

# Endogenously incomplete markets: Macroeconomic implications

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## Abstract

Endogenously incomplete models derive restrictions on asset trading from primitive constraints on the enforcement and monitoring technologies available to societies. They have been applied to a wide variety of macroeconomic problems. This essay reviews some of these applications and the models that underpin them.

## 1 Introduction

An asset trading arrangement is incomplete if it is too restrictive to ensure a fully Pareto optimal allocation of risk. Endogenously incomplete models derive such trading arrangements from primitive frictions. They are to be contrasted with models that *assume* a particular incomplete asset markets structure.

Recent contributions to the endogenous incompleteness literature have emphasized imperfections in the enforcement and monitoring technologies available to societies. They derive endogenous market structures, sometimes supplemented with a tax system, as decentralizations of planning problems in which the planner faces one or both of these imperfections. These market-tax structures ensure that agents are provided with incentives to honor promises that cannot be costlessly enforced or that are contingent on states that cannot be costlessly observed. By construction they admit equilibria that are constrained efficient.

Models with endogenous incompleteness have received a variety of applications in macroeconomics. They have been used to enhance understanding of risk sharing, asset pricing and business cycles; on the normative side they have been applied to analyses of optimal fiscal policy. Below, I review some of these applications and the models that underpin them.

## 2 Limited enforcement

The canonical example of a limited enforcement model is the bilateral insurance game of Kocherlakota (1996). In this, two risk averse agents are endowed with

random and imperfectly correlated income processes. Neither agent can be compelled to deliver resources to the other, even if they have promised to do so in the past.

Equilibrium allocations in this setting can be implemented with strategies that revert to autarky following an agent defection. Agents with high income shocks can be induced to share some of their resources by the threat of such reversion and, when this is insufficient, by promises of extra resources in the future. Such promises introduce additional dynamics into optimal equilibrium allocations; shocks that cannot be smoothed over states are smoothed over time instead, ensuring that individual consumption is persistent even when aggregate consumption is not.

Constrained efficient allocations in limited enforcement economies can be decentralized using a complete set of Arrow security markets coupled with endogenous debt limits (see, Alvarez and Jermann (2000)). Intuitively, agents can only borrow up to the amount that they are willing to pay back in the future given that the penalty for default is consignment to autarky. Thus, the limited enforcement friction provides a micro-foundation for the often-made assumption of a debt limit tighter than that implied by an agent's intertemporal budget constraint. In the limited enforcement case, however, the debt limit is state-contingent; it depends upon the value of autarky to the agent. Since this value is a function of individual and aggregate shocks, the parameters of the shock process and, in richer models, the agent's opportunities for self or public insurance after exclusion from markets, so too is the debt limit.

When agents' endogenous debt limits periodically bind, risk sharing is disrupted; individual consumption, conditional on the aggregate state, is positively correlated with current and past individual income. Qualitatively, such departures from full risk sharing cohere well with evidence on individual consumption. In Alvarez and Jermann's (2001) quantitative analysis of a calibrated limited enforcement model, the endogenous debt limits bind fairly often and permit relatively little risk sharing. This is consistent with evidence on the sharing of low frequency risks. Their analysis also has implications for asset pricing. They obtain a volatile asset pricing kernel and risk premia that are large and time varying. These implications are consistent with asset pricing data, but contrast with those of the benchmark representative agent asset pricing model.

Cross-country consumption data also exhibit apparent departures from full risk sharing. Standard models (with complete markets) imply comovements in consumption that exceed those in output, yet the data suggests the reverse. Kehoe and Perri (2002) show that a limited enforcement model augmented with production and physical capital accumulation can go some way to explaining this anomaly.

Recent papers have considered alternative penalties for default including the confiscation of an endogenously valued collateral asset (see, Lustig, 2005) or the payment of a fixed default cost (Cooley, Marimon and Quadrini (2004)). These contributions illustrate the scope of limited enforcement models: Lustig explores the implications of endogenously valued collateral for asset pricing and obtains a large and time varying price of risk; Cooley, Marimon and Quadrini

examine the role of limited enforcement frictions in propagating business cycle shocks. Cordoba (2005) and Arpad and Cárceles-Poveda (2005), however, sound cautionary notes. They provide calibrated models in which the introduction of collateral relaxes endogenous debt limits so much that agents can fully diversify risk.

### 3 Private information

An alternative line of research has analyzed environments in which risk averse agents privately observe shocks to their endowments, tastes or productivity (see, for example, Atkeson and Lucas (1992)). In this setting, agents must be provided with incentives to reveal information. The socially efficient provision of incentives requires the conditioning of current consumption on an agent's history of shock reports. Intuitively, agents are rewarded for reporting a low current need for resources with the promise of more consumption in the future. Thus, intertemporal consumption smoothing is enhanced and interstate smoothing disrupted.

