

sure is based on columns (3), Table 2 of [1].³ The pure growth effect is somewhat higher in the age group 35–44 than in older age groups but the decline, where it exists, is much smaller than reported by Miller, and in some cases there is actually an increase with age where Miller shows a decline. It is also interesting to note that the group least affected by growth is the group 45–54 and not the oldest group 55–64.

A discussion of the substantive issue of the relation of the economic growth to the shape of the income-age curve is beyond the scope of this note.

YORAM BEN-PORATH*

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³ Miller's annual rates in his Table 2 are one-tenth of the decadal rates.

* The author is a graduate student in the department of economics at Harvard University. He wishes to thank Professor Simon S. Kuznets and Mr. Gur Ofer for their comments.

Engineering Production Functions and Capital-Labor Substitution in Metal Machining: Comment

I. *Engineering vs. Heuristic Production Functions*

For some time the theory of the firm has been subject to an unfortunate division: in the classroom we talk about production functions involving physical inputs and a physical output; in our empirical work we are content to estimate cost or production functions from value and accounting data.¹ The former have come to be called "engineering production functions,"² while the latter have been termed "index numbers of production" [3], "distribution functions" [4], and "surrogate production functions" [13]. Attempts to refine these empirical production functions to bring them closer to the physical production functions of the classroom have led to a mass of literature. The difficulties involve such questions as: Under what conditions may factors be aggregated [5]? Should capital be measured as a stock or service?³ Should there be corrections for price changes? (If so, should these entail weighting by (1) constant prices, (2) moving averages, or (3) labor time equivalents [15]?)

In an attempt to break out of this dualism and its set of problems, Mordecai Kurz and Alan Manne used "engineering data" to estimate

¹ For a review of the literature, see [17]; for a detailed discussion of the Cobb-Douglas function, see [12].

² The definition of an engineering production function is a bit more specific as given by Chenery [1]. Output is a function of engineering variables, i.e., the qualities or characteristics of inputs. Equivalently, an engineering production function is a relationship between a physical output and physical inputs where each input is a vector of characteristics [1, pp. 511–13].

³ For a discussion of these concepts, see Lave [10, Ch. 10, Sec. G].

capital-labor tradeoffs in the metal-machining industry [9]. The novel feature of the analysis was that output was measured by the numbers of operations (of each particular type) that could be performed on a particular machine during a work day. Since these observations consisted of output in physical units and capital in physical units (labor is accommodating), Kurz and Manne hoped to generate capital-labor tradeoffs free from the problems generally encountered in estimating production functions.⁴

In estimating Cobb-Douglas and constant elasticity of substitution production functions, Kurz and Manne measured capital as the purchase price of each type of machine. Inefficient observations, those inside the capital-labor-output frontier, were deleted from the analysis. In estimating the functions, dummy variables were used to take account of the difference between output tasks. For example, there were five classifications for the size of the piece being machined: very small to very large. These classifications, in the form of 0-1 dummy variables, are shown in equation (1):

$$(1) \log Y = b_0 + b_1s_1 + b_2s_2 + b_3s_3 + b_4s_4 + b_5s_5 + \dots + b_k \log K$$

where the s_i indicate the five size classifications, K is the purchase price of a particular machine, and Y is the number of pieces turned out during a normal working day. (Groups of dummy variables for shape, tolerance, and lot size were also used.) An observation might consist of the number (Y) of pieces of size 3 ($s_3 = 1$, $s_1 = s_2 = s_4 = s_5 = 0$) which were turned out by a machine costing \$10,000 during a work day (Y and K are divided by L , the number of men working on the machine). When this equation was fitted to the data, the statistical results were good and the coefficients made economic sense (the implied share of labor in output was compared to Census estimates and the two were quite close).

