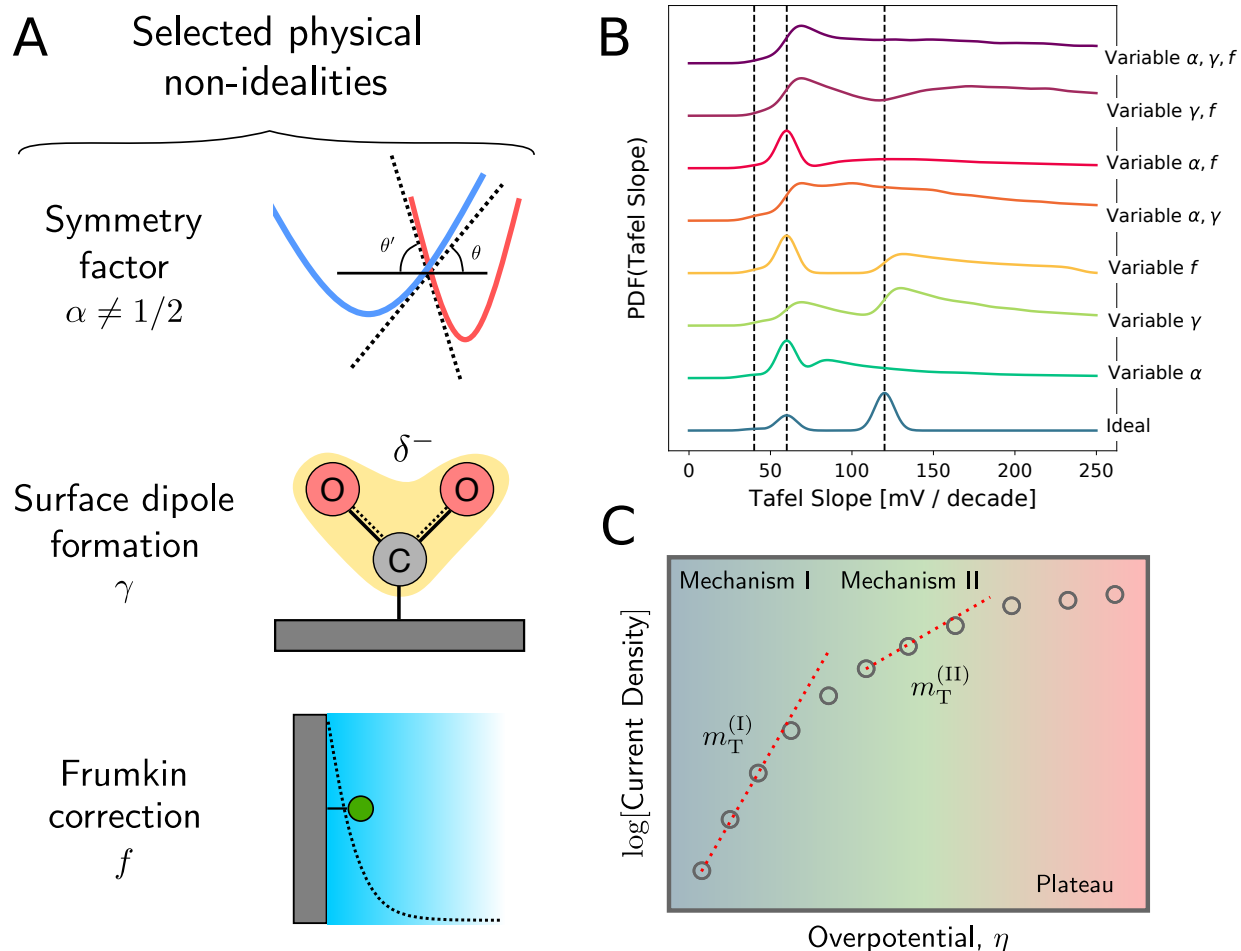


Figure 4: Physical hypotheses for the lack of observed cardinality in literature Tafel slopes. (A) Schematic of three selected physical non-idealities which can affect the measured Tafel slope. (B, blue trace) Synthetic kernel density estimate of the probability distribution over Tafel slopes for a “random” CO_2 reduction catalyst, peaked around the cardinal values predicted by Eq. (5). (B, other traces) Several synthetic kernel density estimates of the probability distributions over the Tafel slope generated from including random values of different parameters governing physical non-idealities. (C) Schematic illustrating the possibility of measuring data across separate kinetic regimes in a Tafel analysis. Due to a switch in mechanism, different overpotential regimes exhibit different Tafel slopes, complicating interpretation of a single Tafel slope value fit straddling both regimes.

