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DISSERTATION

Essays on Asset Pricing and the
Macroeconomy with Limited Stock Market
Participation

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Abstract

In the first essay, I examine the role of personal and corporate income taxation on asset prices in a general equilibrium model featuring limited stock market participation. Taxes are modeled to redistribute income from stockholders, who are relatively richer, to non-stockholders. Under heavier taxation of stockholders, the equity premium rises and the risk-free rate drops. This effect is driven by an increased concentration of consumption risk among stockholders, who then demand a higher premium for bearing the aggregate equity risk. In a version of the model with a realistically calibrated proportional corporate income tax as well as a progressive personal income tax schedule, I find that the redistributive properties of the income tax system are responsible for a one percentage point increase in the after-tax equity risk premium compared to an economy without taxes.

In the second essay (joint with Nam Jong Kim), we study asset prices, exchange rates, and consumption dynamics in a general equilibrium two-country macroeconomic model that features limited stock market participation as well as non-traded goods and distribution cost. The model generates a high price of risk, smooth exchange rates, and makes substantial progress towards explaining the empirically observed low consumption growth correlation between countries. We find that distribution cost play a central role for reducing international consumption co-movement while also amplifying risk premia.

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Chapter 1

Redistribution, Taxation, and Asset Prices with Limited Stock Market Participation

1.1 Introduction

A central feature of the tax system in the U.S. as well as most developed countries is the redistribution of income. Households with higher income or wealth pay higher taxes which are then used to produce public goods or provide transfers. While such a redistributive tax system is a central feature of fiscal policy design, its implications for asset prices are not yet fully understood.

In this paper, I study the effects of personal and corporate income taxes on asset prices in an endowment-economy version of the framework in Guvenen (2009). The model features two types of agents that differ in their access to financial markets. Stockholders can hold shares of stock in addition to trading in the bond market and are the sole capital owners in the economy. Non-stockholders, on the other hand, only have access to the bond market. The role of the government is to collect tax revenue and to provide transfers to all agents. Taxation has the effect of redistributing resources from stockholders, whose income is higher as they receive

corporate distributions in addition to wages, to non-stockholders, whose income is lower as they receive only wages.

The key feature of the model that is responsible for the high equity premium is the lower EIS of non-stockholders compared to stockholders, giving non-stockholders a stronger desire to smooth consumption across time than stockholders. As a result, stockholders are willing to take on a portion of non-stockholders' consumption risk, with the risk transfer happening via trade in the bond market. In good states of the economy, non-stockholders buy bonds to build up precautionary savings; in bad states they sell bonds to supplement their consumption. However, as the bond market can only redistribute consumption between the two agent types, rather than eliminating it, non-stockholders' consumption risk is just shifted to stockholders. The consequence is that trade in the bond market amplifies the consumption growth volatility of stockholders who then demand a high premium to hold the stock.

Redistributive taxation is important for the degree to which this general equilibrium mechanism amplifies stockholders' consumption growth volatility and ultimately risk premia. Under heavier taxation of stockholders, the after-tax equity premium rises and the risk-free rate drops. This effect is driven by an increase in non-stockholders' after-tax income relative to that of stockholders'. Non-stockholders need to make bigger trades in the bond market to reduce the fluctuations in their income, since the after-tax income stream they need to insure is now larger. Furthermore, the trades by non-stockholders have an increased impact on the volatility of stockholders' consumption growth, as they are now larger relative to the decreased level of stockholders' after-tax income. The result is that stockholders' consumption growth volatility increases, leading them to demand a higher compensation to hold the stock.

I first develop a basic model in which I study four types of taxes: lump-sum taxes, proportional taxes on corporate income and distributions, as well as a progressive tax schedule on corporate distributions. I consider four tax counterfactuals that are matched in size to achieve an increase in the average amount of taxes paid by stockholders that is equivalent to 4.6% of average output. This corresponds exactly the difference in the tax burden implied by the

change in the effective tax rate on corporate distribution between 1960 - 1969 and 1990 - 2001 as estimated by McGrattan and Prescott (2005).¹

My results suggest that an increase in the tax burden on stockholders of this magnitude leads to a rise in the after-tax equity premium by about 0.5 percentage points and a decrease in the risk-free rate of about 0.25 percentage points. The size of these effects is the same, irrespective of whether the tax change is implemented as a lump-sum tax, a proportional tax on corporate income or distributions, or a progressive tax on corporate distributions.

While the impact on risk premia after personal taxes is identical for all four types of taxes, there are differences in the effect on the before-tax equity premium. Specifically, increases in the tax on corporate distributions cause the before-tax equity premium to rise by more than the after-tax equity premium. The reason is that the equity premium before personal taxes reflects both the increased risk premium as well as the increased tax burden on corporate distributions.

To assess the overall impact of income taxation on asset prices, I develop an extended version of the model that adds two key features: (i) labor income of both stockholders and non-stockholders is now subject to personal income taxes and (ii) the personal income tax follows a progressive schedule. I then proceed to calibrate the personal income tax schedule to the data and compare asset prices in this more realistically calibrated economy to a counterfactual in which there are no taxes at all. I find that, after the removal of all taxes, the after-tax equity premium drops by about one percentage point and the risk-free rate rises by about half a percentage point.

The rest of my paper is organized as follows. Section 1.2 provides an overview of the related literature. Section 1.3 develops the basic model and Sections 1.4 and 1.5 present the calibration and numerical results. Section 1.6 analyzes the extended model and Section 1.7 concludes.

¹McGrattan and Prescott (2005) estimate the effective tax rate on corporate distributions to be 41.1% from 1960 - 1936 and 17.4% from 1990 - 2001.

1.2 Related Literature

Most closely related to my paper is Veronesi and Pástor (2015), who study income inequality and asset prices under redistributive taxation. In their model, higher taxes are negatively related to expected stock returns. This result is driven by selection effects, through which entrepreneurs are more skilled and less risk-averse, on average, when taxation is heavier. Their study is similar to mine in that it examines redistribution and risk premia jointly. Their paper differs from mine in that selection effects are the primary driver of their results, which are absent in my study. Furthermore, their model delivers qualitative predictions rather than quantitative ones.

Also related is Gomes et al. (2012), who study the effects of capital income taxes and government debt on asset pricing in an overlapping generations model with limited stock market participation. Their analysis differs from mine as i) it does not consider taxation that redistributes income from one type of agents to another and ii) the mechanism generating the equity premium in their model does not depend on the limited stock market participation feature. Hence, the channel generating the asset pricing effects in my model that relies on the redistribution of income from stockholders to non-stockholders and the resulting effects on risk sharing between the two groups is completely absent from their analysis.

Santoro and Wei (2011) study the effects of corporate and dividend taxation on investment and asset returns in a general equilibrium production-based model. They find that corporate income taxes introduce additional risk into the economy by distorting the firm's investment decision. Specifically, corporate taxes amplify the responses of consumption to technology shocks and consequently lead to a lower risk-free rate and a higher equity premium. Their analysis differs from mine as it uses a representative agent setup that does not address the effects of redistribution of income via the tax system.

My paper complements the strand of the literature that focuses on the effects of stochastic taxation on asset prices as in Sialm (2006) and Croce et al. (2012) as well as the literature that studies the implications of more general forms of policy uncertainty on equity returns such as Pastor and Veronesi (2012), Pastor and Veronesi (2013), and Kelly et al. (2015). My analysis differs from these studies in that I focus on the role of a deterministic tax systems.

1.3 Basic Model

I study the effects of taxation on asset prices in a model with limited stock market participation. The setup is an endowment economy version of Guvenen (2009). Abstracting from production allows me to simplify the analysis while preserving the model mechanism that generates realistic equity risk premia. To study the effects of taxation, I further introduce a government sector.²

1.3.1 Households

The economy is populated by a continuum of agents that live forever and whose mass is normalized to one. There are two types of agents: *stockholders*, who make up a fraction $\mu \in (0, 1)$ of the aggregate population and *non-stockholders*, who make up the remainder of the population. Throughout the paper, the superscripts $i = h, nh$ are used to denote individual variables for stockholders and non-stockholders, respectively.

Agents' utility is defined over the private consumption good c . To keep the model simple, I assume that agents have power utility instead of recursive preferences as in Guvenen (2009).³ The relative risk aversion parameter γ^i is allowed to differ between agent types and the time discount rate is β . The utility function then is

$$U^i(c) = \frac{(c)^{1-\gamma^i}}{1-\gamma^i}. \quad (1.1)$$

1.3.2 Endowment and Income

Time is discrete and indexed by $t \in \{0, 1, \dots\}$. There is an aggregate endowment of a single consumption good Y_t that follows an AR(1) process in logs:

$$\log(Y_{t+1}) = \rho \log(Y_t) + \epsilon_{t+1}, \quad \epsilon_t \sim N(0, \sigma^2). \quad (1.2)$$

²Guvenen (2009) studies a production economy with Eppstein-Zin preferences but without a government sector.

³Guvenen (2009) points out that the ability to separate the elasticity of intertemporal substitution from the risk aversion parameter is not central for his results.

The aggregate endowment is split into a part that represents wages and one that represents corporate profits. The parameter $\theta \in (0, 1)$ controls the share of profits in aggregate output and is analogous to the factor share in a production economy. Profits Π_t are given by

$$\Pi_t = \theta Y_t \tag{1.3}$$

and aggregate wages W_t are given by

$$W_t = (1 - \theta) Y_t. \tag{1.4}$$

The distinction between wages and corporate profits allows me to distribute the endowment to agents in a manner that resembles that of a production economy as well as to define a share of stock.

1.3.3 Government

The government's role is to raise tax revenue, which it then redistributes equally to all agents in the form of transfers.⁴ I first study three tax instruments: taxes on corporate income, taxes on corporate distributions, as well as lump-sum taxes. For simplicity, I further assume that only stockholders are taxed. In Section 1.6, I present an extended version of the model in which both agent types are subject to a progressive personal income tax schedule.

The tax rates on corporate income and distributions are τ_C and τ_D , respectively. Per-capita lump-sum taxes levied on stockholders are given by τ_L^h . To capture tax rate schedules, the tax rate on distribution is allowed to depend on the state of the economy.⁵

$$T_t = (1 - \tau_D)(1 - \tau_C)\Pi_t + \mu \tau_L^h \tag{1.5}$$

⁴Santoro and Wei (2011), among others, also assume that the government redistributes all tax proceeds to agents in the form of transfers.

⁵Since corporate profits are exogenous, we can express the tax rate on corporate distributions either as a function of taxable corporate distributions or, equivalently, as a function of the output state of the economy.

1.3.4 Financial Markets and Asset Prices

There are two traded securities: A one-period riskless bond as well as a share of stock. The bond has price P_t^f in period t and pays off one unit of the consumption good in period $t + 1$. The stock is a claim to the firm's dividend stream D_t , which is defined as profits after corporate taxes, i.e.

$$D_t = (1 - \tau_C) \Pi_t. \quad (1.6)$$

The two types of agents differ in the securities they can trade. Both, stockholders and non-stockholders, can trade the bond while only stockholders are allowed to trade the stock. Hence, stockholders are the sole capital owners in the economy. Finally, I use borrowing constraints to prevent Ponzi schemes.