Albanesi and Sleet (2006) and Kocherlakota (2005) show that optimal information-constrained allocations can be implemented with a mixture of non-contingent debt markets and taxes. Thus, these authors derive joint restrictions on the market structure *and* the tax system from primitive informational frictions. Central to their analyses is an "inverted Euler equation". If  $\{c_t^*\}_{t=0}^\infty$  denotes the optimal consumption allocation, this equation is given by:

$$\frac{1}{u'(c_t^*(z^t, \theta^t))} = \beta \lambda_{t+1}(z^{t+1}) E_t \left[ \frac{1}{u'(c_{t+1}^*)} | z^{t+1}, \theta^t \right]. \quad (1)$$

Here  $\theta^t$  denotes an agent's period  $t$  history of privately observed shocks,  $z^t$  and  $z^{t+1}$  denote  $t$  and  $t + 1$  histories of observable aggregate shocks,  $\beta$  is the agent's discount factor and  $u'$  her marginal utility of consumption.  $\lambda_{t+1}$  is a social stochastic discount factor (SSDF) that "prices" resources delivered after each history  $z^{t+1}$ . Golosov, Kocherlakota and Tsyvinski (2003) show that such equations hold in a large class of dynamic moral hazard models. They imply a wedge between an agent's conditional expected intertemporal marginal rate of substitution (IMRS) and the SSDF. This wedge provides a rationale for asset taxation; intuitively, agents must be discouraged from saving at date  $t$  since greater wealth at  $t + 1$  undermines incentives at that date. However, the implications for asset taxation are subtle. The optimal allocation cannot be implemented with an asset tax that merely "matches the wedge" and equates the conditional expectation of an agent's IMRS to the SSDF. Instead, marginal asset taxes at  $t + 1$  are used to generate a positive covariance between the after-tax asset return and the agent's consumption that deters savings. In some cases, the expected asset tax is zero and the wedge is entirely generated by this covariance effect.

Positive analyses of dynamic moral hazard are scarce. Green and Oh (1991) contrast the empirical implications of various incomplete market models, in-

cluding those with moral hazard. Kocherlakota and Pistaferri (2005) identify  $\lambda_{t+1}$  with the market discount factor, assume that utility is CRRA and use (1) to derive expressions for  $\lambda_{t+1}$  in terms of cross sectional moments of the consumption distribution. They then investigate the implications of this dynamic moral hazard model for asset pricing and, in particular, the equity premium and risk free rate. They find that plausible values of the coefficient of relative risk aversion set the equity premium pricing error to zero.

In all of the dynamic moral hazard models described so far, the consumption of agents is observable. An alternative assumption is that agents can undertake asset trades that are hidden from society. Agents must now be given incentives to reveal information *and* save an appropriate amount. This places additional constraints on risk sharing. When agents can control their publicly observable histories and can save at the prices implied by an exogenously given sequence of SSDFs, these constraints are severe. In this case, the optimal allocation is identical to that in an economy with riskless debt (see, Cole and Kocherlakota (2001)). This result is important as it provides a micro-foundation for models that exogenously restrict agents to the trading of such debt.

## 4 Government incentive problems

Governments or mechanism designers may also have difficulty keeping their promises. There is a long tradition of considering government commitment problems in Ramsey models. In these, the government is socially benevolent and has access to a restricted set of tax mechanisms. Such mechanisms typically allow the government to levy linear taxes on income or consumption and to trade claims to resources. Ex ante optimal policy implies implicit promises over future allocations and, in particular, the expected value of the government's future stream of primary surplus values that it is rarely in the government's interests to keep. In particular, if the government can default on its debt, it will, since in this way it can avoid the distortionary taxes necessary for debt repayment. As in the limited enforcement models described above, reversion to autarky after a default can sustain some equilibrium borrowing by the government, though typically it implies a tight endogenous debt limit (Chari and Kehoe (1993)). Sleet (2004) and Sleet and Yeltekin (2006) consider models in which the government's true spending needs are not publicly observable. Although, the government has access to a complete set of contingent claims markets, in equilibrium it is required to adopt a debt-trading policy consistent with truthful revelation of its spending needs. This limits its ability to buy claims against high spending needs states and sell them against low spending needs ones. The outcome is enhanced inter-temporal, as opposed to inter-state, smoothing of taxes.

The optimal allocations, and market-tax implementations, considered in the discussion of dynamic moral hazard above also involve promises from a planner (or government) to an agent. These allocations often imply that almost all agents' are eventually absorbed by a minimal utility immiserating state; they,

thus, place strong demands on the planner's ability to commit. Sleet and Yeltekin (2006b) remove this ability. They show that optimal allocations without planner commitment solve the problems of *committed* planners who discount the future *less heavily* than agents. Coupling this result with the work of Farhi and Werning (2005), who directly assume a planner discount factor in excess of the agents, suggests that constrained optimal allocations can be implemented with non-contingent debt, an income tax and a progressive estate tax. Analysis of dynamic moral hazard models without societal commitment is, however, still in its infancy and much remains to be done.

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