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The Relation of Money Balances to
Optimal Consumption, Payments and Receipts

by

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One of the classical problems of economics is to explain why people hold money. The variety of attempts to derive or explain the existence of a demand for money have made the existence of a non-zero demand depend on legal restrictions, uncertainty, the expectation of price changes and many other factors. The most frequent explanation is the Keynesian explanation which makes the demand for money depend on a number of ostensibly separate "motives" for holding money. This explanation differs from its classical forbearers because a variety of motives are introduced in place of the single classical motive -- the transactions motive. The Keynesian and the classical arguments both suffer from the same defect; they fail to relate the explanation of money holding to economic theory. Holding money is not explained as the result of maximizing behavior by individuals and firms -- the ultimate money holders.

Most recent attempts to derive the demand for money have taken one of three paths that we shall call the Patinkin, the Baumol-Tobin, and the Tobin-Markowitz analyses. The first of these, following Patinkin, relies on the notion of a trading period of "Hicksian week" during which individuals exchange money for goods or goods for money. Individuals

hold money between trading periods so that they can engage in transactions when the markets are open. The Patinkin approach has been extremely useful and has produced agreement and even resolution of some long-standing issues. The explanation of the demand for money, however, fails to capture the significant difference between a monetary economy and a barter economy that uses one commodity as money.

The literature in the Baumol-Tobin tradition eliminates the Patinkin-type assumption of a fixed trading period by substituting the assumption of a fixed payments schedule. Individuals pay out money at a steady rate following a payments schedule that they cannot, or at least do not, change. Money is held only because receipts and payments are not synchronized. This approach has the desirable feature of emphasizing the cost of making transactions. But it rejects the notion that money is held as part of an optimal portfolio of assets, treats money holding as something separate from and independent of the choice of a portfolio of real assets and non-durable consumption goods and conflicts with the available empirical work on money.

The third approach -- based on the Tobin-Markowitz analysis -- has stimulated a large amount of fruitful research on security prices and portfolio choice. However, this approach has left the explanation of money holdings in an unsatisfactory position. Following the Keynesian tradition, the risk and return on money is compared to the risk and return on other securities. Money is said to be "dominated" as an asset because other assets have equivalent risk and higher return.

As a result, money is not held as an asset.

Division along Keynesian lines places securities decisions on one side and consumption-investment decisions on another. Decisions about money are in a third category. This division is puzzling. Why should the analysis relevant to decisions to purchase securities not be just as relevant for decisions to purchase the real assets that produce the income and the return on the financial assets? Although it is mentioned frequently, the risk factor that plays an important role in the demand for securities has no formal role in most analyses of the demand for real assets. It seems useful to relate decisions about consumption, investment, securities and money within a single framework and to let the decision-making process within households independent of the type of asset chosen.

We believe that any explanation of the demand for money must start from the view that money is both a producer and a consumer good. Money facilitates exchange by eliminating barter as has long been recognized. The elimination of barter permits individuals to allocate time or other resources used in carrying out barter transactions to more productive uses or to leisure. Allocating both time and goods more efficiently permits individuals and society to produce more from given resources or to enjoy more leisure with the same output. An essential property of money, the property that distinguishes money from other commodities, is that money permits transactions to be carried out at lowest cost. This view which one of us [Brunner, 1961] emphasized several years ago has been recognized by monetary theorists and plays a leading role in Cagan's paper

on the role of money in open market operations [Cagan, 1958]. Nevertheless, this view of the role of money is not fully integrated into the theory of money. It has been used to explain the fact that money is held and used, but it has not been shown to be either a necessary or sufficient condition for holding or using money.

Within the last several years there has been discussion of another, related reason for holding money -- the notion that individuals receive utility directly from money, so that money is held for the "consumption services that it provides." This view, frequently mentioned by Friedman, receives its most useful formal development in the recent book by Pesek and Saving. [Pesek and Saving] They show that if individuals consume the services of money, maximization of a general, intertemporal utility function implies that individuals hold money.

The Pesek and Saving formulation emphasizes the utility or consumption aspect. Although their book breaks important new ground, their work ignores the productive aspect of money and does not consider explicitly the most important "service" provided by money -- the opportunity to rearrange the payment schedule.

In this paper, we separate the analysis of money-using from the analysis of money-holding, first showing why money is used and why society reaches the consensus that a particular medium-of-exchange becomes and remains money. Then we show that individuals desire to hold money as part of their portfolios because it helps them to arrange their payments schedules and to achieve their optimal consumption plans.

A brief discussion of the effects of changing the quantity of nominal money and a conclusion complete the paper.

Reasons for Using Money

Attempts to analyze the demand for money generally start with an assertion about the purpose or purposes for which people use money and go on to discuss the reasons individuals and firms hold money. Two views of the services provided by money -- the uses of money -- have received most attention in recent years. One, put forward by Friedman, identifies money as the asset used as "a temporary abode of purchasing power." The second which became popular following the publication of Tobin's well-known paper, insists that money is held only because receipts and payments are not synchronized.^{1/} Neither statement is useful. As others have noted, many assets can be described as "temporary abodes of purchasing power," so unless some other distinguishing characteristic is offered, the first definition does not distinguish money from other assets. On the other hand, it is no more helpful to say that money is held because receipts and payments are not synchronized than it is to say that receipts and payments are not synchronized because there is money.