Since only stockholders can trade both the stock and the bond, their marginal utility growth must price both assets. Thus, the stochastic discount factor m_{t+1} in the economy is given by

$$m_{t+1} = \beta \left(\frac{c_{t+1}^h}{c_t^h} \right)^{-\gamma^h}. \quad (1.7)$$

The return on the stock before taxes on corporate distributions is $R_t^S = \frac{P_{t+1}^S + D_{t+1}}{P_t^S}$ and the after-tax return is $\tilde{R}_t^S = \frac{P_{t+1}^S + (1 - \tau_D) D_{t+1}}{P_t^S}$. The risk-free rate is $R_t^f = \frac{1}{P_t^f} - 1$. The equity premium before and after personal taxes is then given by $R_t^{ep} \equiv R_t^S - R_t^f$ and $\tilde{R}_t^{ep} \equiv \tilde{R}_t^S - R_t^f$, respectively.

1.3.5 Agents' Dynamic Problem

In this section, I describe the dynamic programming problem that agents solve, using the common convention to denote next-period variables by "prime". The state of the economy is completely described by the aggregate bond holdings of non-stockholders B and the output level Y . I denote the vector of the two aggregate state variables as $\Upsilon = (B, Y)$. Individual financial wealth is ω and individual bond and stock holdings are b' and s' . The borrowing

limit is \underline{b} . The aggregate law of motion for non-stockholders' bond holdings is $B' = \Gamma_B$. The dynamic programming problem of a stockholder then is

$$V^h(\omega; \Upsilon) = \max_{c, b', s'} \left\{ U^h(c) + \beta \mathbb{E} [V(\omega'; \Upsilon') | Y] \right\} \quad (1.8)$$

s.t.

$$c + P^f(\Upsilon) b' + P^S(\Upsilon) s' \leq \omega + W(\Upsilon) - \tau_L + T(Y) \quad (1.9)$$

$$\omega' = b' + s' (P^S(\Upsilon') + (1 - \tau_D(Y')) D(\Upsilon')) \quad (1.10)$$

$$B' = \Gamma_B(\Upsilon) \quad (1.11)$$

$$b' \geq \underline{b}^h, \quad (1.12)$$

The problem solved by non-stockholders is analogous with the only difference that stock holdings are equal to zero, i.e. $s' \equiv 0$, and that the superscripts h are replaced by nh .

1.3.6 Equilibrium and Solution Method

A stationary recursive competitive equilibrium for this economy consists of the following objects: value functions $V^i(\omega; \Upsilon)$ for $i = h, nh$; consumption and bond holding decision rules $c(\omega^i; \Upsilon)$ and $b^{i'}(\omega^i; \Upsilon)$; stock holding decision rule for stockholders $s'(\omega^i; \Upsilon)$; stock and bond price functions $P^S(\Upsilon)$ and $P^f(\Upsilon)$; a law of motion for aggregate bond holdings of non-stockholders $\Gamma_B(\Upsilon)$, such that the following are true:

1. Given the pricing functions and the laws of motion, the value function and decision rules of each agent solve the agent's dynamic problem.
2. All markets clear: $\mu b^{h'}(\bar{\omega}^h; \Upsilon) + (1 - \mu) b^{nh'}(\bar{\omega}^{nh}; \Upsilon) = 0$ (bond market) and $\mu s'(\bar{\omega}^h; \Upsilon) = 1$ (stock market), where $\bar{\omega}^i$ denotes the wealth of each agent corresponding to the aggregate state Υ .
3. The aggregate law of motion for non-stockholders' bond holdings is consistent with their individual decision rule: $B' = (1 - \mu) b^{nh'}(\bar{\omega}^{nh}; \Upsilon)$

Table 1.1: Non-Tax Parameters

This table summarizes the model calibration at quarterly frequency. The borrowing limits are expressed as a multiple of the average wage rate in the economy, \bar{W} .

Description	Parameter	Value
Relative risk aversion of stockholders	γ^h	4
Relative risk aversion of non-stockholders	γ^{nh}	10
Time discount	β	0.99
Participation rate	μ	0.2
Factor share	θ	0.3
Borrowing limit of stockholders	\underline{b}^h	$12\bar{W}$
Borrowing limit of non-stockholders	\underline{b}^{nh}	$2\bar{W}$
Standard deviation of shock (%)	σ	1.8
Persistence of aggregate shock	ρ	0.95

- There exists an invariant probability measure \mathbf{P} defined over the ergodic set of equilibrium distributions.

I solve for the recursive competitive equilibrium using numerical methods. Since markets are incomplete, using a social planner’s problem to solve for the equilibrium is not feasible. Instead, I solve for allocations and pricing functions simultaneously. To ensure a globally accurate solution, I approximate all equilibrium functions over the entire state space using multidimensional cubic splines. I use a version of the algorithm in Guvenen (2009) to solve my model. Appendix A.1 contains a detailed description of the numerical procedure.

1.4 Calibration

I calibrate the model at quarterly frequency. Table 1.1 summarizes the non-tax parameters and Table 1.2 lists the tax parameters for the base parametrization as well as for a set of tax counterfactuals that I study in Section 1.5.2.

Non-Tax Parameters

Following Guvenen (2009), I set the factor share θ to 0.3 and the time discount rate β to 0.99, which are standard values in the literature. I chose the persistence of the endowment shock to be 0.95 to match the persistence of the Solow residual. The AR(1) process for output is discretized using a 5-state Markov chain.

I set the participation rate μ to 0.2 as in Guvenen (2009), who points out that this corresponds roughly to the average rate from 1962 to 1992. While stock market participation has since increased to about 0.5, Poterba and Samwick (1996) shows that about 20% of households still hold more than 98% of stocks.

I set the stockholder borrowing limit \underline{b}^h equal to 12 quarters of average labor income while I set the non-stockholder borrowing limit \underline{b}^{nh} equal to 2 quarters of average labor income. The larger borrowing limit for stockholders reflects the fact that they own the aggregate capital stock which they can use as collateral to increase borrowing. In the simulations, these borrowing constraints never bind and allow me to conveniently rule out Ponzi schemes while not materially affecting any of the model results.

Since agents in my model have power utility, rather than Epstein-Zin utility, I cannot calibrate the elasticity of intertemporal substitution (EIS) separately from relative risk aversion. Hence, I need to set the agents' risk aversion parameters to capture the most important features of the calibration in Guvenen (2009). In the case of non-stockholders', the central feature of his calibration is that they have a very low EIS. First, as summarized in Guvenen (2006), there is strong empirical evidence that poor households (and non-stockholders) have an EIS that is close to zero while wealthy households (and stockholders) have a much higher EIS. Secondly, the mechanism generating the high price of risk in Guvenen (2009) relies on non-stockholders' strong desire to smooth consumption over time. To capture both the empirical and theoretical relevance of the low EIS, I set non-stockholders' risk aversion γ^{nh} to 10, implying an EIS of 0.1 as in Guvenen (2009). Regarding the relative risk aversion of stockholders', I chose a value of $\gamma^h = 4$ to strike a balance between the calibration in Guvenen (2009) who uses a risk aversion of 6 and an EIS of 0.3 (corresponding to a relative risk aversion of 3.33). My

Table 1.2: Taxes in the Basic Model

This table summarizes the calibration of the tax system in the basic model. The table contains the baseline calibration as well as four tax counterfactuals. The aggregate tax revenue from the lump sum tax in counterfactual (2), $\mu\tau_L^h$, is expressed as a multiple of the average aggregate endowment, \bar{Y} .

Description	Base	Tax Counterfactuals			
		(1)	(2)	(3)	(4)
Corporate income tax rate τ_C	0.353	0.353	0.539	0.353	0.353
Corporate distributions tax rate τ_D	0.174	0.174	0.174	0.411	$\left[\begin{array}{c} 0.461 \\ 0.436 \\ 0.411 \\ 0.383 \\ 0.348 \end{array} \right]$
Lump-sum taxes $\mu\tau_L^h$	0	$0.046\bar{Y}$	0	0	0

calibration captures that stockholders have a substantially higher EIS than non-stockholders while also making stockholders risk averse enough to generate a realistic price of risk.

The last parameter to calibrate is the volatility of the endowment shock σ . I set this parameter to strike a balance between matching the volatility of Hodrick-Prescott filtered quarterly output and the annual equity premium. Setting $\sigma = 1.8\%$ produces a volatility of the Hodrick-Prescott filtered endowment of 2.3% (compared to output volatility of 1.89% in the data) and an equity premium of 5.94% (compared to 7.08% in the data).

Tax Parameters

The calibration of the tax rates is summarized in Table 1.2. In my baseline calibration, I use the tax rates that McGrattan and Prescott (2005) estimate for the period covering 1990 - 2001. The tax rate on corporate income τ_c they obtain is 35.3%. It is measured as the ratio of the corporate profits tax liability in the U.S. national income and product accounts (NIPA) to before-tax corporate profits.⁶ The tax liability they measure includes federal, state, and local

⁶McGrattan and Prescott (2005) exclude the Federal Reserve Banks from their calculation because all of their profits are passed on to the U.S. Treasury, making their tax rate 100%.

taxes, so that their tax rate captures the total U.S. tax burden. The tax rate on corporate distributions τ_D they obtain is 17.4%. This tax rate is an effective tax rate that measures the average tax burden on corporate distributions in the form of dividend payments as well as capital gains. Their calculation takes into account the marginal tax rates for all groups of recipients, weighted by the fraction of receipts of each group. Lump-sum taxes are set to zero.

To illustrate the effects of tax changes on asset prices, I then study four tax counterfactuals by changing one tax rate at a time. All of the counterfactuals are matched in size to achieve an increase in the average amount of taxes paid by stockholders that is equivalent to 4.6% of average output. This corresponds exactly to the increased tax burden implied by the higher tax on corporate distributions that McGrattan and Prescott (2005) estimate for the earlier time period from 1960 - 1969.

Counterfactual (1) implements this tax experiment directly as a lump-sum tax on stockholders, i.e. $\mu\tau_L^h = 0.046\bar{Y}$. Counterfactual (2) implements the same average tax burden on stockholders by increasing the corporate income tax to $\tau_C = 53.9\%$. Counterfactual (3) uses the estimated tax rate on corporate distributions for 1960 - 1969 of $\tau_D = 0.411$. Finally, counterfactual (4) implements a tax schedule imposed on corporate distributions with the goal of studying the effect of pro-cyclical taxation separately from any change in the average tax burden. To achieve this, I impose a tax schedule with the rate for the middle endowment state equal to $\tau_D = 0.411$, which is the same as for counterfactual (3). I then pick the tax rates in the other states so that i) the average tax burden on dividends remains unchanged and ii) the tax rates in the second highest and highest endowment states are 2.5 and 5 percentage points higher, respectively, than the rate for the middle state.

1.5 Model Results

Table 1.3 summarizes the results for the baseline calibration and shows that the model generally fits the data well. In terms of asset prices, the model generates a realistic equity premium

Table 1.3: Model Results

This table summarizes unconditional moments of asset returns, the price-dividend ratio, consumption, and wealth for the data and the model. The historical asset return moments are calculated using data from the online appendix to Sialm (2009) and cover 1913 - 2006. The historical moments for the distributions of wealth and consumption are from Wolff (2000) and Guvenen (2006), respectively.