391 operate in CO₂ reduction electrocatalysis; our goal is simply to show that these effects *can*
392 spoil a preference for cardinal values of the Tafel slope, not to single out these particular
393 effects as the only non-idealities present in CO₂ reduction.

394 First, we consider the possibility that the symmetry coefficient $\alpha \neq 1/2$. Such deviations
395 could arise, for example, due to disparate local slopes of the Marcus free energy surfaces at
396 their crossing point, or reactant species position fluctuations in the electrochemical double
397 layer.^{32–35} Second, we examine the effect of partial charge transfer or surface dipole formation
398 in the adsorption of CO₂ to the electrode surface, phenomena that have been hypothesized
399 and characterized in prior studies on CO₂ reduction.^{29,36} Mathematically, the formation of
400 a surface dipole introduces a potential dependence in the adsorption equilibrium constant
401 which mathematically resembles a partial charge transfer parameter γ . Third, we introduce
402 a possible Frumkin correction f originating from the protrusion of an electrode-adsorbed
403 species into the electrochemical double layer, which attenuates the applied potential due to
404 electrostatic screening effects.¹ The Frumkin correction is most important at low supporting
405 salt concentration (and hence large resultant electrolyte screening length), but it has been
406 considered in the context of CO₂ reduction electrocatalysis.²⁹ Each of these non-idealities
407 depends on the value of a non-ideality parameter. Of course, we have no idea, at least *a*
408 *priori*, about the distribution of values these non-ideality parameters can take in typical CO₂
409 reduction catalyst systems. In the absence of information, we make the maximally ignorant
410 choice, and assume that the non-ideality parameters are drawn from uniform distributions
411 within a reasonable set of bounds. Once we postulate these uniform distributions, we can
412 examine how randomly selected non-ideality parameters deform a distribution of Tafel slopes
413 that begins concentrated around cardinal values.

414 The blue trace in Fig. 4B shows a distribution of Tafel slopes concentrated around the
415 cardinal values predicted by Eq. (5). The relative heights of the cardinal value peaks are
416 selected by artificially binning the distribution over MAP Tafel slopes into buckets centered
417 around the cardinal values. The effects of randomly drawn physical non-idealities on this

418 distribution can be examined using Monte Carlo simulation. Briefly, we sample a Tafel slope
419 from the distribution depicted in the blue trace, sample the values of one or more non-ideality
420 parameters, and finally calculate the resultant Tafel slope in the presence of non-idealities
421 (see Methods section for additional details). Repeating the sampling procedure several times
422 yields distributions over the Tafel slope, depicted as multicolored traces in Fig. 4 for different
423 sets of non-idealities. Evidently, even rather mundane pieces of additional physics like the
424 ones discussed above can produce stark changes in the distribution of Tafel slopes, spanning
425 a range of behavior from moving certain peaks away from cardinal values to smearing out
426 the entire distribution. While we cannot prove with certainty that physical non-idealities
427 are responsible for the lack of observed Tafel cardinality in the literature, the results in Fig.
428 4 demonstrate that this is at least a plausible explanation for the observed behavior.

429 Alternatively, the observed lack of cardinality may also be a consequence of interpreting
430 current-voltage data measured under several disparate kinetic regimes through the lens of
431 Eq. (2), which cannot capture these intricacies. As illustrated schematically in Fig. 4C, the
432 kinetic regimes may exhibit different Tafel slopes; in this case, mechanistic interpretation
433 of a single Tafel slope extracted by fitting Eq. (2) to the data is inappropriate. Indeed, as
434 examined in more detail in the SI, fitting synthetic data generated from a model with multiple
435 cardinal Tafel slope regimes using Eq. (2) can produce an off-cardinal Tafel slope value. This
436 underscores the need to rigorously characterize the several physical complexities present
437 in catalytic systems for CO₂ reduction, as they can complicate mechanistic interpretation
438 guided solely by the Tafel slope.