Eirik Furubotn argued that equation (1) was not an engineering production function since capital is defined in terms of price, not physical units [6]. He also argued that the measure of capital (machine purchase price) is inadequate and "... the quantitative results obtained are open to question ..." [6, p. 515]. Furubotn contended that the analysis could not be corrected ("... the censoring rule is invalid as stated by Kurz and Manne, and incapable of useful revision" [6, p. 615]) since the valuation of capital is necessarily subjective depending on the expectations of entrepreneurs.

Is it possible to estimate an engineering relation of the sort defined in the classroom? Just what did Kurz and Manne estimate? How might their procedure have been changed?

II. *Theoretical vs. Empirical Production Functions*

Furubotn emphasizes the distinction between the physical production function of the classroom and the sort of function estimated in practice:

... if a *production function* is to be established, information on prices is not needed. The efficient technical alternatives can be separated from the total array of processes on the basis of objective physical criteria;

⁴ Other work along these lines includes [1], [2], [7], [14].

the problem here is technical rather than economic. Needless confusion is created whenever the distinction between these two levels of choice is blurred [6, p. 515].

In practice the technological relationships can not be kept so pure. As Chenery points out, the engineer is faced with a mushrooming set of possible input combinations, owing to the large number of possible materials that are relatively close substitutes [1, p. 509]. Unless alternatives can be ruled out, the set becomes so large that there is no possibility of handling it empirically. (For example, a desk might be made of wood, steel, gold, diamonds, or many other materials; if the engineer attempted to consider all materials that would make a feasible desk, the number of alternatives would be too large to evaluate.) The engineer, according to Chenery, rules out inputs by assuming a set of prices. In contrast, the economist limits alternatives by aggregating inputs into two or three sets of factors, e.g., labor, capital, raw materials [1, p. 510].

Each of these simplifications leaves its mark on the estimated relation. The engineer's production function will continue to describe the efficient process only as long as relative prices are approximately constant. The economist's relation depends on each factor behaving as a Hicksian "composite commodity" (relative prices within each factor category must be constant) [8, pp. 33-34]. In addition, all relative technological relations between capital, labor, and raw materials must be essentially constant.

With a large number of inputs, and a moderately large book of technology "blueprints," it is extremely unlikely that a single, reasonably simple function would describe all efficient input-output combinations. Yet, it is just such a simple function which must be estimated in practice. Even neglecting the question of the constancy of the book of blueprints (neglecting technological change), only a part of the technology can be described, and that only approximately. In estimating a production function, one is estimating an approximation; it is important that the estimation procedure tailor the approximation to a particular use. Only if the biases and limitations of the data and estimation procedure are unimportant to the particular context can a useful relation be estimated.

III. *Capital-Labor Tradeoffs in Metal Machining*

To come back to the metal-machining context, the pure, physical production functional can be none other than the entire set of relations collected by Markowitz and Rowe [11]. These data, with which Kurz and Manne began their analysis, describe various physical outputs in terms of the labor and time required to produce them on a given machine; they are derived independently of prices. Unfortunately, the data are not very useful in this form. Without a scheme for aggregating or comparing the various machine tools, there is no way of summarizing the data and they remain 1,143 observations on 115 machines performing 129 tasks. Since these data were collected under the assumption of fixed labor-machine proportions, one cannot even get useful isoquants.

Simplifying assumptions are necessary before the Markowitz-Rowe data

can be used to estimate capital-labor substitution. Machines must be compared and so some sort of price weights are necessary. Kurz and Manne used purchase price of each machine tool, and so their relation gives an estimate of the tradeoff between labor and machine purchase price. This tradeoff is an extremely limited one, as Furubotn points out, since a machine costing one dollar more than another, but lasting five times as long, would be labeled "inefficient" (assuming both give the same output per labor hour). Such a limited tradeoff might be relevant for a firm with an extremely limited amount of funds for purchasing equipment.

To estimate capital-labor tradeoffs, Furubotn argues, present value, not purchase price, is the correct measure for comparing machines. Indeed, one must agree that purchase price is not likely to be a good surrogate for capital's services. But one must specify the nature of the required relation before capital's services can be measured.