	Data	Basic Model				Extended Model		
		Base	Tax Counterfactuals			Schedule	No Taxes	
			(1)	(2)	(3)			(4)
Panel A: Asset Returns								
$E(R^{ep})$	7.08	5.94	6.35	6.34	8.25	8.25	6.50	4.32
$\sigma(R^{ep})$	19.37	21.55	22.41	22.29	22.38	22.43	21.90	19.76
$E(R^{ep}) / \sigma(R^{ep})$	0.37	0.28	0.28	0.28	0.37	0.37	0.30	0.22
$E(\tilde{R}^{ep})$	5.61	5.10	5.51	5.50	5.49	5.49	5.29	4.32
$\sigma(\tilde{R}^{ep})$	19.22	21.54	22.32	22.28	22.35	22.40	21.88	19.76
$E(\tilde{R}^{ep}) / \sigma(\tilde{R}^{ep})$	0.29	0.24	0.25	0.25	0.25	0.25	0.24	0.22
$E(R^f)$	1.29	1.09	0.86	0.86	0.86	0.86	0.97	1.55
$\sigma(R^f)$	5.31	3.89	4.04	4.03	4.03	4.04	3.97	3.56
Panel B: Price-Dividend Ratio								
$E(P^S/D)$	26.6	22.5	22.7	22.6	16.1	16.2	20.9	26.8
$\sigma(\log(P^S/D))$	43.5	28.0	29.4	29.0	29.1	29.2	28.5	25.5
Panel C: Consumption and Wealth								
$E(\omega^h / \omega^A)$	0.88	0.97	0.98	0.96	0.96	0.93	0.96	0.98
$E(C^h / C^A)$	0.32	0.35	0.31	0.31	0.31	0.31	0.33	0.44

of 5.94% and a low and smooth risk-free rate of 1.09%. The implied Sharpe ratio of 0.28 is somewhat lower than the 0.37 in the data. The average price-dividend ratio is similar to its data counterpart, reaching 22.5 in the model and 26.6 in the data. The price-dividend ratio is, however, considerably smoother in the model than in the data, with volatilities of 28.0% and 43.5%, respectively. Finally, the model matches the average share of stockholders' consumption in aggregate consumption reasonably well at 0.35 but produces a share of financial wealth for stockholders that is somewhat too high, reaching 0.97 in the model compared to 0.88 in the data.

1.5.1 Mechanism for the Equity Premium

Before proceeding to analyze how taxes affect asset prices in this model economy, it is useful to recall the mechanism that generates the high equity premium in Guvenen (2009) and which is also at work in this model. The key feature of the model that is responsible for the high equity premium is the lower EIS of non-stockholders compared to stockholders, giving non-stockholders a stronger desire to smooth consumption across time than stockholders. As a result, stockholders are willing to take on a portion of non-stockholders' consumption risk, with the risk transfer happening via trade in the bond market. In good states of the economy, non-stockholders buy bonds to build up precautionary savings; in bad states they sell bonds to supplement their consumption. However, as the bond market can only redistribute consumption between the two agent types, rather than eliminating it, non-stockholders' consumption risk is just shifted to stockholders. The consequence is that trade in the bond market amplifies the consumption growth volatility of stockholders, which it does in a pro-cyclical fashion. Stockholders then demand a high premium to hold the stock, whose dividends are large in booms and small in recessions.

Stockholders' consumption hence has two components, with the first driven by their after-tax factor income and the second driven by trade in the bond market. To quantify the relative importance of both of these for stockholders' consumption growth volatility, it is useful to express their consumption as

$$c_t^h = \underbrace{\left(W_t + \frac{(1 - \tau_D)(1 - \tau_C)\Pi_t}{\mu} - \tau_L^h \right)}_A + T_t - \underbrace{\left(P^f b_t^h - b_{t-1}^h \right)}_B. \quad (1.13)$$

This expression provides a decomposition of stockholders' consumption into a part A that is due to aggregate sources, i.e. their labor and dividend income adjusted for taxes and transfers, and a part B that measures the net payments made to non-stockholders through the bond market.

The variance of stockholders' consumption growth can then be approximated as

$$\sigma^2 \left(\Delta \log c^h \right) \approx \sigma^2 \left(\Delta \log A \right) + \sigma^2 \left(\Delta \frac{B}{A} \right) + 2 \text{Cov} \left(\Delta \log A, -\Delta \frac{B}{A} \right), \quad (1.14)$$

Table 1.4: Sources of Consumption Growth Volatility

This table shows the contribution of aggregate sources and bond market payments to the variance of stockholders' consumption growth. Aggregate sources are reflected by the first term in equation 1.14 and payments in the bond market are captured by the sum of the second and third terms in that equation.

	Basic Model					Extended Model	
	Base	Tax Counterfactuals				Schedule	No Taxes
		(1)	(2)	(3)	(4)		
$\sigma(\Delta C^h)$ (%)	3.02	3.14	3.13	3.13	3.13	3.07	2.78
Aggregate sources	0.37	0.44	0.34	0.34	0.14	0.33	0.44
Bond trade	0.63	0.56	0.66	0.66	0.86	0.67	0.56

with the first term measuring the contribution from aggregate sources and the second and third terms measuring the impact from trade in the bond market. Table 1.4 shows that, for the baseline calibration, 63% of the variance in stockholders' consumption growth is due to trade in the bond market and only 37% is directly due to aggregate fluctuations in their labor and dividend income (after adjusting for taxes and transfers). This is true despite the fact that the magnitude of the payments in the bond market relative to stockholders' average consumption is rather small. Specifically, the average payments to non-stockholders make up only about 1.3% of stockholders' average consumption, i.e. $E[B]/E[c^h] = 0.013$.

1.5.2 The Role of Taxes

In this section, I present results for four tax counterfactuals that illustrate the differential affect that lump-sum taxes, proportional taxes, and progressive tax schedules have on asset prices. The central channel through which taxation acts in this economy is the redistribution of income from stockholders to non-stockholders. Specifically, the after-tax equity premium is a function of the after-tax income distribution between stockholders and non-stockholders. To match the magnitude of the income redistribution across all tax counterfactuals, I implement each counterfactual to represent an increase in the average tax burden on stockholders of

4.6% of average output while holding the tax burden on non-stockholders constant.⁷ Since the government uses tax revenue to pay transfers to all agents (in equal per-capita amounts), this means that all tax counterfactuals redistribute $(1 - 0.2) \times 4.6\% = 3.7\%$ of average output from stockholders to non-stockholders.⁸ Tables 1.3 and 1.4 show the model-implied moments and the decomposition of stockholders' consumption growth for the baseline calibration as well as the four tax counterfactuals.

Lump-Sum Taxes

The first counterfactual I study is the lump-sum tax equal to 4.6% of GDP. The tax change results in slight increases in the before-tax equity premium from 5.94% to 6.35% and in the after-tax equity premium from 5.10% to 5.51%. Part of the increase in the equity premium is driven by a drop in the risk-free rate from 1.09% to 0.89%. The after-tax Sharpe ratio increases by one percentage point from 0.24 to 0.25.

The increase in risk premia and the drop in the risk-free rate is driven by an increase in non-stockholders' after-tax income relative to that of stockholders'. Non-stockholders, who have a low EIS and hence a strong desire to smooth their consumption across periods, now need to make bigger trades in the bond market to reduce the fluctuations in their income, since the after-tax income stream they need to insure is now larger. Furthermore, these larger trades by non-stockholders have an increased impact on the volatility of stockholders' consumption, as stockholders have lower after-tax income. The result is that stockholders' consumption growth volatility increases from 3.02% to 3.14%, leading to a higher price of risk.

Proportional Taxes on Corporate Income

I next study an increase in the corporate income tax rate from 35.3% to 53.9%, which is the second counterfactual. This generates the same average increase in income redistribution

⁷In the basic model, non-stockholders do not pay taxes but they receive transfers.

⁸Tax revenue is distributed to all agents equally in the form of transfers. This means that a fraction $(1 - \mu)$ of all tax revenue goes to non-stockholders while the remainder is rebated back to stockholders.

from stockholders to non-stockholders as the lump-sum counterfactual and results in virtually the same increase in risk-premia.

While the effect on asset prices and stockholders' consumption growth volatility is the same for the lump-sum tax as it is for the corporate income tax, there is a difference in the source of stockholders' consumption growth volatility. Specifically, Table 1.4 shows that 44% of stockholders' consumption growth volatility is due to after-tax income volatility in the case of lump-sum taxes, whereas it is only 37% in the case of corporate income taxes. Since the corporate income tax is proportional, an increase in the tax does not affect the volatility of after-tax income *growth* compared to the baseline calibration. The lump-sum tax, however, is lower in the good states and higher in the low states, when measured *relative* to income. Hence, we can think of the lump-sum tax as being regressive. It amplifies the volatility of after-tax income growth for stockholders. The flip-side is that, since a fraction $(1 - \mu)$ of the tax revenue are redistributed to non-stockholders, the lump-sum tax lowers the after-tax income growth volatility of non-stockholders. As a result, they need to trade less in the bond market to achieve the same volatility of their consumption stream. Hence, a larger fraction of stockholders' consumption growth comes from aggregate sources and a smaller proportion from the bond market, compared to the case where corporate income taxes are raised.

As non-stockholders need to rely on the bond market more heavily, they need to increase the amount of precautionary savings and stockholders' share in aggregate financial wealth drops from 98% to 96% as a result of the tax being proportional rather than lump-sum.

Proportional Taxes on Corporate Distributions

The third counterfactual is an increase in the corporate distributions tax rate from 17.4% to 41.1%. This case is very similar to an increase in the corporate income tax rate. All after-tax returns are almost identical to those for the corporate income tax counterfactual and the composition of stockholders' consumption growth is the same as well. The two taxes differ, however, in the implied *before-tax* equity premium and the level of the price-dividend ratio. An increase in the corporate distributions tax raises the *before-tax* equity premium by more

than the *after-tax* equity premium to compensate stockholders for the increased personal tax burden. Similarly, as corporate distributions are taxed more heavily, the stock price falls (as the stock price is the present value of *after-tax* dividends). Since the price-dividend ratio is measured as the ratio of (before-tax) dividends to the stock price, this ratio falls mechanically.

A Progressive Tax Schedule on Corporate Distributions

Finally, the fourth counterfactual studies the impact of a progressive tax schedule on corporate distributions. In order to examine the effects of pro-cyclical taxation separately from any change in the average tax burden, I chose the tax schedule to imply the same average tax burden on corporate distributions as the previously studied case with a fixed tax rate. To achieve this, I impose a tax schedule with the rate for the middle endowment state equal to the fixed rate in the previous counterfactual, i.e. 41.4%. I then pick the tax rates in the other states so that i) the average tax burden on dividends remains unchanged and ii) the tax rates in the second highest and highest endowment states are 2.5 and 5 percentage points higher, respectively, than the rate for the middle state. The resulting tax schedule raises the same average amount of taxes as the third counterfactual while being progressive and charging higher rates in the better states and lower rates in the worse states.

The results show that both the before and after-tax equity premium and risk-free rate are identical to the proportional tax on corporate distributions. As the tax is progressive, it reduces the volatility of stockholders' after-tax income which now only comprises 14% of stockholders' consumption growth volatility. The transfers received by non-stockholders, on the other hand, are higher in booms and lower in recessions, increasing the volatility of their after-tax income and requiring them to rely more heavily on trade in the bond market to smooth consumption. Consequently, 86% of stockholders' consumption growth volatility is now due to bond trade.

As non-stockholders need to rely on the bond market more heavily, they need to increase the amount of precautionary savings and stockholders' share in aggregate financial wealth drops from 96% to 93% as a result of the procyclicality of the tax.