439 Taken in their entirety, the results presented in Figures 3 and 4 present some compelling
440 reasons for the community to rethink the current approach of deducing mechanistic informa-
441 tion purely from Tafel slope data. Indeed, the prevalence of Tafel slopes in the CO₂ reduction
442 literature that do not fall neatly on cardinal values suggests that physical non-idealities, omit-
443 ted by Eq. (5), may be commonplace in typical catalytic systems. The familiar approach
444 of “rounding” experimentally-measured Tafel slopes to their nearest cardinal value to guide

445 mechanistic interpretation, then, leaves one prone to interpreting experimental data in an
446 overly simplistic manner. Rather than hand-wave away the physical complexities present in
447 catalytic systems with strong ideality assumptions, we believe it is important to interpret
448 Tafel data alongside several other pieces of experimental data (e.g. more diverse electro-
449 chemical kinetic data, surface-sensitive spectroscopy, materials characterization, etc.). In
450 this manner, one can take a non-cardinal Tafel slope (and its associated uncertainty based
451 on the data) at face value, and build a holistic physical picture that attempts to explain the
452 deviation from cardinality in a manner consistent with all other experimental observations.
453 Ideally, all these observations can be interpreted in the context of a richer model that allows
454 one to determine the true breadth of physical phenomena present across a wide range of
455 operating parameters, as has been done in some select studies in the literature.^{2,28,37} We
456 believe our Bayesian data analysis approach will be equally useful for rigorously quantifying
457 parametric uncertainties in the suggested new paradigm for kinetic data interpretation.

458 **Methods**

459 **Data Mining and Re-Analysis**

460 We built up a dataset of Tafel measurements reported in the literature by manually extract-
461 ing figures from published papers and digitizing them using the WebPlotDigitizer tool.³⁸ A
462 full accounting of all papers and corresponding figures can be found in the SI. When select-
463 ing datasets to analyze, we excluded those that reported continuous current-voltage data,
464 because it is difficult to ascertain the underlying data density associated with a continuous
465 curve, because our method is meant to address the unique challenges of estimating Tafel
466 slopes with a small amount of data, and because continuous current-voltage data may be
467 unreliable because product selectivity is not always 100%, especially in CO₂ reduction. We
468 also excluded datasets that reported current-voltage data but did not report an explicit value
469 of the Tafel slope. We assumed that all datasets were collected with appropriate experimen-

470 tal techniques (IR-correction for solution resistance has already been applied, etc.), and did
471 not modify or omit any data from a figure during the digitization process. After digitiza-
472 tion, each dataset was tagged with manually entered metadata to facilitate re-analysis. A
473 full accounting of the metadata fields, as well as a complete record of all scraped data and
474 metadata, is available in the SI.

475 Re-analysis of the data was carried out using the Python-based `julius` package, de-
476 veloped in-house to handle data collation and Bayesian posterior sampling workflows. In
477 order to determine prior distributions for the parameters, we first find an optimal set of
478 parameters θ^* for the limiting current model using an implementation of the trust-region re-
479 flective (TRF) algorithm implemented in the optimization and root finding package included
480 in SciPy.³⁹ For each model parameter θ_i , we select uniform prior distributions supported on
481 the interval $[0, a \times \theta_i^*]$, with $a = 10$ (note that all parameters we fit in this study are strictly
482 non-negative). For all models studied here, we find that the posterior distribution does not
483 depend on the value of a , indicating that the data imposes strong preferences on the optimal
484 model fit (see SI for detailed sensitivity analysis). For each dataset, we draw $N = 4 \times 10^4$
485 total samples (10^4 samples from four independent chains, each burning their first 2000 sam-
486 ples) from the posterior distribution using the No-U-Turn Hamiltonian Monte Carlo sampler
487 (NUTS) implemented in the PyMC3 probabilistic programming package.⁴⁰

488 In total, we re-analyzed 344 distinct Tafel datasets. Figure 3A restricts to both reported
489 and MAP Tafel slopes $m_T \in [0, 200]$, which comprises 300 distinct Tafel datasets. A corre-
490 lation plot including all analyzed Tafel datasets is reported in the SI.

491 **Kernel Density Estimation**

492 We use kernel density estimation (KDE) to estimate probability distributions given a finite
493 set of samples. KDEs are used in the distributional visualizations in Figs. 2, 3, and 4. We
494 use the Gaussian KDE function in the statistics package included in SciPy, and use Scott's
495 rule for bandwidth selection in Figs. 2 and 3. Since the estimates in Fig. 4 are meant to

496 emulate the result of a single simulated experimental observation with some associated error,
497 here we use a pre-specified bandwidth of 6 mV/decade.