^{1/} James Tobin, "Liquidity Preference as Behavior Toward Risk," Review of Economic Studies, ---, pp. ----. Tobin shows that (1) if individuals have quadratic utility functions and (2) if there is an asset with higher mean return and no higher variance of return, individuals prefer to hold non-money assets. Since individuals hold money, their demand for money must be explained in another way. Tobin, and those who use his argument, fall back on the "lack of synchronization of receipts and payments". See, inter alia, Don Patinkin, Money, Interest, and Prices (New York: Harper and Row, 2nd. ed., 1965), pp. 14-15.

Since money is a stock, someone must hold it. As we noted above, discussions of why money is held have become entangled with the discussion of why money is used. As a result, analyses of the demand for money have relied on one or more "motives for holding money" rather than on a comparison of the costs and benefits of the services yielded by money.

In an earlier paper, we offered a preliminary, formal statement of the net benefit that results from using money.^{2/} We emphasized the role of transactions costs of a very broad type -- costs of acquiring information, the use of time and specialized personnel, etc. -- as a characteristic distinguishing assets and argued that the money is used ~~because it reduces~~ the costs of acquiring, storing and processing information. In this section, we resume the discussion and show that cost-minimizing behavior leads individuals to use money and that changes in costs affect the choice of the item used as money. In the following sections, we show that individuals hold money as part of their chosen consumption-investment plan.

An example helps to clarify the above discussion of the uses of money. Suppose Company A, whose shares are traded actively on an organized security exchange, acquires control of Company B by offering to pay a fixed price for each share of B that is offered in exchange. Instead of selling additional shares of its own stock in the market and

^{2/} See our, "Some Further Investigations of Demand and Supply Functions for Money," Journal of Finance (May, 1964), esp. pp. 258-262.

using the proceeds to pay money for the shares of B, the directors of A offer shares of A to holders of B's shares. The price or exchange ratio is fixed in terms of shares and variable terms of market prices. Holders of B must now decide whether they believe the expected future market price of A is higher or lower than the present price. Among the holders of shares in B are some who possess information about the expected prices of the shares of various companies. Suppose these individuals sell the shares of B before the exchange and immediately purchase shares of C, without receiving money. Other holders complete the exchange of B for A, sell the shares of A, receive money (M), search, and later decide to exchange money for shares of C. Still others search and acquire information prior to the exchange and instruct their brokers to make the exchange of B for A, sell the shares of A and immediately thereafter purchase shares in C.

Decisions of this kind are made frequently. We can represent the three decisions by three transaction chains.

- (1) B → C
- (2) B → A → M → C
- (3) B → A → C

In our example, each decision starts and ends with the same asset. It is not useful to distinguish money from the shares of A by describing M but not A as a "temporary abode of purchasing power." Once it is known that all of B's shares are to be acquired by company A, B's shares have become a

"temporary abode of purchasing power." Similarly, it is pointless to describe some of the above transaction chains as synchronized and others as lacking synchronization. Moreover, there is no gain from describing four of the six transactions in the three chains as barter, because "barter" has been used traditionally to describe exchanges in which the participants negotiate the exchange rate at which two commodities are traded as part of the transaction. None of the transactions in our example involves the transactor in negotiation of an exchange rate.

Money is used in each of the transactions in our example as a unit-of-account, but it is used as a medium-of-exchange in only one of the transaction chains. The frequency with which the same unit serves as medium-of-exchange and as unit-of-account suggests that it is efficient for money users and money issuers to combine the two, but it is not necessary to do so. We shall discuss the functions separately and consider, below, some of the conditions under which the two become separated after they have been combined.^{3/}

The use of a unit-of-account increases efficiency by reducing search. Within a given market, the gain in efficiency -- or reduction in cost -- is a one-time, permanent increase resulting from the elimination of random selection of the units used to express exchange ratios, a

^{3/} See Patinkin, op. cit., p. 15 for references to some literature on some of the units that have been used to perform each function.

characteristic of barter.^{4/} The source of the gain in efficiency can be shown easily. If there are n commodities in a barter economy, anyone wishing to organize or participate regularly in a market must know (or be able to obtain) each of the $\frac{n(n+1)}{2}$ independent exchange ratios in the barter-exchange matrix. The choice of a unit-of-account reduces the matrix to an $n \times 1$ vector of exchange ratios, expressed in the unit of account, and thus reduces the cost of acquiring, storing and processing information. This gain is analogous to the gain that comes from introducing a common unit to measure weight, height or temperature.^{5/}

Within a market, the gain from using a unit-of-account is limited by the size of the market. Once an accounting unit is accepted, resources devoted to trade receive higher returns (or incur lower costs), so resources previously devoted to the production and consumption of goods, services or leisure are reallocated to trade. The market expands and additional private and social benefits result from the spread of trade and

4/ We ignore the additional reduction in the cost of transacting that comes from the replacement of a less efficient by a more efficient unit-of-account. Abstract units have a comparative advantage in the unit-of-account function and drive out more specific commodities such as cows or rocks. We assume that an abstract unit is chosen initially.