Kurz and Manne decided to estimate an optimal efficiency locus rather than an "efficiency in practice" or "average efficiency" locus. This decision simplifies the analysis since it makes the problem of measuring capital more tractable. Which of the two measures (stock or service) will give the better approximation to capital's role and usefulness? Since it is the efficiency locus we are considering, we may assume that capital will be used efficiently. With full utilization and a fixed rate of interest, the two measures will be equivalent except for expectations about the future: what will be the rates of obsolescence, price change, and technological change? Furubotn is correct in arguing that entrepreneurs will answer these questions differently, and so the value of capital will vary between entrepreneurs. However, Kurz and Manne are again free to choose the "correct" or "perfect foresight" expectations since they are estimating the efficiency locus.

The Markowitz-Rowe observations were generated independently of price data; they represent an attempt to derive an exhaustive list of the operations that might be performed on 115 tools. No matter what set of prices is used to compare machines, some of these observations will be inefficient. What place should an observation inside the production frontier have in estimating capital-labor tradeoffs? As in a theoretical treatment, these observations must be deleted before fitting the production relation. However, the censoring rule must be specified carefully and must be relevant to the use to which the resulting estimates will be put. The estimates are likely to be quite sensitive to the criteria of efficiency.⁵

IV. *Changing the Kurz-Manne Procedure*

In an empirical study of the Kurz-Manne sort, a number of assumptions and approximations are required. An essential part of the study involves

⁵ The set of "correct" expectations will be manifested in the model by the choice of capital (machine) values. Machines whose values render them "inefficient" will be deleted from the analysis. A machine will be purchased only if its price is less than or equal to the present discounted value of the profit that would be realized by using this machine. Thus, an independent hypothesis emerging from the analysis might be that inefficient machines, ones deleted from the analysis for all tasks, should be left unsold in the market.

testing the implications of the estimates. Kurz and Manne tested their estimates by comparing the share of capital in value added (implied by their estimates) with capital's share as given in the U.S. *Census of Manufacturers*. The *Census* data are not pure and so the two ". . . cannot be compared directly . . ." [9, p. 677]; but the figures are comparable and lend some confidence to the estimates.

A more direct way to test the estimated model might have involved assuming a price of labor (the price of capital is already in the model) and getting the implied prices and price differentials of each "task" (performing each machining operation). If the metal-machining industry is competitive and efficient, the prices implied by the model should be found in the market. Even if there is imperfect competition in the market, some of the prices ought to be correct (some of the tasks will be priced competitively) and all other prices should be above the predicted ones. Such a comparison (more than one hundred prices for comparison would emerge) should prove a sensitive one in testing the twin hypotheses of (a) competition and efficiency in metal machining, and (b) the reasonableness of the estimated model.

The last two comments are related to the Kurz-Manne statistical analysis. Dummy variables are used to take account of differences in size, shape, tolerance, and lot size of the piece being machined. As specified, the model cannot be estimated since it would give rise to a singular variance-covariance matrix [16]. Kurz and Manne follow the general procedure of excluding one variable from each of the sets of dummy variables and then using simple least squares. However, they do not note that the coefficients which are estimated will be the difference between the coefficient of each remaining variable and the coefficient of the excluded variable. Changing the excluded variable would have changed the number of "significant" coefficients.⁶

Finally, it should be noted that their estimation procedures are part of a covariance analysis. They have asked questions about the relation between the capital coefficient and the various tasks. Such issues are part of the general question of which groups of observations may be pooled to obtain parameter estimates of wider scope. Questions of interest include: (a) What sort of aggregation across machines or tasks is statistically permissible? In other words, is there a single model that describes all machine tools? (b) Just how many different capital coefficients and different intercepts should there be? (c) Is there really capital-labor substitution between different machines? Covariance analysis would provide answers to these questions.