498 **Bootstrap Resampling**

499 We carry out a bootstrap resampling procedure to quantify the degree of variability of the
500 results in Fig. 3 associated with our choice of a specific subset of literature data.³¹ Essentially,
501 we posit that the observed distribution over the Tafel slope is a good estimate of the true
502 underlying distribution, and then resample several datasets of the same size as the original
503 dataset from this distribution with *replacement* (i.e. samples can show up more than once, or
504 not at all). The error intervals presented in Figs. 3A and 3B are gleaned from one standard
505 deviation of 20 such bootstrapped resamples.

506 **Monte Carlo Simulation**

507 We use Monte Carlo simulation to estimate the distributional changes precipitated in a Tafel
508 slope distribution by the physical non-idealities identified in the main text. To carry out
509 this procedure, we begin with the distribution presented in Fig. 4A, which is generated
510 by artificially bucketing the MAP Tafel slopes from the literature analysis into the bins
511 $\{[0, 50), [50, 90), [90, \infty)\}$. We sample the physical non-ideality parameters according to $\alpha \sim$
512 $\text{Unif}[0.20, 0.80]$, $\gamma \sim \text{Unif}[0, 1]$, $f \sim \text{Unif}[0.50, 1.00]$, where $\text{Unif}[a, b]$ signifies a uniform
513 distribution supported on the interval $[a, b]$. For each set of non-idealities, we draw $N =$
514 4×10^4 total posterior samples (10^4 samples from four independent chains, each burning
515 their first 500 samples). The equations governing modifications to the Tafel slope based
516 on physical nonidealities are worked out in the SI. A sensitivity analysis of the Tafel slope
517 distributions including non-idealities with respect to the bounds of the uniform distributions
518 over non-ideality parameters is also reported in the SI. All Monte Carlo simulation is again
519 carried out using the PyMC3 probabilistic programming package.⁴⁰

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526 **Author Contributions**

527 Conceptualization: AML, APW, and KM; Methodology: AML, JSZ, APW, and KM; Soft-
528 ware: AML; Reproduction: JSZ and AML; Investigation: AML, APW, and KM; Writing
529 (Original Draft): AML; Writing (Review and Editing): AML, JSZ, APW, and KM; Super-
530 vision: APW and KM.

531 **Competing Interests**

532 The authors have no competing interests to declare.

533 **Data and Code Availability**

534 Code that supports the findings of this study is available under the MIT License ([https://](https://opensource.org/licenses/MIT)
535 opensource.org/licenses/MIT) in Zenodo (<http://doi.org/10.5281/zenodo.3995021>).
536 Data that supports the findings of this study is available under CC BY 4.0 ([https://](https://creativecommons.org/licenses/by/4.0/)
537 creativecommons.org/licenses/by/4.0/) in Zenodo ([http://doi.org/10.5281/zenodo.](http://doi.org/10.5281/zenodo.3995021)
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References

- (1) Bard, A. J.; Faulkner, L. R.; Leddy, J.; Zoski, C. G. *Electrochemical methods: fundamentals and applications*; Wiley New York, 1980; Vol. 2.
- (2) Marshall, A. T. Using microkinetic models to understand electrocatalytic reactions. *Current Opinion in Electrochemistry* **2018**, *7*, 75–80.
- (3) Shinagawa, T.; Garcia-Esparza, A. T.; Takanabe, K. Insight on Tafel slopes from a microkinetic analysis of aqueous electrocatalysis for energy conversion. *Scientific reports* **2015**, *5*, 13801.
- (4) Fang, Y.-H.; Liu, Z.-P. Tafel kinetics of electrocatalytic reactions: from experiment to first-principles. *ACS Catalysis* **2014**, *4*, 4364–4376.
- (5) Dunwell, M.; Luc, W.; Yan, Y.; Jiao, F.; Xu, B. Understanding surface-mediated electrochemical reactions: CO₂ reduction and beyond. *ACS Catalysis* **2018**, *8*, 8121–8129.
- (6) Gattrell, M.; Gupta, N.; Co, A. A review of the aqueous electrochemical reduction of CO₂ to hydrocarbons at copper. *Journal of Electroanalytical Chemistry* **2006**, *594*, 1–19.
- (7) Chang, X.; Wang, T.; Zhao, Z.-J.; Yang, P.; Greeley, J.; Mu, R.; Zhang, G.; Gong, Z.; Luo, Z.; Chen, J., et al. Tuning Cu/Cu₂O interfaces for the reduction of carbon dioxide to methanol in aqueous solutions. *Angewandte Chemie* **2018**, *130*, 15641–15645.
- (8) Zhang, L.; Zhao, Z.-J.; Gong, J. Nanostructured materials for heterogeneous electrocatalytic CO₂ reduction and their related reaction mechanisms. *Angewandte Chemie International Edition* **2017**, *56*, 11326–11353.
- (9) Lee, C. W.; Cho, N. H.; Yang, K. D.; Nam, K. T. Reaction mechanisms of the electrochemical conversion of carbon dioxide to formic acid on tin oxide electrodes. *ChemElectroChem* **2017**, *4*, 2130–2136.