5/ Patinkin, op. cit., p. 15 distinguishes between the unit-of-account and medium-of-exchange functions of money and reserves the term "money prices" to refer to exchange ratios stated in terms of the medium-of-exchange. We will follow his terminology and avoid further use of the word "money" until we discuss the medium-of-exchange.

the development of the market system.^{6/}

Initially, the gain from introducing a unit-of-account is limited by the extent to which information about the unit is not widespread but narrowly circumscribed. As the size of the market expands, information about a particular unit-of-account becomes more widespread, additional exchange ratios are expressed in the unit, and it becomes efficient to use the unit where other units were used previously.^{7/} Evidence of this is found in the use of the dollar as a unit-of account in trade between third countries, e.g. between the U.S.S.R. and eastern European countries or Japan. Conversely, where trade between countries is expected to remain relatively small and infrequent, it is not profitable to devote resources to negotiations about the unit-of-account, and barter predominates.

The elimination of barter does not imply that the unit-of-account is used as the medium-of-exchange. Our earlier example suggests that the medium-of-exchange does not enter some transaction chains even in markets (or countries) where the same unit performs both functions. Furthermore, the transaction chains that include the medium-of-exchange are not always

^{6/} The use of the dollar as an international unit-of-account after World War II or the use of the U.S. dollar in place of the various state of regional units in the first half of the nineteenth century suggests the gains from selecting a unit-of-account. The examples are not clearcut, however, since the dollar served also as the medium-of-exchange in both instances. More recently, gold has been used increasingly as the medium-of-exchange in international transactions, although the dollar has continued to be used as the unit-of-account.

^{7/} An example is the spread of the decimal and dollar system (or the metric system) to countries where the pound (feet) has long served as a standard.

most efficient. The efficiency of a particular transaction chain, in our example, depends on the time at which individuals acquire information about the commodities purchased and sold and on the cost of acquiring the information during a particular time span. For some transactors -- such as brokers and dealers in a particular asset -- the marginal cost of acquiring information and making decisions promptly is small for transactions in particular assets. These transactors find most efficient those transaction chains in which the dominant or conventional medium-of-exchange does not enter. Transactors for whom the marginal cost of acquiring information is higher use the dominant medium-of-exchange.

Our discussion and our earlier example suggest that, where relative prices and costs of transacting differ, transactors differ in their choice of a medium-of-exchange. Costs of transacting -- including the cost of acquiring information or forming expectations about the properties of the asset to be received in payment -- vary with the volume of transactions, the speed with which the transactor wishes to change portfolio, and perhaps with the wealth of the transactor.^{8/} For each transactor in each transaction there is a cost, Z_i , of making transactions which depends on the asset (i) chosen by the transactor (s) as medium-of-exchange. The asset chosen as the medium-of-exchange for a particular transaction is "money" for that transaction.

^{8/} A formal analysis of the determinants of an individual's payment schedule and of the relation of the payments schedule and the demand for money is left to a later section.

It is the use of money in transaction chains that makes money a productive asset.

The notion of the productivity of money and its relation to transaction costs requires elaboration. Suppose an individual has a bundle of assets A_t that he wishes to transform into a bundle A_{t+1} . There are a number of transaction chains that can be used to make the transformation. The act of transforming is productive or utility increasing since it permits the individual to obtain a higher expected real income and/or to reach a preferred position. Money appears in many of the transaction chains or transformations in part because money is a substitute for the time used in search or in learning about the properties of assets. Once the resource costs of acquiring information and the cost of time used in making transactions are recognized, there is always a cost in making transactions. The productivity of money results from the fact that it is resource or time saving in some transaction chains because z is lower when money is used as the medium-of-exchange, which is to say that there is a function $F(A_t, A_{t+1})$ that is the most productive way of obtaining a new portfolio.

In developed societies, many assets are used as mediums-of-exchange. Gold is a medium-of-exchange in transactions between central banks; Federal funds are a medium-of-exchange in many of the transactions between commercial banks or in the market for Treasury bills; securities are, frequently, the medium-of-exchange in corporate acquisitions.

Even in the most primitive societies, however, one asset (or a few assets) become(s) the dominant medium-of-exchange. This asset, money, acquires the property of general acceptability. Our problem is to show why some assets are selected and used as money and why some ~~assets cease being used as money or are replaced by others.~~

The answer is found in resource-saving or cost-minimizing behavior and in generalized risk aversion.^{9/} As we have emphasized, cost-minimizing alone does not assure that the same asset is used as the medium-of-exchange in all transaction chains or that the same asset is used as money for long periods of time. On the contrary, in the ~~absence of risk-aversion, cost-minimization leads to a number of mediums-~~ of-exchange which vary with the cost incurred in the particular transaction by the particular transactor. The social consensus to use a particular asset as money requires further analysis.