1.6 Extended Model with Personal Income Tax Schedules

The goal of this section is to quantify the aggregate importance of income taxation for asset prices. I first present an extended version of the model that adds two key features: (i) labor income of both stockholders and non-stockholders is now subject to a personal income tax and (ii) the personal income tax follows a progressive schedule. I then proceed to calibrate the personal income tax schedule to the data and compare asset prices in this more realistically calibrated economy to a counterfactual in which there are no taxes at all.

1.6.1 Model Extension

In the extended model, both, labor income as well as corporate distributions are subject to personal income taxes at rate τ_P^i . In order to allow for tax schedules, the personal tax rate can depend on the state of the economy and it can differ by agent type i . Corporate profits continue to be taxed at the fixed rate τ_C , as the corporate income tax code exhibits very little progressivity. The expression for aggregate transfers in Equation 1.5 is then replaced by

$$T_t = \left(1 - \tau_P^h\right) (1 - \tau_C) \Pi_t + \mu \tau_P^h W_t + (1 - \mu) \tau_P^{nh} W_t. \quad (1.15)$$

The budget constraint in Equation 1.9 of the dynamic programming problem of stockholders becomes

$$c + P^f(\Upsilon) b' + P^S(\Upsilon) s' \leq \omega + \left(1 - \tau_P^h(Y)\right) W(\Upsilon) + T(Y) \quad (1.16)$$

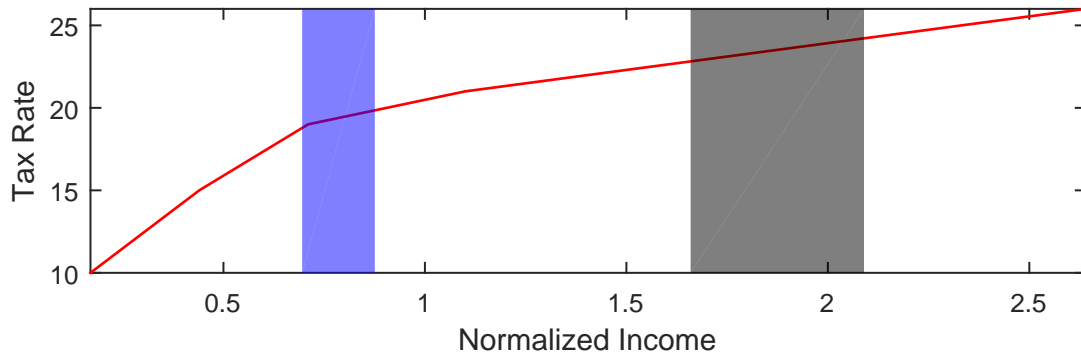
and the law of motion for stockholders' wealth in Equation 1.10 now is

$$\omega' = b' + s' \left(P^S(\Upsilon') + \left(1 - \tau_P^h(Y')\right) D(\Upsilon') \right), \quad (1.17)$$

As in the basic model, the problem solved by non-stockholders is analogous with the only difference that stock holdings are equal to zero, i.e. $s' \equiv 0$, and that the superscripts h are replaced by nh .

Figure 1.1: Estimated Personal Income Tax Schedule

This figure plots estimated effective personal income tax rates as a function of normalized income. Income is normalized by the cross-sectional average income in the economy. The data are from the Congressional Budget Office and cover 1979 to 2010. The shaded areas show the support of the ergodic income distribution for non-stockholders (blue) and stockholders (gray) in the model.



1.6.2 Calibration

I estimate the personal income tax schedule using data from the Congressional Budget Office (CBO). The dataset contains market income as well as federal taxes paid for each year from 1979 to 2010 and provides cross-sectional averages as well as quintile values.⁹ Market income is the sum of labor and capital income.

To estimate an average personal income tax schedule over the sample period, I proceed as follows. First, I compute the effective tax rate for each income quintile as the ratio of income taxes to market income. Next, I normalize the income level of each quintile by the cross-sectional average income for that year. These two steps yield income tax schedules for each year as a function of normalized income. Finally, I average the tax rates and normalized income values across years.

Figure 1.1 plots the resulting personal income tax schedule. Agents earning 17% of the average market income pay the lowest tax rate of 10%. Rates then increase, reaching 26% for agents

⁹The data is from the supplement information provided in the Congressional Budget Office's December 2013 report "The Distribution of Household Income and Federal Taxes, 2010".

Table 1.5: Taxes in the Extended Model

This table summarizes the calibration of the tax system in the extended model. The personal income tax schedules contain the tax rates for the five endowment states with the rate for the highest endowment at the top.

Corporate Income Tax Rate	Personal Income Tax Schedules	
$\tau_C = 0.353$	$\tau_P^h =$	$\tau_P^{nh} =$
	$\begin{bmatrix} 0.242 \\ 0.238 \\ 0.235 \\ 0.231 \\ 0.228 \end{bmatrix}$	$\begin{bmatrix} 0.188 \\ 0.191 \\ 0.194 \\ 0.196 \\ 0.198 \end{bmatrix}$

that earn 264% of the average market income. Since the tax schedule is normalized to be a function of the multiple of average income, I can directly apply the schedule to the model economy. The blue and gray shaded areas indicate the boundaries of the ergodic distribution of non-stockholders' and stockholders' income and the associated tax rates in the economy. Table 1.5 lists the exact tax rates corresponding to each state of the economy.

I assume that the corporate income tax rate is constant at 35.3%. As both the federal and state corporate income tax schedules exhibit virtually no progressivity, this appears as a reasonable simplification.

1.6.3 Results

Table 1.3 summarizes the results for the extended model with the calibrated tax schedule. The model generates a realistic equity premium of 6.50%, a low and smooth risk-free rate of 0.97%, as well as a high Sharpe ratio of 0.30. The price-dividend ratio is close to its data counterpart, both in its average level of 20.9 as well as in its volatility of 28.5. The model also matches the average share of stockholders' consumption in aggregate consumption almost perfectly, producing a consumption share of 0.33, compared to 0.32 in the data. This indicates that the calibrated income tax schedule provides a good fit for the average amount of redistribution from stockholders to non-stockholders via the tax system. The share of total

financial wealth held by stockholders is 0.96, which is somewhat high compared to 0.88 in the data.

When all taxes are eliminated, both, before and after-tax risk premia fall considerably. The reason is that removing taxes alleviates the disproportionate tax burden on stockholders. The share of stockholders' consumption rises from 0.33 to 0.44, indicating that about 11% of income is redistributed from stockholders to non-stockholders when the tax code is in place.

The largest changes to asset prices affect the before-tax equity premium, which drops by almost one third from 6.50% to 4.32%, resulting in a large drop in the Sharpe ratio from 0.30 to 0.22. The drop in the after-tax equity premium is much more moderate. It falls only by about one percentage point, showing that the large decline in the before-tax equity premium is mostly due to the elimination of the tax penalty imposed by the taxation of corporate distributions and not to an increase in compensation for risk.

Table 1.4 shows that the reduction in the after-tax equity premium is driven by a lower volatility of stockholders' consumption growth. As stockholders' share in aggregate consumption is larger without taxes, the contribution of trade in the bond market to stockholders consumption growth is reduced from 67% to 56% after taxes are eliminated.

1.7 Conclusion

I study the effects of personal and corporate income taxes on asset prices in an endowment economy with limited stock market participation. Taxation has the effect of redistributing income from stockholders, who are relatively richer, to non-stockholders. Under heavier taxation of stockholders, the equity premium rises and the risk-free rate drops. This effect is driven by an increased concentration of consumption risk among stockholders, who then demand a higher premium for bearing the aggregate equity risk.

I study four types of taxes: lump-sum taxes, proportional taxes on corporate income and distributions, as well as a progressive tax schedule on personal income. All of these types

of taxes deliver the same effect on after-tax risk premia, while increases in the taxation of corporate distributions lead to an additional increase in the before-tax equity premium. In a version of the model with a realistically calibrated proportional corporate income tax as well as a progressive personal income tax schedule, I find that the redistributive properties of the income tax system are responsible for a one percentage point increase in the after-tax equity risk premium compared to an economy without taxes.

In future research, it would be interesting to extend the model to include endogenous production and labor supply. This would allow me to account for adjustments in output, hours worked, and wages in response to changes in taxes.

Chapter 2

Limited Stock Market Participation and Goods Market Frictions: A Potential Resolution for Puzzles in International Finance

2.1 Introduction

It has long been a challenge for macro-finance models to jointly explain i) the high equity premium, ii) relatively smooth exchange rates, and iii) the low international correlation of consumption growth.¹ We propose a general equilibrium two-country macroeconomic model that features limited stock market participation as well as non-traded goods and distribution cost to address these salient features of the data.

Our model brings together two strands of literature. Firstly, we build on work showing that the nature of goods markets is an essential determinant of the correlation of consumption growth between countries in general equilibrium models. In particular, Corsetti et al. (2008) show that modeling goods markets to feature non-tradable goods and distribution cost helps

¹See, for example, Brandt et al. (2006).

decrease the strong international correlation of consumption. These features moderate the international risk sharing mechanism shown in Cole and Obstfeld (1991), in which a country hit by an adverse productivity shock benefits from a natural hedge, as its goods become more scarce and appreciates in price. We model distribution services that are produced with the intensive use of local inputs, allowing the model to generate deviations from the law of one price. The role of such distribution cost in explaining real exchange rate movements has been emphasized by a growing empirical literature, e.g. Crucini et al. (2005).

Secondly, we draw from the literature on limited stock market participation. We adopt the asset market structure of Guvenen (2009), where only a fraction of the population has access to the stock market and the remainder of agents are restricted to trade in a bond.² This type of setup has two appealing features for our analysis. First, it is known to generate a realistic price of risk, i.e. Sharpe ratio, which we need in order to jointly study asset prices, international consumption co-movements, and exchange rates. Secondly, it introduces market incompleteness. Brandt et al. (2006) argue that the equilibrium condition linking marginal utility growth to the rate of depreciation in the exchange rate that results in complete market models makes it impossible to generate high risk premia, smooth exchange rates, and moderately correlated consumption growth simultaneously. Allowing for market incompleteness in the form of limited stock market participation breaks this link, making it feasible - at least in principle - for the model to match the three stylized data facts.

There have been previous attempts in the literature to generate the joint dynamics of asset prices, consumption, and exchange rates. Colacito and Croce (2011) study a model that combines cross-country-correlated long-run risk with Epstein-Zin preferences. In their paper, the two countries' exogenous consumption growth processes are calibrated to be moderately correlated but include a persistent predictable component that is highly correlated across countries. This leads to moderate consumption growth correlation and high pricing kernel correlation, allowing the model to successfully match the stylized data facts. Stathopoulos

²See also Vissing-Jorgensen (2002).

(2012) uses preferences with external habit formation and home-bias in consumption to address the puzzle. The present paper differs from previous work in that we do not resort to non-standard preferences. Rather, we ask whether goods market frictions that have been studied in the international macroeconomics literature can generate a similar result. Our results also do not rely on exogenous driving processes such as long-run risk or external habits that are empirically difficult to observe.

2.2 The Role of Asset and Goods Markets

The stylized fact in the data that our model attempts to rationalize is the co-existence of moderately correlated consumption growth between countries with smooth exchange rates and high equity risk premia. Our model features two main ingredients that help it address these data facts. On the asset market side, we assume that only a limited fraction of agents in each country can participate in equity markets. On the goods market side, we model a traded and non-traded sector in each country featuring distribution cost for the consumption of tradables.