- 566 (10) Lu, Q.; Rosen, J.; Jiao, F. Nanostructured metallic electrocatalysts for carbon dioxide
567 reduction. *ChemCatChem* **2015**, *7*, 38–47.
- 568 (11) Lee, C. W.; Cho, N. H.; Im, S. W.; Jee, M. S.; Hwang, Y. J.; Min, B. K.; Nam, K. T.
569 New challenges of electrokinetic studies in investigating the reaction mechanism of
570 electrochemical CO₂ reduction. *Journal of Materials Chemistry A* **2018**, *6*, 14043–
571 14057.
- 572 (12) Corbin, N.; Zeng, J.; Williams, K.; Manthiram, K. Heterogeneous molecular catalysts
573 for electrocatalytic CO₂ reduction. *Nano Research* **2019**, 1–33.
- 574 (13) Lu, Q.; Jiao, F. Electrochemical CO₂ reduction: Electrocatalyst, reaction mechanism,
575 and process engineering. *Nano Energy* **2016**, *29*, 439–456.
- 576 (14) Schmickler, W.; Santos, E. *Interfacial electrochemistry*; Springer Science & Business
577 Media, 2010.
- 578 (15) Clark, E. L.; Resasco, J.; Landers, A.; Lin, J.; Chung, L.-T.; Walton, A.; Hahn, C.;
579 Jaramillo, T. F.; Bell, A. T. Standards and protocols for data acquisition and reporting
580 for studies of the electrochemical reduction of carbon dioxide. *ACS Catalysis* **2018**, *8*,
581 6560–6570.
- 582 (16) Williams, K.; Corbin, N.; Zeng, J.; Lazouski, N.; Yang, D.-T.; Manthiram, K. Pro-
583 tecting effect of mass transport during electrochemical reduction of oxygenated carbon
584 dioxide feedstocks. *Sustainable Energy & Fuels* **2019**, *3*, 1225–1232.
- 585 (17) Singh, M. R.; Clark, E. L.; Bell, A. T. Effects of electrolyte, catalyst, and membrane
586 composition and operating conditions on the performance of solar-driven electrochemi-
587 cal reduction of carbon dioxide. *Physical Chemistry Chemical Physics* **2015**, *17*, 18924–
588 18936.

- 589 (18) Singh, M. R.; Goodpaster, J. D.; Weber, A. Z.; Head-Gordon, M.; Bell, A. T. Mechanistic insights into electrochemical reduction of CO₂ over Ag using density functional
590 theory and transport models. *Proceedings of the National Academy of Sciences* **2017**,
591 *114*, E8812–E8821.
592
- 593 (19) Zhang, B. A.; Ozel, T.; Elias, J. S.; Costentin, C.; Nocera, D. G. Interplay of homoge-
594 neous reactions, mass transport, and kinetics in determining selectivity of the reduction
595 of CO₂ on Gold electrodes. *ACS Central Science* **2019**, *5*, 1097–1105.
- 596 (20) Brown, S. M. Catalysis and reactor engineering for the electrochemical conversion of
597 carbon dioxide to carbon monoxide. Ph.D. thesis, Massachusetts Institute of Technol-
598 ogy, 2019.
- 599 (21) Voiry, D.; Chhowalla, M.; Gogotsi, Y.; Kotov, N. A.; Li, Y.; Penner, R. M.;
600 Schaak, R. E.; Weiss, P. S. Best practices for reporting electrocatalytic performance of
601 nanomaterials. 2018.
- 602 (22) Rosenfeld, M. J. OLS in Matrix Form. [https://web.stanford.edu/~mrosenfe/soc_](https://web.stanford.edu/~mrosenfe/soc_meth_proj3/matrix_OLS_NYU_notes.pdf)
603 [meth_proj3/matrix_OLS_NYU_notes.pdf](https://web.stanford.edu/~mrosenfe/soc_meth_proj3/matrix_OLS_NYU_notes.pdf).
- 604 (23) Manthiram, K.; Beberwyck, B. J.; Alivisatos, A. P. Enhanced electrochemical metha-
605 nation of carbon dioxide with a dispersible nanoscale copper catalyst. *Journal of the*
606 *American Chemical Society* **2014**, *136*, 13319–13325.
- 607 (24) Deen, W. *Analysis of Transport Phenomena*; Topics in chemical engineering; Oxford
608 University Press, 2012.
- 609 (25) Hsu, S.-H.; Stamatis, S. D.; Caruthers, J. M.; Delgass, W. N.; Venkatasubramanian, V.;
610 Blau, G. E.; Lasinski, M.; Orcun, S. Bayesian framework for building kinetic models of
611 catalytic systems. *Industrial & engineering chemistry research* **2009**, *48*, 4768–4790.