Suppose that, at some point in time, there is a systematic search for the asset that minimizes the cost of making transactions. An asset is selected, for the time, as the medium-of-exchange and prices are expressed in terms of the price of that asset. If there are n commodities, there are $n-1$ independent price ratios expressed in terms of each of the alternative mediums-of-exchange

$$p_i/p_j \quad i, j = 1, \dots, n-1 ; \quad i \neq j \quad .$$

^{9/} Again, we point out that we are concerned with the choice of the asset to be used as money, not with the choice of assets to be held as part of a portfolio. Clearly, if money is used, money must be held in some portfolio. Analysis of the determinants of the demand for money is in a later section.

$$z_j = z_j(\sigma_p^2) \dots \text{ or } R_j = R_j(\sigma_p^2) \dots$$

the variance of the price ratios, so that

Our analysis makes the cost of transacting depend, *inter alia*, on

available for use in transaction chains.

generalized transaction cost minimizing asset, the most productive asset

When a medium-of-exchange asset is used generally, it becomes money, the

and minimum variance σ_p^2 , becomes widespread and is acquired at low cost.

properties of the asset, including the properties of "general acceptability"

of-exchange in other transactions. The reason is that knowledge of the

medium-of-exchange, the lower the cost of using that asset as a medium-

The longer and the more frequently the j^{th} asset is used as the

lower and the productivity in exchange will be higher.

medium-of-exchange because in general transactions costs will be

aversion is general, it pays non-risk-aversers to use the dominant

lowest for those who use the asset as the medium-of-exchange. If risk

offer this asset in transactions. Transaction costs are, consequently,

exchange and will offer more favorable exchange ratios for those who

Risk averters will desire to retain the j^{th} asset as the medium-of-

This sum will generally be a minimum when j is the medium-of-exchange.

$$\sigma_p^2 = \frac{1}{n-1} \sum_{i=1}^{n-1} \text{Var. } \frac{p_i}{p_j}$$

If for each j , we compute the sum

Thus, the analysis implies that when the variance of the prices expressed in terms of money -- the money prices -- becomes large, risk averters will search for a new medium-of-exchange.

Hyperinflation provides one example because at such times the variance of the money prices increases substantially more than the variance of prices expressed in terms of goods, the real (relative) prices. Risk averters refuse to accept the previously dominant medium-of-exchange, thereby raising the transaction cost for those who seek to use the existing money in transactions. Graphically, segment AB of curve ABC in Figure 1 disappears. The variances of exchange ratios expressed in terms of the assets grouped from A to B have increased and the cost of acquiring information about these price ratios has increased also. Now, the real asset located at point B is used most frequently as medium-of-exchange and becomes money. Note, however, that the former money, A, generally continues to be used as the unit-of-account long after it ceases to be used as the medium-of-exchange.

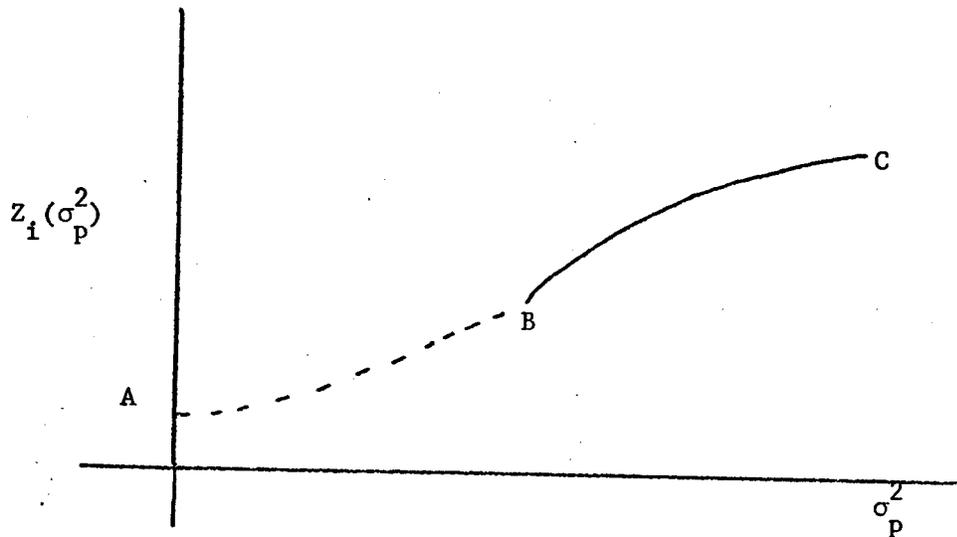


Figure 1

The effect of hyper-inflation must be distinguished from the effect of inflation. The cost of holding money increases during inflation and hyper-inflation, but generally there is no change in the asset used as money during mild inflations. This suggests that the cost function in Figure 1 increases steeply as we move some distance away from A, and this is consistent with observations. The same asset is used as the medium-of-exchange in most countries for long periods of time. Substitutes appear that reduce the cost of transacting, but new mediums-of-exchange are not introduced frequently. The introduction of deposit-banking is one of the last important changes in the assets used as money.

Another example of the effect of the variances on transaction costs and the productivity of money is provided by deflation. Our analysis implies that the hyper-inflation and severe deflation have similar effects on the asset used as money. In severe deflation, the variance of money prices rises just as it does in hyper-inflation, so the cost of using money rises. The return from holding money increases in deflation, and this time the two effects work together to reduce the use of the former money and to cause one or more real assets to be used as mediums-of-exchange.^{10/}

A final example of the importance of stability comes from the recent discussions of international monetary policies and the emphasis given to the maintenance of "confidence" in reserve currencies under the gold

^{10/} In the Purchasing Power of Money, Fisher reports that barter increases during deflations.

exchange standard. Confidence varies inversely with the actual or expected variance of exchange rates expressed in reserve currency units. As the variance of the exchange ratios increases, confidence declines and the reserve currency ceases to perform its former function as well as some other asset, e.g. gold. Some governments and individuals demand that gold be used as the medium-of-exchange. The demand for gold as a medium-of-exchange and the desire to hold gold as a reserve asset have similar effects on the demand for the reserve currency.