In this section, we motivate the choice of these two key model components and provide intuition for why they help the model come closer to the data. Since an analytical solution is not available for the full model, we conduct this analysis with respect to two benchmarks. First, we study a complete markets model. This analysis highlights the importance of introducing some form of market incompleteness - in our case limited stock market participation - as a necessary condition for models of a wide class to be able to fit the data. Then we study another benchmark, a model *without* financial markets and with just traded goods. In that setup, we revisit the result from Cole and Obstfeld (1991) that consumption tends to co-move strongly between countries even in the absence of financial markets and provide intuition for how non-traded goods with distribution cost weaken this effect.

2.2.1 Asset Markets

To motivate our modeling of asset markets, we first consider a representative agent model with complete financial markets in which agents have power utility $U(C)$. In this type of framework, one of the equilibrium conditions requires that exchange rates depreciate by the difference between foreign and marginal utility growth,

$$\ln \frac{Q_{t+1}}{Q_t} = \ln U' \left(\frac{C_{t+1}^*}{C_t^*} \right) - \ln U' \left(\frac{C_{t+1}}{C_t} \right). \quad (2.1)$$

Here, C_t denotes the home consumption index at time t and the $*$ superscript indicates that a variable refers to the foreign country (as we will adopt throughout the paper). The real exchange rate is $Q = \frac{P_F}{P_H}$, where P_H denotes the consumer price index for the aggregate domestic consumption basket and P_F denotes the same index for the foreign consumption basket.

Brandt et al. (2006) use equation 2.1 to document a tension that exists between the data and the theory. On the one hand, we know that observed high risk premia in financial markets require marginal utility growth to be highly volatile at home and in the foreign country. On the other hand, observed exchange rates are comparatively smooth. With regards to equation 2.1, the only way that exchange rates on the left hand side can exhibit low volatility while the two marginal utility terms on the right hand side can exhibit high volatility is if the marginal utilities are highly correlated. This implication of theory is not borne out by the data, however, as the correlation of consumption growth across countries is around 0.6 (depending on the measure and country pair), much lower than required to satisfy equation 2.1.

In order to break the tight link between exchange rates and consumption growth in equation 2.1, we assume that only a fraction of the population in each country has access to the stock market. Hence, we introduce a particular type of market incompleteness in which stockholders have access to more complete markets than non-stockholders. In addition, using a limited stock market participation setup has proven successful in explaining high equity risk premia. The model in Guvenen (2009) represents the current state-of-the-art framework for asset

pricing with limited stock market participation and we will adopt his specification of the asset market in our full model below.

While this risk sharing condition in equation 2.1 only arises if financial markets are complete, it turns out that it still holds approximately true even in incomplete markets. The goods market frictions that we turn to next play an important role in that respect.

2.2.2 Goods Markets

While having incomplete markets is necessary to reconcile asset prices with consumption and exchange rate dynamics, it is not sufficient. In particular, Cole and Obstfeld (1991) famously point out that consumption growth tends to be highly correlated across countries even without financial markets. We will in turn summarize their argument in a model with no financial markets and only traded goods. Note that the exact relationships derived below will not hold in our model. However, analyzing the role of distribution cost in a simple framework that is analytically tractable is useful to build intuition for the results from our full model that we present in the next section.

There are two symmetrical countries, called “home” and “foreign”. Aggregate consumption (also referred to as the consumption index) in the home county is given by the constant elasticity of substitution (CES) goods aggregator

$$C = C_T \equiv \left[a_D^{1-\rho} (c_H)^\rho + (1 - a_D)^{1-\rho} (c_F)^\rho \right]^{1/\rho}, \quad \rho < 1, \quad (2.2)$$

where c_H denotes domestic consumption of the home tradable good and c_F denotes domestic consumption of the foreign tradable good. The elasticity of substitution between the two varieties of traded goods is given by $\omega = \frac{1}{1-\rho}$ and the weight of home tradables in aggregate consumption is a_D . Further, define the terms of trade $\tau = \frac{p_F}{p_H}$ as the ratio of the price of the foreign tradable p_F to that of the domestic tradable p_H . Corsetti et al. (2008) show that the response of domestic demand for the home good to a fall in its price (increase in τ) can be

decomposed into a substitution effect (SE) and income effect (IE), such that

$$\frac{\partial C_H}{\partial \tau} = \omega \underbrace{\frac{a_D (1 - a_D) \tau^{-\omega}}{[a_D + (1 - a_D) \tau^{1-\omega}]^2} Y^T}_{\text{SE}} - \underbrace{\frac{a_D (1 - a_D) \tau^{-\omega}}{[a_D + (1 - a_D) \tau^{1-\omega}]^2} Y^T}_{\text{IE}}, \quad (2.3)$$

where Y^T is the home endowment of the domestic tradable good.

Corsetti et al. (2008) illustrate that Equation 2.3 allows us to analyze how supply shocks propagate between countries. If the elasticity of substitution between domestic and foreign tradables ω is larger than 1, then the substitution effect dominates the income effect. This is the case studied by Cole and Obstfeld (1991). If the home country is hit with a negative supply shock, the home good needs to become more expensive (τ decreases, the terms of trade improve) for domestic demand to fall and match supply. This partially compensates domestic agents for the adverse supply shock by raising the value of their tradable endowment in international goods markets. Hence, with a large trade elasticity, goods market prices endogenously adjust to provide insurance against negative supply shocks. For this reason, consumption growth will be highly correlated across countries even in the absence of financial markets. This mechanism is amplified by the fact that foreign demand for the home good unambiguously decreases in its price, i.e. $\frac{\partial C_H^*}{\partial \tau}$, as the substitution and income effects are both positive regardless of the trade elasticity.

Now consider the case where the trade elasticity is below one. Then, the income effect dominates. If the home country is hit with a negative shock, the price of the home tradable has to fall in order to induce home agents to reduce their demand to match the restricted supply. Hence, the value of their income drops further, amplifying the effect of the negative endowment shock on their consumption. This is the key mechanism that pushes the consumption growth correlation below unity. Note, however, that the domestic income effect not only has to be stronger than the domestic substitution effect - it also has to outweigh the positive income and substitution effects of the foreign agents.

In the next section, we develop the full model whose goods market side also includes non-tradable goods and distribution cost. It turns out that distribution cost amplify the size of

the income effect relative to the substitution effect and play a quantitatively important role for our results. We will discuss the role of the distribution cost in more detail in Section 2.6.2. Further, recall that equation 2.3 only holds without financial markets. The full model will have nearly complete asset markets, allowing agents to share consumption risk more effectively between countries. The quantitative results from the full model will hence shed light on the question of how much of the income effect discussed in this section survives after adding financial markets.³

2.3 Model

This section presents the full model featuring limited stock market participation and distribution cost with non-traded goods. Each country is now endowed with *two* Lucas trees, producing a country-specific tradable and non-tradable good, respectively. Furthermore, each country is inhabited by two types of agents that differ in the set of financial securities they are allowed to trade. A fraction μ of agents in each country has access to the home and foreign stocks as well as an international bond, a fraction $1 - \mu$ can only hold the international bond. In the interest of brevity, we focus our presentation on the domestic economy with the understanding that the foreign counterparts are defined symmetrically.

2.3.1 Preferences

As above, the consumption index for tradable consumption C_T is given by equation 2.2. Aggregate consumption now consists of traded and non-traded goods with CES aggregator

$$C \equiv \left[a_T^{1-\phi} (C_T)^\phi + (1 - a_T)^{1-\phi} (c_N)^\phi \right]^{1/\phi}, \quad \phi < 1,$$

³Note that although our model features essentially the same goods market frictions as in Corsetti et al. (2008), the financial market structure is vastly different as the only tradable asset in their study is a contingent international bond.

where c_N is domestic consumption of the home non-tradable good. The elasticity of substitution between tradables and non-tradables is $\frac{1}{1-\phi}$ and agents assign a weight of a_T to tradables in aggregate consumption.

Agents have power utility and maximize

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \theta_t \frac{C_t^{1-\gamma}}{1-\gamma} \right],$$

where γ controls relative risk aversion. The discount factor θ_t is endogenous and evolves as $\theta_{t+1} = \theta_t \omega C_t^{-\eta}$, with $0 \leq \eta \leq \gamma$ and $0 < \omega \bar{C}^{-\eta} < 1$ and where \bar{C} denotes the steady-state value of consumption.⁴

2.3.2 Distribution Cost and Goods Prices

In addition to distinguishing between tradable and non-tradable goods, our economy features distribution cost such that for every unit of either the home or foreign tradable good consumed in the home (foreign) country, ν units of the home (foreign) non-tradable good are needed to distribute the tradable good to consumers. This drives a wedge between prices for tradable goods at the producer and consumer level. Taking consumption of the home tradable in the home country as an example,

$$p_H = \bar{p}_H + \nu p_N$$

gives the relation between the producer price of the home tradable, \bar{p}_H , its consumer price, p_H , and the price of the home non-tradable, p_N .

The utility-based consumer price indices for the home basket of tradable goods is

$$P_T = \left[a_D (p_H)^{\rho/(\rho-1)} + (1 - a_D) (p_F)^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho}.$$

⁴Endogenizing the discount factor in this way pins down a unique steady state for the distribution of wealth in the presence of incomplete financial markets. Otherwise, the model would exhibit a unit-root and not be amenable to standard numerical solution techniques. The use of this discount factor is standard in such settings. See Schmitt-Grohé and Uribe (2003), Corsetti et al. (2008), and Devereux and Sutherland (2011) for further discussions.

Similarly, the utility-based consumer price index for the aggregate home consumption basket is

$$P_H = \left[a_T (P_T)^{\phi/(\phi-1)} + (1 - a_T) (p_N)^{\phi/(\phi-1)} \right]^{(\phi-1)/\phi}.$$

We choose the home consumption basket as the numeraire, so that $P_H \equiv 1$. The exchange rate is given by $Q = \frac{P_F}{P_H}$, where P_F denotes the consumer price index for the aggregate foreign consumption basket.

2.3.3 Endowments and Asset Markets

The home country endowments of the tradable good, Y^T , and the non-tradable good, Y^N , evolve as

$$\begin{aligned} \ln Y_t^T &= \Psi^T \ln Y_{t-1}^T + \epsilon_t^T \\ \ln Y_t^N &= \Psi^N \ln Y_{t-1}^N + \epsilon_t^N, \end{aligned}$$

where ϵ_t^T and ϵ_t^N are iid normally distributed disturbances.

Labor income Y_L and capital income Y_K are given by

$$\begin{aligned} Y_{L,t} &= \theta_L (\theta_T p_{H,t}^T Y^T + (1 - \theta_T) p_{H,t}^N Y^N) \\ Y_{K,t} &= (1 - \theta_L) (\theta_T p_{H,t}^T Y^T + (1 - \theta_T) p_{H,t}^N Y^N), \end{aligned}$$

where the parameter θ_L controls the labor share and θ_T controls the share of traded goods.

Denoting asset prices by Z with the appropriate sub and superscripts, the return to a claim to the home capital income is $r_{A,t+1} = \frac{Z_{A,t+1} + Y_{K,t+1}}{Z_{A,t}}$ and the corresponding return to the foreign capital income is $r_{A,t+1}^* = \frac{Z_{A,t+1}^* + Y_{K,t+1}^*}{Z_{A,t}^*}$. Furthermore, there is an international bond which pays off half a unit of the home tradable and half a unit of the foreign tradable with return $r_{B,t+1} = \frac{\frac{1}{2}(p_{H,t+1} + p_{F,t+1})}{Z_{B,t}}$.