V. Conclusion

The Kurz-Manne analysis represents a path-breaking attempt to estimate characteristics of the production function from engineering data. As with most pioneering studies, there were a number of difficulties and un-

⁶ Suppose $Y = b_0 + b_1s_1 + b_2s_2 + b_3s_3$, while $s_1 + s_2 + s_3 = 1$. Then $S'S$ will be singular and estimation by simple, least-squares techniques will be impossible. However, substitution is possible: $s_3 = 1 - s_1 - s_2$ and so $Y = b_0 + b_1s_1 + b_2s_2 + b_3(1 - s_1 - s_2)$; which simplifies to $Y = (b_0 - b_3) + (b_1 - b_3)s_1 + (b_2 - b_3)s_2$. Thus, one would estimate $Y = a_0 + a_1s_1 + a_2s_2$. Testing the significance of a_1 is equivalent to testing whether b_1 is significantly different from b_3 . If one chose to exclude an s_i whose b_i was quite different from the other b_i 's, all of the a_i would be "significant."

necessary limitations. Some of these were corrected by Furubotn; others are suggested here. In large part the problems stem from the failure to set out a specific question for analysis. It is not possible to estimate a general capital-labor substitution relation for metal machining, even given the existing technology. To render the data susceptible to analysis, some sort of machine prices are necessary, a censoring rule must be specified, and many other assumptions are needed. Since a range of plausible alternatives exists for each of the assumptions, the one actually chosen must be justified as being particularly appropriate to the purpose at hand.

LESTER B. LAVE*

* The author is assistant professor of economics at Carnegie Institute of Technology. While not responsible for any remaining errors, Leland Case, Robert Strotz, Judith Rice, and Robert Eisner provided helpful criticism.

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Engineering Production Functions and Capital-Labor Substitution in Metal Machining: Reply

A point of universal agreement is that the production function of pure theory raises the most difficult problems for empirical work. If the concept is interpreted rigorously, an impasse is reached because the set of efficient technical alternatives for the typical commodity is virtually limitless and, hence, incapable of capture by any practical econometric technique. Professor Lave has emphasized, correctly, the need for empirical studies to focus on what is, in effect, a *partial* production function—i.e., a function defined subject to a series of price conditions and able to show some subset of the efficient alternatives. However, as the Kurz-Manne paper [3] makes clear, the introduction of prices does not eliminate all difficulties; indeed, new problems arise which seem almost as intractable as those from which escape is sought. And it is these latter problems that deserve further discussion.

I. Alternate Censoring Rules and their Deficiencies

The K-M censoring rule is misleading because it asserts that, whenever a number of machines have the same output per labor hour, the best choice is necessarily the machine which has the lowest price and involves the least investment outlay.¹ To escape the bias this efficiency criterion imparts to the production model, an alternate rule is needed. One possibility is to base the selection of technical alternatives on a comparison of machine-tool prices with the corresponding present values which can be estimated for the separate tools.² The latter approach permits choices to be made from the standpoint of optimization over time. Thus, reasonable weight can be given to factors which operate throughout the whole life of a capital asset and influence its effective economic contribution to the firm.³

The use of the present-value concept may help to resolve one difficulty, but immediately raises another. Obviously, the present value attached to any machine tool depends on the expectations held concerning the economic environment in which the input will perform. However, since different observers are almost certain to see future technical and market developments

¹ Thus, any quality of a machine which tends to raise its supply price, without simultaneously altering its one-period productivity, must be viewed as undesirable.

² There are, of course, problems connected with the use of the present value approach. In general, machines will involve different investment costs and have different lifespans; thus, additional assumptions about the investment plan are necessary to insure comparability of the alternatives. Further, questions arise about replacement policy and the extent of the planning period.

³ One implication is that choice of relatively high-priced machines is conceivable. The production function obtained need not be shaped by the odd strategy of investment minimization.