In this section, we have discussed the the main functions performed by money and have shown that individuals choose a unit-of-account to avoid the high costs of barter and choose a standard or common medium-of-exchange to avoid risk and further reduce transaction costs. Observation suggests that the same unit generally performs both functions, and our analysis implies that where this is so, the unit will continue to perform both functions in the absence of instability in the price ratios expressed in terms of the medium-of-exchange.

Our analysis implies also that the dominant medium-of-exchange asset appears in most transaction chains. While there may be several specialized mediums-of-exchange that are used for particular types of transactions, neither the choice of these specialized monies nor the choice of the dominant money is the result of random selection or fiat.

The use of the dominant money in transaction chains is a consequence of the productivity of money. The invention of money reduces the amount of resources devoted to search and to acquiring information. It is for this reason that money is used in all but the most primitive societies.

The fact that money is used and is productive does not imply that money is held as part of a portfolio of assets, as contemporary theory has emphasized. Frequently, money is said to be dominated in portfolios by other assets and held only during the interval between receipts and payments. In our earlier writing, we have insisted that the services provided by money -- the productivity of money -- gives rise to a demand for money. In the next two sections, we discuss the relation between transactions costs, the payments receipts schedules and the demand for money as a productive asset.

Some Basic Relations in a Money-Using Economy

One essential difference between a money-using and a barter economy is that in a money-using economy, exchanges take place in organized markets. This is not an accident as we noted in the previous section. The development of the market system and of individual markets requires that the cost of transacting be reduced below some minimum level. The invention of money facilitated the development of the market system by reducing transactions costs.

All decisions are not made in markets. Some are made privately on the basis of market information. Equation (1) distinguishes the consumption of goods and services acquired in a market during the current period, n_t , consumption resulting from the ownership of durable goods, δ_t , and consumption of the services of money, v_t . The separation of durable goods into money and other assets is made to distinguish between the flow

of consumption services that is received principally through depreciation and the services of money, a durable asset that does not depreciate with use. However, we wish to note that the term v_t is included for completeness. Our analysis does not require that v is non-zero.

$$c_t = n_t + \delta_t + v_t \quad (1)$$

Differences in transactions costs explain also why there are organized markets for the services provided by some assets but not others. Where the services of the assets can be purchased or the asset rented, consumers of services are able to choose between renting and buying. Such decisions depend, of course, on relative prices, so the distribution of consumption, c_t , between n and δ in any period depends on prices or costs. If the relative price of obtaining a service by owning the asset rises, consumers will rent the asset or rent the services of the asset, perhaps delaying the purchase to some future time. Rentals are included in n_t since the essential property of n is that items are acquired as they are used.

Since durables do not have to be acquired at the rate they are used and since it is generally costly to do so, individuals choose to run down their stock of durables by depreciating the existing stock and make replacements at discrete intervals. Indeed, costs of transacting require that for most durable goods the decision to purchase must be separated from the decision to consume. Decisions to purchase or acquire goods in

the market, a_t , in real terms is defined as

$$a_t = n_t + \Delta A_t \quad (2)$$

where ΔA_t is the gross amount of real durables acquired during the period. The gross value of assets acquired during the period may be positive, negative or zero for any individual, since individuals may sell some of the assets included in A for money or sell their asset and rent an alternative asset or the services of an alternative asset. For the community as a whole, gross acquisition cannot be negative.

The third equation defines payments, X_t , as the nominal expenditure required to obtain the real acquisitions, a_t . P_t and p_t are the prices of assets and current output.

$$X_t = p_t n_t + P_t \Delta A_t \quad (3)$$

Individuals need not and generally do not pay for assets currently. Payments are deferred by borrowing, that is, by increasing the real value of liabilities at the same time that one increases the real value of assets. In equation (4), changes in liabilities, ΔL are treated as an addition to current receipts, R_t , and repayment of debt is treated as a reduction of current receipts. This last is a convenience and has no analytic importance. The symbols y and Π

$$R_t = p_t y_t + \Pi_t \Delta L_t \quad (4)$$

represent current real income and the price of liabilities, respectively.

Nothing in the equations suggests that receipts and payments must be equal during any period, and generally they will not be. Each individual must choose one or more assets or liabilities that he uses to adjust the gap between receipts and payments. Some individuals obtain a line of credit, if borrowing costs are low relative to costs of adjusting money or other assets. Others allow short-term securities to decline or grow less than planned. As we have indicated, a variety of assets are used in transaction chains. Money is the asset that appears most frequently in transaction chains, because it is the asset that generally minimizes the cost of transactions. For much the same reason, money is used generally to adjust current discrepancies between receipts and payments. This is one of the services provided by a medium-of-exchange.