While both the stock market participants and non-participants receive labor income, the capital income is endowed only to the stock market participants.⁵ Recalling that the non-participants can only invest in the international bond, the budget constraints for the two types of home agents can be written as

$$\begin{aligned} W_t^p &= r_{B,t}W_{t-1}^p + \alpha_{1,t-1}^p (r_{A,t} - r_{B,t}) + \alpha_{2,t-1}^p (r_{a,t}^* - r_{B,t}) - C_t^p + \frac{1}{\mu}Y_{K,t} + Y_{L,t} \\ W_t^{np} &= r_{B,t}W_{t-1}^{np} - C_t^{np} + Y_{L,t}, \end{aligned}$$

where $W^p = \alpha_1^p + \alpha_2^p + \alpha_3^p$ and $W^{np} = \alpha_3^{np}$ denote net financial wealth of domestic participants and non-participants, respectively. The net amounts invested by an agent are denoted by α_1 for the home stock, α_2 for the foreign stock, and α_3^p for the international bond, with superscripts p and np referring to participants and non-participants, respectively.

Note that since we defined wealth and asset positions as net positions, asset market clearing is given by

$$\begin{aligned} \mu (W_t^p + W_t^{p*}) + (1 - \mu) (W_t^{np} + W_t^{np*}) &= 0 \\ \alpha_{1,t}^p + \alpha_{1,t}^{p*} &= 0 \\ \alpha_{2,t}^p + \alpha_{2,t}^{p*} &= 0, \end{aligned}$$

where foreign variables are denoted by $*$.⁶

2.4 Model Solution

The model is challenging to solve. It is not amenable to standard global solution techniques such as value function iteration because the incompleteness of financial markets requires that

⁵As pointed out in Guvenen (2009), the 20% of US households who participate in the stock market own 90% of the economy's wealth.

⁶As an example, consider the case where the home agents has a zero net position in both stocks and the bond ($\alpha_1^p = \alpha_2^p = W^p = 0$). His gross position would correspond to holding all of the home equity and none of the foreign stock.

we solve for the decentralized equilibrium directly. Due to the large number of state variables, the computational burden of doing this is prohibitive.

The method proposed by Chien et al. (2011), who solve incomplete market economies with heterogeneous trading technologies using stochastic Lagrange multipliers and measurability constraints, is inapplicable in our setup as well. The reason for this is that their aggregation result for consumption fails due to the differentiated goods in our setup and the home bias in preferences.⁷

For these reasons, we solve the model using second-order linearization techniques. This requires solving for the steady-state and first-order portfolio choice, for which we use the method suggested in Devereux and Sutherland (2010). We rely on Dynare to implement this approach.

2.5 Data and Calibration

We follow a conservative calibration strategy in that we do not choose the structural parameters of our model to directly match the stylized facts for asset price, consumption, and exchange rate dynamics. Rather, we set the structural parameters to the values that are typically used in the literature. Similarly, the moments of traded and non-traded output are calibrated to directly match their empirical counterparts.

The next section describes the data and summarizes some empirical regularities regarding international production, consumption, and exchange rates. We then discuss the model calibration.

2.5.1 Data

Our main data sources are the National Accounts database provided by the OECD and the International Financial Statistics and Direction of Trade Statistics databases offered by the

⁷While extending their method to accommodate differentiated goods and heterogeneous specifications of goods aggregators might be possible, doing this would require a substantial methodological contribution.

Table 2.1: Data Summary

Panel A shows covariances between U.S. and foreign traded production (row 1), U.S. and foreign non-traded production (row 2), and U.S. traded and foreign non-traded production (row 3). The moments refer to real, per-capita production that has been logged and hp-filtered. Panel B shows the correlation of real per-capita consumption growth between the US and the foreign countries for non-durable consumption (row 1) and total consumption (durables plus non-durables and services, row 2). Panel C shows the volatility of the real exchange rate between the US and the foreign countries. Panel D shows the average share of a country’s trade with the US as a percentage of total US trade between our set of countries. The “Average” column uses the trade weights from Panel D. Data is annual and covers 1970 to 2012 with the exception of non-durable consumption, which starts in the 1990’s for some of our countries and is not available for the UK at all. See Appendix B.1 for more details.

	Average	Canada	France	Germany	Italy	Japan	UK
Panel A: Correlation of Output by Sector							
Traded	0.74	0.85	0.68	0.59	0.68	0.61	0.85
Non-traded	0.61	0.75	0.78	0.24	0.43	0.47	0.71
Traded / non-traded	0.46	0.58	0.62	-0.05	0.09	0.38	0.84
Panel B: Correlation of Consumption Growth							
Non-durable	0.67	0.85	0.75	0.53	0.81	0.39	N/A
Total	0.52	0.61	0.49	0.39	0.29	0.39	0.72
Panel C: Real Exchange Rate							
Volatility (%)	9.5	6.8	11.1	11.4	11.0	11.8	11.8
Panel D: Trade Weights							
		0.43	0.06	0.11	0.05	0.26	0.10

IMF. We draw on these international datasources rather than on national ones in order to have data measures that are comparable across countries. Furthermore, we use annual data which allows us to analyze time series that start early, ranging from 1970 to 2012. The only time series that is not available for this time horizon is that for non-durable consumption, which starts in the 1990’s for most of our countries. Table 2.1 summarizes the data.

We analyze the data from the perspective of the US as the home country and focus on the other G7 economies, including Canada, France, Germany, Italy, Japan, and the UK, as the foreign countries. We then use the trade-weights reported in Panel D to average the moments with respect to the foreign countries.

While international co-movements in traded and non-traded production have been previously analyzed in the literature, (e.g. Stockman and Tesar, 1995) the data used in these studies only extends to 1990. Since then, rapid technological progress has facilitated international trade. We hence find it important to study more recent data.

We start by analyzing the international co-movement of output in the traded and non-traded sectors among the G7 countries. Following the methodology of Kravis et al. (1982) and Stockman and Tesar (1995), we assign output to be either tradable or non-tradable depending on the sector of production. We consider agriculture, fishing, mining, manufacturing, electricity and utilities, retail, hotels, and transportation to be tradable.⁸ The remaining categories, including construction, finance, real estate, and other services are assigned to the non-tradable sector.

Panel A shows the resulting correlations for real per-capita output in the two sectors that has been logged and hp-filtered. The first row shows the correlation between traded output in the US and traded output in the foreign country. The correlations range from 0.59 for Germany to 0.85 for Canada and the UK. The trade-weighted average, which takes into account the relative importance of a country for US trade, is quite high at 0.74. This is due in large part to the high correlation with Canada, which is responsible for nearly half of US trade among the G7 countries.

The second row of the panel shows the correlation of US non-traded output with foreign non-traded output. For all our countries, these correlations are lower than those for traded output, with a trade-weighted average of 0.61. Finally, while the correlations within the same sectors between countries tend to be quite high, the correlation of traded output in the US with non-traded output abroad reported in row 3 is significantly lower for most countries and even slightly negative for Germany. The trade-weighted average for this correlation between sectors

⁸While electricity and utilities are arguably non-tradable, in particular as they refer to the associated distribution services, they are reported together with manufacturing, which is a large component of tradable goods. Since electricity and utilities only make up a small fraction of output, classifying them as tradable rather than non-tradable does not have a significant bearing on the results.

is only 0.46. Overall, these results are quite comparable in magnitude to what Stockman and Tesar (1995) find for their earlier sample.

One of our main moments of interest is the correlation of consumption growth between countries. Since all consumption in our model is non-durable, the appropriate moment to match is the correlation of real per-capita consumption growth between countries. From row 1 of Panel B, this correlation ranges from 0.85 with Canada to 0.39 with Japan, averaging 0.67. Since data on non-durable consumption is only available since 1990 for most countries and unavailable for the UK, we also compute the correlation from total household final consumption expenditure which is available over the period from 1970 to 2012. Row 2 of the panel shows that this correlation is significantly lower for all countries, averaging 0.52.

We are also interested in the volatility of real exchange rate growth. Panel C shows that this volatility is around 11% for all countries except for Canada, where it is almost half, at 6.8%. Since Canada is the most important trade partner for the US, we find the trade-weighted average volatility of real exchange rate growth to be 9.5%.

2.5.2 Calibration

We calibrate the endowment processes for tradable and non-tradable goods to their empirical counterparts in the data. For the within-country moments, we calibrate to US data. This leads us to setting the persistence parameter equal to $\Psi^T = 0.27$ for the tradable sector and $\Psi^N = 0.45$ to the non-tradable sector, matching the first order autocorrelation of hp-filtered US production. Similarly, we choose the volatility of the endowment shocks so that the implied standard deviation of traded output, $\text{std}(Y^T) = 0.023$, and non-traded output, $\text{std}(Y^N) = 0.010$, match that from US data. We calibrate the size of the traded sector to match the average share of traded goods in US production, leading to a tradables share of $\theta_T = 0.35$. Finally, we choose the covariance of the shocks to traded and non-traded goods within a country to match the correlation of traded and non-traded production in the US, setting $\text{cor}(Y^N, Y^T) = 0.64$.

Table 2.2: Calibration

This table summarizes the model calibration at annual frequency.

	Parameter	Source
Risk aversion		
Participants	$\gamma_p = 3$	Guvenen (2009)
Non-participants	$\gamma_{np} = 10$	Guvenen (2009)
Weight on traded goods	$a_T = 0.55$	Corsetti et al. (2008)
Home bias in tradables	$a_D = 0.72$	Corsetti et al. (2008)
Elasticity of substitution		
Home and foreign traded goods	$\frac{1}{1-\rho} = 0.85$	Corsetti et al. (2008)
Traded and non-traded goods	$\frac{1}{1-\phi} = 0.74$	Corsetti et al. (2008)
Endogenous discount factor		
Curvature	$\eta = 0.1$	
Steady-state discount rate	$\omega \bar{C}^{-\eta} = 0.95$	Guvenen (2009)
Distribution cost	$\nu = 0.85$	
Labor share	$\theta_L = 0.7$	Corsetti et al. (2008)
Tradables share	$\theta_T = 0.35$	
Stock market participation rate	$\mu = 0.3$	Guvenen (2009)
Endowments		
Autocorrelation of tradables	$\Psi^T = 0.27$	
Autocorrelation of non-tradables	$\Psi^N = 0.45$	
Implied moments	$\text{std}(Y^T) = 0.023$	
	$\text{std}(Y^N) = 0.010$	
	$\text{cor}(Y^N, Y^T) = 0.64$	
	$\text{cor}(Y^T, Y^{T*}) = 0.74$	
	$\text{cor}(Y^N, Y^{N*}) = 0.61$	
	$\text{cor}(Y^N, Y^{T*}) = 0.46$	

We calibrate between-country moments of output to the average correlation between US and foreign production for a given sector. The moments we match are $\text{cor}(Y^T, Y^{T*}) = 0.74$ for the correlation of traded production between countries, $\text{cor}(Y^N, Y^{N*}) = 0.61$ for the correlation of non-traded production between countries, and $\text{cor}(Y^N, Y^{T*}) = 0.46$ for the correlation of domestic non-tradable output with foreign tradable output.