- 612 (26) Grinstead, C. M.; Snell, J. L. *Introduction to probability*; American Mathematical Soc.,
613 2012.
- 614 (27) Chandler, D.; Wu, D. *Introduction to Modern Statistical Mechanics*; Oxford University
615 Press, 1987.
- 616 (28) Zeng, J. S.; Corbin, N.; Williams, K.; Manthiram, K. Kinetic Analysis on the Role of Bi-
617 carbonate in Carbon Dioxide Electroreduction at Immobilized Cobalt Phthalocyanine.
618 *ACS Catalysis* **2020**, *10*, 4326–4336.
- 619 (29) Ringe, S.; Morales-Guio, C. G.; Chen, L. D.; Fields, M.; Jaramillo, T. F.; Hahn, C.;
620 Chan, K. Double layer charging driven carbon dioxide adsorption limits the rate of
621 electrochemical carbon dioxide reduction on Gold. *Nature Communications* **2020**, *11*,
622 1–11.
- 623 (30) Lim, R. J.; Xie, M.; Sk, M. A.; Lee, J.-M.; Fisher, A.; Wang, X.; Lim, K. H. A review
624 on the electrochemical reduction of CO₂ in fuel cells, metal electrodes and molecular
625 catalysts. *Catalysis Today* **2014**, *233*, 169–180.
- 626 (31) Efron, B.; Tibshirani, R. J. *An introduction to the bootstrap*; CRC press, 1994.
- 627 (32) Bockris, J. O.; Nagy, Z. Symmetry factor and transfer coefficient. A source of confusion
628 in electrode kinetics. *Journal of Chemical Education* **1973**, *50*, 839.
- 629 (33) Chandler, D. Electron transfer in water and other polar environments, how it happens.
630 Classical and Quantum Dynamics in Condensed Phase Simulations. 2010; pp 25–49,
631 DOI: 10.1142/9789812839664_0002.
- 632 (34) Guidelli, R.; Compton, R. G.; Feliu, J. M.; Gileadi, E.; Lipkowski, J.; Schmickler, W.;
633 Trasatti, S. Defining the transfer coefficient in electrochemistry: An assessment (IUPAC
634 Technical Report). *Pure and Applied Chemistry* **2014**, *86*, 245–258.

- 635 (35) Limaye, A. M.; Willard, A. P. Modeling interfacial electron transfer in the double
636 layer: the interplay between electrode coupling and electrostatic driving. *The Journal*
637 *of Physical Chemistry C* **2019**,
- 638 (36) Guidelli, R.; Schmickler, W. *Modern Aspects of Electrochemistry*; Springer, 2005; pp
639 303–371.
- 640 (37) Adanuvor, P. K.; White, R. E. Analysis of Electrokinetic Data by Parameter Estimation
641 and Model Discrimination Techniques. *Journal of the Electrochemical Society* **1988**,
642 *135*, 1887.
- 643 (38) Rohatgi, A. WebPlotDigitizer. 2017.
- 644 (39) Virtanen, P.; Gommers, R.; Oliphant, T. E.; Haberland, M.; Reddy, T.; Cournau-
645 peau, D.; Burovski, E.; Peterson, P.; Weckesser, W.; Bright, J., et al. SciPy 1.0: fun-
646 damental algorithms for scientific computing in Python. *Nature Methods* **2020**, *17*,
647 261–272.
- 648 (40) Salvatier, J.; Wiecki, T. V.; Fonnesbeck, C. Probabilistic programming in Python using
649 PyMC3. *PeerJ Computer Science* **2016**, *2*, e55.