Since synchronization of payments and receipts has been offered frequently as a reason for holding money, it is essential to note that nothing in our present discussion implies that money is held because it is used to synchronize receipts and payments. The discussion in the previous paragraph could be carried on in terms of decisions to increase or reduce money balances that cause payments and receipts of non-money assets to be unequal, to lack synchronization. As equation (5) attempts to make clear, the change in nominal money, ΔM , is equal to the difference between receipts and payments solely because we have defined receipts and payments so as to exclude money from the right side of the equation. Again, money is not held because it bridges the gap between receipts and payments; money is used to bridge this gap because it

minimizes the cost of transactions. Equation (5) is simply a consequence of our previous definitions.

$$\Delta M_t = R_t - X_t = p_t(y_t - n_t) + \pi_t \Delta L_t - p_t \Delta A_t \quad (5)$$

Equation (6) defines the stock of real balances held at time

$$\frac{M}{P} T = \sum_{t=0}^T \left[y_t - n_t + \frac{(\pi_t \Delta L_t - p_t \Delta A_t)}{p_t} \right] \quad (6)$$

One part of our problem is to find the desired money balance and to show its relation to the receipts and payments schedules and to the optimal consumption plan. This is the subject of the following section.

A Model of Optimal Behavior

During his lifetime, the remainder of his lifetime, or up to some horizon, T , an individual engages in four separate but related activities: consuming, acquiring, receiving and paying. Just as the individuals may separate the act of consuming from the act of acquiring, acquiring can be separated in time from paying, and paying from saving. Individuals issue debts and acquire money balances as part of the process of acquiring goods, receiving income, making payments and consuming.

Part of a household's time and other resources must be devoted to deciding on the appropriate rate at which to acquire goods, borrow, pay bills or repay debts. There are a variety of ways in which payments and

receipts can be arranged so as to alter the costs borne by the household. Among all of the payments and receipts and acquisitions schedules that a household can choose that are consistent with its consumption plan, the household desires to choose schedules that minimize cost. Search for cost minimizing schedules is one of the main productive activities in which households qua households engage. The desire to minimize cost is expressed in equation (7), where z is the cost function.

$$\min z = z \left(\frac{R}{p}, \frac{X}{p}, \frac{\hat{R}}{p}, \frac{\hat{X}}{p}; M \right) \quad (7)$$

discussed in our analysis of money using and M is included in the equation to recall that in general transaction costs are minimized when money is used as the medium-of-exchange, $\frac{R}{p}$, $\frac{X}{p}$ are current and $\frac{\hat{R}}{p}$, $\frac{\hat{X}}{p}$ future receipts and payments.

Minimizing transactions cost is one part of the individual's problem, and maximizing the utility of consumption over time is the other. Equation (8) states the decision to maximize the utility of present and future consumption, c and \hat{c} , and equation (9) introduces the usual constraint, the total value of lifetime resources available to the individual or expected by him.

$$\max U = U(c, \hat{c}) \quad (8)$$

$$\text{s. t.} \quad W_T + \sum_{t=T}^T \frac{y_t}{(1+r)^t} = \sum_{t=\tau}^T \frac{c_t}{(1+r)^t} + \frac{W_T}{(1+x)^{T-T}} \quad (9)$$

Forming the Lagrangean λ , maximizing U , minimizing z , setting the derivatives equal to zero and eliminating λ between pairs of equations, we obtain an optimal value for c , R/p , and X/p for every year from τ to T . The optimal values are the equilibrium values obtained from the consumption, payments, and receipts schedules and depend on relative prices, including the discount rate, expected income and the desired terminal wealth position.

In general, the receipts schedule and the payments schedule are not identical and receipts do not match payments in every period. Hence the optimal change in money balances is obtained as a by-product of the optimal payments and receipts schedules as shown in equation (5) above and thus the individual's optimal real money balance is derived as part of his lifetime consumption and acquisition plan.

Since desired money balances are obtained from the optimal receipts and payments schedules, the demand for money depends on the variables that enter as arguments in these schedules, wealth or expected income, relative prices and asset yields -- the variables we have used in a number of previous empirical papers. The point to be emphasized, here, however, is not the precise arguments that appear in the function but the role that money balances play in the household's optimization and the reason money is held. Money balances are held because they are productive. The service yielded by money -- the productivity of money -- results from the power that money provides to alter payments, receipts and acquisitions schedules. In a money-using economy, money is held by those who wish to enjoy these services. The amount held depends of course on the productivity

of money relative to the cost of holding money, or more concretely relative prices and real wealth.

Solutions for other variables of interest are obtained using the definitions introduced above. Since prices and income are assumed to be known and the household's expectations are given, we can use the solution for optimal receipts in equation (4) to solve for the optimal value of ΔL in the present and each future period. This is the optimal increase or decrease in real indebtedness that is required to achieve the optimal consumption plan at minimum transaction cost.

The fact that there is a desired change in debt position and by summation a desired stock of liabilities suggests by implication that households have optimal debt/equity ratios, a point that requires emphasis because some recent literature has suggested the opposite.^{11/} Since households are not indifferent to the timing of receipts and payments in our analysis, they are not indifferent between contracts requiring the payment of a fixed sum per period and contracts of equivalent current market value. Their expectations and opportunities may differ from those expressed in prevailing market prices or in expected future prices. In fact, borrowing and lending are a principal means by which individuals adjust their payments and acquisitions schedules so as to achieve the optimal consumption plan. From equations (7) - (9), we have the condition for an optimal change in liabilities.