We follow the working paper version of Guvenen (2009) in calibrating relative risk aversion,

the time discount rate, and stock market participation.⁹ Specifically, we set relative risk aversion to $\gamma_p = 3$ for stockholders and $\gamma_{np} = 10$ for non-stockholders. The steady-state discount rate is $\omega\bar{C}^{-\eta} = 0.95$.¹⁰ We calibrate the stock market participation rate to $\mu = 0.3$. While Guvenen (2009) uses a parameter value of 0.2, he also points to recent evidence that stock market participation has increased. We take this into account by choosing a slightly higher participation rate than him as we calibrate the model to more recent data.

The remaining utility parameters refer to agents' preferences over the different types of goods. Here, we follow Corsetti et al. (2008). Like them, we set the utility weight of tradables to $\theta_T = 0.35$, matching the share of traded goods in the US consumption basket, and the home bias in tradable goods to $a_D = 0.72$. Similarly, we choose the elasticity of substitution between the two traded goods to be $\frac{1}{1-\rho} = 0.85$ and the elasticity between the traded and non-traded good to be $\frac{1}{1-\phi} = 0.74$, which the authors obtain by performing a method of moments estimation on a model whose goods market structure is similar to ours.

A key parameter in our model is the distribution cost parameter v . As will become apparent in the next section, the distribution cost are quantitatively the most important feature of the model in reducing the correlation of consumption growth between countries. The higher v , the lower the consumption correlation. While we want to restrict the magnitude of the distribution cost to be consistent with results in previous studies, we choose it to be on the higher end of that spectrum. This allows us to evaluate how far the present model can go in matching the low consumption correlation in the data. When we study the quantitative importance of distribution cost for our results, we will then conduct extensive sensitivity analysis with regards to this parameter. There are several studies that estimate the distribution margin, which is defined as $\kappa = v\frac{p_N}{p_H}$. Burstein et al. (2003) find that the share of the retail price accounted for by distribution services is between 40% to 50% in the US, depending on the industry. Anderson and Wincoop (2004) find that distribution cost average more than 55%

⁹The working paper version of Guvenen (2009) differs from the published paper in that it uses power utility (as this paper) instead of recursive Epstein-Zin preferences.

¹⁰We use a value of $\eta = 0.1$ for the curvature of the endogenous discount factor, which is reasonably small while still producing a stable model solution.

Table 2.3: Results

This table summarizes unconditional moments of asset returns, exchange rate growth, and consumption growth. All moments are annual and in percent.

	Model	Data	Source
Asset markets			
Equity premium	3.06	6.17	Güvönen (2009)
Volatility of equity premium	9.81	19.40	Güvönen (2009)
Risk-free rate	0.73	1.94	Güvönen (2009)
Volatility of risk-free rate	9.94	5.44	Güvönen (2009)
Sharpe ratio	0.31	0.32	Güvönen (2009)
Exchange rate growth volatility	3.52	9.50	
Consumption growth			
Aggregate volatility	1.25	1.95	
Volatility participants/non-participants	3.30	> 2	Güvönen (2009)
Cross country correlation	0.75	≈ 0.6	

among industrialized countries. Considering this evidence, we set $\nu = 0.85$, which implies a steady-state distribution margin of 64% in our model.

2.6 Results

2.6.1 Full Model

Table 2.3 summarizes the moments implied by the fully featured model. The model matches asset prices rather well. The Sharpe ratio of 0.31 is nearly identical to that in the data. While the model produces a realistic *price* of risk, the equity premium is only 3.06%, about half of what it is in the data. This is not surprising, however, given that there is no financial leverage in the model and hence the *quantity* of risk is less than in the data. This is also reflected in the fact the the equity premium is about half as volatile in the model as in the data. The risk-free rate is 0.73%, which is just slightly lower than in the data. The standard deviation of the risk-free rate is higher than in the data, with 9.94% in the model compared to 5.44% in the data.

Table 2.4: Distribution Cost

The table shows the correlation of aggregate consumption growth between countries, the Sharpe ratio, and the volatility of the exchange rate in the benchmark model for varying degrees of distribution cost ν .

	Distribution cost ν					
	0	0.2	0.4	0.6	0.8	0.85
Consumption correlation	0.99	0.98	0.97	0.93	0.81	0.75
Sharpe ratio	0.23	0.23	0.22	0.22	0.27	0.31
Exchange rate volatility	1.57	1.83	2.27	3.01	3.90	3.52

The model-implied correlation of consumption growth is 0.75, which is somewhat larger than the value of 0.67 that is implied using non-durable consumption data and considerably larger than the value of 0.52 that we measure using total household final consumption expenditure. The model hence makes substantial progress in generating less than perfect consumption co-movement between countries though it falls short of fully explaining the low consumption correlation in the data.

Finally, we find that the volatility of exchange rate growth is low. It is less than half of what we measure in the data. This finding is consistent with much of the literature, e.g. the international real business cycle model of Backus et al. (2008).

We proceed by analyzing the role of distribution cost for the model mechanism in Section 2.6.2 and then quantifying the importance of all our main model ingredients for our results in Section 2.6.3.

2.6.2 The Importance of Distribution Cost

Table 2.4 shows how the model results change for varying values of the distribution cost parameter ν . The comparative statics illustrate the importance of distribution cost for the model's ability to produce a consumption correlation below unity. In fact, without distribution cost, consumption co-moves nearly perfectly between countries despite the existence of non-tradable goods and a low trade elasticity. The correlation only drops significantly for values of

the distribution cost that are on the high end of the empirically observed spectrum, reaching a correlation of 0.75 in our benchmark calibration with $\nu = 0.85$.

To provide intuition for the effect of distribution cost on the correlation of consumption, we return to our analysis from Section 2.2.2. In particular, we focus on how distribution cost lower the effective elasticity of substitution between the traded goods at the consumer level and hence amplify the magnitude of the income effect studied in that section.

To see how introducing distribution cost helps lower the correlation of consumption growth, consider the equivalent of Equation 2.3 with distribution cost,

$$\frac{\partial C_H}{\partial \tau} = \underbrace{\omega (1 - \kappa) (1 - a_D) \left(\frac{P_F}{P_H} \right)^{1-\omega}}_{\text{SE}} - \underbrace{(1 - a_D) \left(\frac{P_F}{P_H} \right)^{1-\omega} - \kappa a_D}_{\text{IE}}.$$

Similar to above, this equation shows the response of domestic demand for the home good to a fall in its price (increase in τ) in a model without financial markets. The expression here however takes account of distribution cost, which are linear in the distribution margin $\kappa = v \frac{P_N}{P_H}$. We see that an increase in the distribution margin lowers the magnitude of the substitution effect and increases the (negative) importance of the income effect.

Next, we turn to the importance of distribution cost for the price of risk in the economy. Without the cost, the Sharpe ratio is 0.24. It increases to 0.31 for the high value of the cost in our benchmark calibration. The reason is that the utility-based consumption index becomes more volatile. This increase in volatility due to the distribution cost is also reflected in the volatility of the exchange rate, which increases in the cost as well. It is, however, low compared to the data for the entire range of the parameter studied here.

2.6.3 Relative Importance of Model Ingredients for the Results

In this section, we quantify the relative importance of our main model ingredients, limited stock market participation, non-traded goods, and distribution cost, for our main results. Table 2.5 shows our main moments of interest, the consumption growth correlation, the Sharpe

Table 2.5: The Contribution of the Model Ingredients

The table shows the correlation of aggregate consumption growth between countries, the Sharpe ratio, and the volatility of exchange rate growth for six different model specifications. The two asset market specifications permit either full stock market participation ($\mu = 1$) or limited stock market participation ($\mu = 0.3$). The goods market either includes only traded goods (T), traded and non-traded without distribution cost (T/NT), or the fully featured goods market specification with traded and non-traded goods as well as distribution cost (T/NT/Dist).

	T	T/NT	T/NT/Dist	T	T/NT	T/NT/Dist
	Consumption correlation			Sharpe ratio		
$\mu = 1.0$	0.99	0.97	0.74	0.03	0.04	0.06
$\mu = 0.3$	1.00	0.99	0.75	0.18	0.24	0.31
	Exchange rate volatility					
$\mu = 1.0$	0.64	1.37	2.14			
$\mu = 0.3$	0.67	1.57	3.52			

ratio, and the exchange rate volatility for six different model specifications. We study three special cases with respect to the goods market: only traded goods, traded and non-traded goods but without distribution cost, and traded and non-traded goods with distribution cost. For each of these three cases, we solve a version of the model with and without limited stock market participation.

First, we find that virtually all of the reduction in the consumption growth correlation comes from distribution cost. Irrespective of the financial market setup, we find that the consumption growth correlation is around 0.75 with distribution cost and close to unity without.

With regards to the Sharpe ratio, we find that the model with full stock market participation only produces a small price of risk that does not exceed a Sharpe ratio of 0.06. However, once limited stock market participation is introduced, the price of risk increases significantly. If all goods are tradable, the Sharpe ratio reaches 0.18 and increases significantly both with the addition of non-traded goods and by introducing distribution cost. The goods market setup matters for asset prices as both non-tradability and distribution cost raise the volatility of the utility based consumption index.

Finally, the model produces very smooth exchange rates in all versions, ranging from a stan-

Table 2.6: The Relative Importance of the Model Ingredients

The table shows the percentage contribution of each of the model ingredients for the Sharpe ratio, consumption correlation, and exchange rate volatility. The percentage contributions are produced as we start from a frictionless model with only tradable goods and full asset market participation and then introduce the frictions one-by-one. The order of the model specifications studied is: 1) all tradable goods and full stock market participation 2) all tradable goods and limited stock market participation 3) non-tradable goods and limited stock market participation, and 4) non-tradable goods, distribution service costs, and limited stock market participation. Note that for the Sharpe ratio and the exchange rate volatility, we report the relative contribution to the total *increase* from specification 1) to 4). For the consumption growth correlation, we report the relative contribution to the total *reduction* from 1) to 4). LP = limited participation, NT = non-traded goods, Dist = distribution cost.

	LP	NT	Dist	Sum
Sharpe ratio	55%	21%	24%	100%
Consumption correlation	-8%	8%	100%	100%
Exchange rate volatility	2%	34%	64%	100%

dard deviation of 0.64% in the model with full stock market participation and all traded goods to a standard deviation of 3.52% in the model with limited stock market participation and non-traded goods with distribution cost.

To further formalize the relative contribution of the model ingredients for the three target moments, we provide another decomposition in Table 2.6. In the table, the relative contributions sum to one for each of the target moments. The percentage contributions are produced as we start from a frictionless model with only tradable goods and full asset market participation and then introduce the frictions one-by-one. The order of the model specifications studied is: 1) all tradable goods and full stock market participation 2) all tradable goods and limited stock market participation 3) non-tradable goods and limited stock market participation, and 4) non-tradable goods, distribution service costs, and limited stock market participation.¹¹

¹¹Note that the relative contributions depend on the order in which the model ingredients are introduced. The differences are small, however, and do not materially affect the relative importance of the model ingredients.

2.6.4 Consumption Dynamics by Agent Type

Our analysis thus far has focused on the correlation of aggregate consumption growth between countries and we have shown that the model is capable of producing a correlation as low as 0.75. We next analyze the consumption dynamics in more detail by focusing on the different agent types.