^{11/} F. Modigliani and M. Miller, "The Cost of Capital."

$$\frac{du_i}{du_j} = \frac{dz_i}{dz_j} = -(1 + r_{ij})$$

Households borrow up to the point at which the marginal utility of consumption between any two periods, ij , is equal to the marginal cost of transactions or lending in period i and repaying in period j . In turn, these ratios must be equal to the rate of interest on loans beginning in period i and repayable in period j . More generally, the equilibrium position depends on the expected future rates as expressed in the term structure of interest rates prevailing at time τ .

The desired rate of acquiring assets can be obtained in several ways once we know the desired change in indebtedness. First, we solve for the desired rate of consumption of existing assets, δ , and use the value of δ to solve for n . The desired rate of consumption of durables depends on the size of the stock that is held and on d , the rate of depreciation per unit of assets per period. The rate at which depreciation takes place is not constant; the rate varies with use and thus depends on the relative prices of assets and output and the prices expected to prevail in future periods.

Equations (10) reflect these assumptions and our earlier discussion of the role of relative prices in determining the rate at which assets are used rather than rented.

$$\delta_t = dA_{t-1} ; \quad d = d(P,p) \quad (10)$$

Since prices are assumed to be known or expected, A_{t-1} is given, and since total consumption, c , is determined in the optimization, there are two remaining variables in equation (1), n and v . Let

$$v_t = \frac{kM_t}{p_t}$$

where the value of k may be small or zero for some individuals. We can now solve for n and Δ_A using the known prices and the optimal X/p .

An alternative procedure is to define saving, Δ_{w_t}

$$\Delta_{w_t} = \Delta_A - \Delta_L - \delta + \Delta_{Mp} .$$

Once we have made our assumption about δ , as in equation (10), we can solve for Δ_A , the real value of gross investment and $\Delta_A - \delta$, the desired rate of net investment in assets.

One consequence of our analysis is that optimal money holding, receipts, payments, borrowing, lending, consumption and investment decisions are obtained simultaneously. The reason is that we have taken explicit account of the differences between consumption, acquisitions and expenditures. We need not restate the importance of these distinctions, although we will draw some additional implications in a later section. However, we wish to make clear that the formal structure of the analysis in this section is identical to the traditional statement of capital theory presented by Fisher.

The formal structure of our problem, call this equations A, is

$$\max U(c, \hat{c}) \quad \min z(R/p, \hat{R}/p, X/p, \hat{X}/p) . \quad (A)$$

The formal structure of the traditional, Fisher problem is

$$\max F(c, \hat{c}) \quad \max U(c, \hat{c}) \quad . \quad (B)$$

In both formulations, the constraint is the same, the present value of the expected income stream or total lifetime resources.

The two approaches give the same general result, as they should. The function $U(c, \hat{c})$ in A is replaced by $F(c, \hat{c})$ in B. Instead of finding the utility, maximizing value of consumption when there is no lending or borrowing, as in A, we find from B the amount saved per period that maximizes the present value of consumption or wealth over time. In our analysis, we first maximize utility subject to the constraints imposed by expected resources, while in the Fisher analysis, we start by maximizing present value subject to the constraints imposed by technology and knowledge of opportunities.

The second half of each problem is then the reverse of the first. In A we now use the cost function or alternatively a production function to find the minimum cost or maximum gain from changing payments and receipts and by borrowing or lending. In case B, we follow Fisher and maximize the utility of consumption by borrowing and lending. Since it is entirely arbitrary which activity is called utility maximizing and which activity is called present value maximizing or cost-minimizing, there is no difference in the formal structure.

The important difference between the two is in the productive activities in which households engage. The Fisher analysis concentrates

on the allocation of resources over time and emphasizes the role of the household in this allocation process. Our analysis retains this emphasis on acquiring assets but makes more explicit the role of payments and receipts in a money using economy. In our analysis, the allocation of resources over time through variations in desired payments and receipts becomes the main productive activity of the household. Leisure must be sacrificed and labor supplied to acquire information and to choose payments and receipts schedules. Income in the form of services is earned and costs are reduced so that wealth position and utility increase. Because money permits households to adjust the payment schedule more cheaply and more rapidly and to carry on the activities we have discussed with smaller sacrifice of leisure, money is productive. For these reasons money appears frequently in a household's portfolio.^{12/}

Some Further Implications

In current discussions, monetary policy is said to affect economic activity by changing market interest rates or bond prices. Changes in interest rates affect investment, and the change in investment affects output, so monetary policy affects output by changing investment. If

^{12/} Our point is illustrated by the fact that some individuals hire personal managers to perform many of the services associated with making payments, "shopping," and choosing the consumables and durable assets used in the household. Within households these activities are allocated so as to reduce the total resource cost to the household. On the last point, see Gary Becker, "The Economics of Time," Economic Journal

the monetary policy is a fiat increase in money, any effect on output is transitory and disappears. Fiat changes in money affect the price level; permanent effects on interest rates are obtained only if open market operations are used to change the volume of securities held by the public. The broad outlines of this discussion are familiar and require little further elaboration.^{13/}

These conclusions of conventional analysis are obtained from a framework in which payments and receipts made in money play a very minor role, or no role at all. Our analysis raises the receipt and expenditure decisions to a position of equality with consumption decisions, so it suggests a different and more pervasive effect of monetary policy.

Monetary changes affect the receipt and payment schedules and cause individuals to desire to change receipts and payments. Payments and receipts are related to -- and derived as a part of -- an acquisition plan that is in turn a part of the optimal consumption plan. Attempts to change payments or receipts cause relative prices to change and thus lead to changes in the rate at which goods are acquired. Individuals whose optimal plan called for renting rather than owning durable assets revise plans when faced with new relative prices or differences between the prevailing relative prices and those that were expected to prevail at the time the plan was drawn. If market rates are reduced, temporarily, as a result of a monetary change and are expected to increase again, it pays individuals to increase their current receipts by borrowing in the present rather than in the future and acquire durables now rather than later. Such changes alter other prevailing relative prices and induce

^{13/} Patinkin, op. cit., and Lloyd Metzler, "Wealth, Saving and the Rate of Interest," JPE 1951 ?

others to change their payments, receipts, purchases, borrowing, lending and money holding.

The above discussion can be made more explicit. Suppose that individuals have a lifetime or horizon plan of consumption, receipts and payments. In the absence of any change in money or any new issue of debt, it is true by definition that receipts and payments are equal for the community as a whole. If there is a change in the quantity of money, this equality no longer holds. Someone, or some group, must absorb the increase or decrease in money, so for the community as a whole,

$$\sum_{n=1}^N R_{t_1 n} - \sum_{n=1}^N X_{t_1 n} = \Delta M_t$$

If the change in M is positive, receipts now exceed payments and are larger than planned. Individuals attempt to eliminate excess receipts, the new money, by making payments. Their attempts take a variety of different channels and affect each of the broad categories of variables that appear in the receipts and payments schedules, non-durables, durables, securities, etc. In this way, the change in money affects consumption decisions and by affecting decisions between labor and leisure, the change in money affects output. Eventually, the structure of relative prices is readjusted, the price level rises and the value of real money balances falls toward its previous value.

The long-run effect of a change in the quantity of money will not alter the value of anyone's real consumption over life (or to his horizon) if the discounted value of the future reduction in consumption equals

the change in current consumption. But unless prices adjust instantaneously, there will be some effect on consumption and output as individuals attempt to adjust their payments and receipts.

The effect of an open market operation is similar and occurs mainly because of the change in relative prices. This time, however, the payment schedule is affected directly since ΔA is smaller or larger than anticipated in the aggregate schedule of the community. If the open market operation is a purchase of bonds in exchange for money, X_t is reduced in the aggregate by the sale of debt and nominal money balances are increased. The open market operation affects the relative prices, altering Π relative to P and p . As before, these relative price changes cause changes in individual payment, receipt acquisition and consumption schedules of the type we have described.

Up to this point, our discussion has treated households and ignored firms. With some slight changes, the framework can be adapted to analyze decisions within firms and to derive the optimal investment or long-term capital acquisition and the optimal holding of money from a set of simultaneous equations. The receipts and payments schedules remain as before. The main differences are that a profit function replaces the household's consumption function and payments for labor and other non-storables used in the process of production replace non-durable consumption, n . With these changes, the framework is adapted to firms.

The consequences of our analysis of the role of money balances that we have stressed for households apply as well to firms. Money is held

by firms because it is productive, and it is productive because it permits the firm to adjust payment schedules and take advantage of opportunities which, though unforeseen, are profitable. Examples of such opportunities are numerous and frequent in an economy in which relative prices change and in which expectations are not always realized.

Conclusion

Contemporary theory uses several different artifices to explain why people hold money. Hicksian weeks, the indirect utility of the commodities that may be purchased at some future time, or lack of synchronization are invoked to explain why households or firms hold money.

In our analysis, money is productive. The special property of money that makes money productive -- for most transactors in most transactions -- is that money is the commodity that permits exchanges to be made most efficiently, i.e., at minimum cost. This property explains why a medium-of-exchange is used, and as we tried to show, generalized risk aversion explains why society generally selects a single money as the dominant money to be used in most exchanges.

The productivity of money explains also why money is held. As we argued in several earlier papers, it is not the volume of transactions but the cost of making them that underlies the demand for money. The costs of making transactions are not simply the amount paid to a broker for purchasing securities as is commonly suggested but include the costs of

acquiring information and the costs of time used to acquire the information, to decide on the appropriate or desired payment and receipts schedules and to arrange a payments schedule that is consistent with the optimal consumption plan. These costs are reduced for most transactors by using money.

There is nothing in our analysis, however, that requires individuals to hold money, and there is no implication that an individual desires to hold money at a particular point in time if he holds it at another point. Our analysis does not "prove" that individuals hold money, just as it does not "prove" that individuals hold bonds, shoes, or any other asset. During parts of his life, individuals or households may have an opportunity set or a utility function that makes it desirable to be shoeless or moneyless, debtor or creditor. Decisions about the assets or liabilities held are matters that depend on tastes, opportunities, information and relative prices.

Just as there is no reason to explain the motives for owning shoes, there is no reason for imposing "motives for holding money" on monetary analysis. Money is an asset. Individuals hold it as part of their lifetime utility and/or present value maximizing plans, which is to say, as a part of their program of allocating over time at minimum cost. The productivity or utility of real money balances is no different in this respect from the productivity or utility yielded by shoes. Both raise productivity by reducing the cost of carrying out the household's consumption-production plan. Where interest payments on money are prohibited and shoes are owned, not rented, both yield services only in kind. These services provide utility.