Here, we find that the current model has the unappealing implication that consumption growth between stockholders and non-stockholders is nearly perfectly *negatively* correlated. The reason for this is the low persistence of output that we measure in the data and to which we calibrate our endowment processes. As agents are hit by a positive supply shock, they expect output to mean-revert quickly. Hence they expect negative future consumption growth. Non-stockholders would like to increase their precautionary savings to smooth the expected reversion of output. Stockholders, on the other-hand, are reluctant to increase their borrowing substantially. As a result, the interest rate falls and reduces the value of non-stockholders' bond holdings (which are positive, on average). This reduction in wealth forces non-stockholders to reduce their consumption despite the positive endowment shock. While calibrating output to be highly persistent with an auto-correlation above 0.95 annually resolves this problem, we find that output is just not nearly this close to a unit root in the data.

As expected, we find that the correlation of consumption growth between foreign and domestic stockholders is unity as they have access to virtually complete financial markets. The above fact that consumption of non-stockholders is almost perfectly negatively correlated with that of stockholders then implies that the correlation of consumption growth between foreign and domestic non-stockholders is nearly unity as well. Consumption for both groups of non-stockholders hence moves together and against that of stockholders. The result that the *aggregate* consumption growth correlation between countries is only 0.75 despite consumption for stockholders and non-stockholders co-moving nearly perfectly then comes from the cross-correlation of stockholders consumption in one country with that of non-stockholders in the other. This correlation is almost perfectly *negative*, driving down the aggregate consumption growth correlation.

It is worth pointing out that the low aggregate consumption growth correlation the model achieves still obtains if we allow all agents to participate in the stock market, as can be seen from Table 2.5. Hence, this results does not hinge on the unappealing co-movement of consumption between the different groups of agents. That said, we regard improving the model to ameliorate its implications along this dimension as a crucial next step for future research.

2.7 Conclusion

We propose a general equilibrium two-country macro-finance model that features limited stock market participation, non-traded goods and distribution cost. The model makes significant progress towards rationalizing the coexistence of three stylized data facts that have been a challenge for theory thus far: i) The high equity premium, ii) relatively smooth exchange rates, and iii) the low international correlation of consumption growth.

Consistent with closed-economy models, the limited stock market participation friction produces a high and realistic price of risk. We further find that distribution cost play a central role for reducing international consumption co-movement while also amplifying risk premia. The model naturally produces a low exchange rate volatility that is even lower than in the data, irrespective of the severity of the frictions we study.

Future research will need to focus on resolving the stark implications for the consumption dynamics between agent types that are implied by the model.

Appendix A

First Appendix

A.1 Solution Method

This appendix contains an outline of the solution algorithm. Both the algorithm and the exposition closely follow Guvenen (2009), but are adapted to the model in this paper.

1. Choose grids for each agent's wealth holdings ω^h and ω^n , as well as for Υ . Off-grid values for all functions are obtained using cubic spline interpolation. Let superscript j index the iteration number.
2. Solve each agent's dynamic problem, defined by equations 1.8 - 1.12, using value function iteration. For stockholders, I enforce the market clearing condition by imposing $s' \equiv 1$. This simplification has two advantages: (i) it reduces the dimensionality of stockholders' optimization problem and (ii) it avoids having to solve for the stock price at each iteration. Instead, I use shadow pricing to solve for the stock price in step 5.
3. Update the bond pricing function, $P^{f,j-1}$. To do this, first solve

$$\begin{aligned} \tilde{V}^h(\omega; \Upsilon, \hat{q}) &= \max_{b', s'} \left\{ U^h(c) + \beta E \left[V^{h,j}(\omega'; \Upsilon) \mid Y \right] \right\} \\ &\text{s.t.} \\ c + \hat{q}b' + P^{S,j}(\Upsilon) s' &\leq \omega + W(\Upsilon) - \tau_L + T(Y) \end{aligned}$$

and equations 1.10 - 1.12 as well as the equivalent problem for non-stockholders. The difference between this dynamic problem and the original Bellman equation in the main text is that here the individual views the current period bond price as some arbitrary parameter \hat{q} . This problem generates bond holding rules $\tilde{b}^h(\omega; \Upsilon, \hat{q})$ and $\tilde{b}^{nh}(\omega; \Upsilon, \hat{q})$, which are functions of \hat{q} . Then, at each grid point, search over values of \hat{q} to find the market clearing q^* . Then, use this to update the bond pricing function, i.e. $P^{f,j}(\Upsilon) = q^*(\Upsilon)$.

4. Iterate on 1. to 3. until convergence.

5. Obtain the stock price as the solution to

$$P^S(\Upsilon) = \text{E} [m(\Upsilon') ((1 - \tau_D(\Upsilon')) D(\Upsilon') + P^S(\Upsilon'))], \text{ using value function iteration.}$$

Note that aggregate transfers are given by 1.5 and are not a function of individual decisions. The reason is that wages and corporate profits, which together form the aggregate tax base, are exogenous.

Appendix B

Second Appendix

B.1 Data

Our main data sources are the OECD National Accounts and the IMF International Financial Statistics. To ensure that we have long time series, we use data at the annual frequency. For most measures, we are able to obtain data from 1970 to 2012. Table 2.1 provides summary statistics for the data.

B.1.1 Production of Tradables and Non-Tradables

We use annual data covering 1970 to 2012 from the OECD on value-added by sector for the US, Canada, France, Germany, Italy, Japan, and the UK. We then categorize output to be either tradable or non-tradable, depending on its sector, as described in the main text.

The data we retrieve is measured in constant prices and PPPs fixed in the OECD base year (2005). We then divide by each country's population (also from the OECD) to get sectoral output in per-capita terms. Finally, we detrend the data by taking logs and applying an hp-filter with smoothing parameter 6.25. Table 2.1 shows the resulting correlations of detrended output across sectors between the US and our set of six foreign countries.

B.1.2 Consumption

We use two different measures of consumption provided by the OECD, one that measures only non-durable consumption and one that measures total final household consumption.

Our measure for non-durable consumption is the sum of household final consumption expenditure for non-durables and services. These time series are in constant prices (OECD base year = 2005) and hence are additive. This measure is available starting in 1981 for Canada, 1959 for France, 1991 for Germany, 1995 for Italy, 1994 for Japan, and 1970 for the US. It is not available for the UK.

To have a longer time series of consumption, we also obtain final household consumption expenditure in constant prices of the OECD base year. This measure is available from 1970 to 2012 for all countries.

For both measures, we then divide by the country's population, take logs, and compute the growth rate. Panel B of Table 2.1 shows the correlation of US consumption growth with our set of foreign countries for the two measures.

B.1.3 Exchange Rates

We retrieve annual end-of-period nominal exchange rate data from the IMF IFS Database. Exchange rates are in terms of foreign currency to USD. To convert the nominal exchange rates to real terms, we use the deflator for household final consumption expenditure provided by the OECD. Panel C of Table 2.1 shows the volatility of real exchange rate growth that we obtain. The data cover 1970 to 2012 for all countries.

B.1.4 Trade Weights

We obtain data on imports and exports between the US and our set of foreign countries from the IMF Direction of Trade Statistics database. The data are in USD terms and span 1970

to 2012 for all countries. Then, for every year, we determine a country's trade weight with the US as the sum of imports and exports between that country and the US divided by the sum of all imports and exports between the US and our complete set of foreign countries. This procedure yields one trade weight for every country in every year. We then compute the time-series averages for every country. The resulting trade weights are reported in Panel D of Table 2.1.

B.2 Goods Market Clearing

The goods market clearing conditions are for domestic tradables, domestic non-tradables, foreign tradables, and foreign non-tradables (in that order) are

$$\begin{aligned} \mu (c_H^p + c_H^{p*}) + (1 - \mu) (c_H^{np} + c_H^{np*}) &= \theta_T Y^T \\ \mu (c_N^p + \nu c_H^p + \nu c_F^p) + (1 - \mu) (c_N^{np} + \nu c_H^{np} + \nu c_F^{np}) &= (1 - \theta_T) Y^N \\ \mu (c_F^p + c_F^{p*}) + (1 - \mu) (c_F^{np} + c_F^{np*}) &= \theta_T Y^{T*} \\ \mu (c_N^{p*} + \nu c_H^{p*} + \nu c_F^{p*}) + (1 - \mu) (c_N^{np*} + \nu c_H^{np*} + \nu c_F^{np*}) &= (1 - \theta_T) Y^{N*}, \end{aligned}$$

where * denotes foreign variables.

B.3 Portfolios Choice

This section outlines how we adapt the method in Devereux and Sutherland (2010) and Devereux and Sutherland (2011) to the case with multiple assets and a non-zero exchange rate.

B.3.1 Portfolio Choice Equations

In what follows, we denote the base asset, corresponding to the international bond in the main text, as asset 4. The exchange rate is denoted by E and all returns are converted to units of the home consumption basket.

For every asset m , the home and foreign portfolio choice equations are

$$\begin{aligned} \mathbb{E} \left[C_{t+1}^{-\gamma} (R_{m,t+1} - R_{4,t+1}) \right] &= 0 \\ \mathbb{E} \left[C_{t+1}^{*-\gamma} \frac{1}{E_{t+1}} (R_{m,t+1} - R_{4,t+1}) \right] &= 0. \end{aligned}$$

B.3.2 Steady-state portfolio

Expanding the portfolio choice equations to the second order accuracy and taking the difference yields

$$\mathbb{E} \left[\left(\hat{C}_{t+1} - \hat{C}_{t+1}^* - \hat{E}_{t+1}/\gamma \right) \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] = 0 + O(\epsilon^3).$$

The steady-state portfolio is the one that satisfies this equation as outlined in Devereux and Sutherland (2011).

B.3.3 First-order portfolio

Following Devereux and Sutherland (2010), we expand the third-order portfolio choice equations to the third order. For the domestic country, this is

$$\begin{aligned} \mathbb{E} \left[\bar{C}^{-\gamma} (\bar{R}_m - \bar{R}_4) - \gamma (\bar{R}_m - \bar{R}_4) \bar{C}^{-\gamma} \hat{C}_{t+1} + \bar{C}^{-\gamma} \bar{R} \hat{R}_{m,t+1} - \bar{C}^{-\gamma} \bar{R} \hat{R}_{4,t+1} - \gamma \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1} \hat{R}_{m,t+1} \right. \\ \left. + \gamma \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1} \hat{R}_{4,t+1} + \frac{1}{2} (\bar{R}_m - \bar{R}_4) \gamma^2 \bar{C}^{-\gamma} \hat{C}_{t+1}^2 + \frac{1}{2} \bar{C}^{-\gamma} \bar{R} \hat{R}_{m,t+1}^2 - \frac{1}{2} \bar{C}^{-\gamma} \bar{R} \hat{R}_{4,t+1}^2 \right. \\ \left. + \frac{3}{6} \gamma^2 \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1}^2 \hat{R}_{m,t+1} - \frac{3}{6} \gamma^2 \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1}^2 \hat{R}_{4,t+1} - \frac{3}{6} \gamma \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1} \hat{R}_{m,t+1}^2 + \frac{3}{6} \gamma \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1} \hat{R}_{4,t+1}^2 \right. \\ \left. - \frac{1}{6} \gamma^3 \bar{C}^{-\gamma} (\bar{R}_m - \bar{R}_4) \hat{C}_{t+1}^3 + \frac{1}{6} \bar{C}^{-\gamma} \bar{R} \hat{R}_{m,t+1}^3 - \frac{1}{6} \bar{C}^{-\gamma} \bar{R} \hat{R}_{4,t+1}^3 \right] = 0 + O(\epsilon^4) \end{aligned}$$

The third-order expansion of the foreign portfolio choice equation is (where C^* is replaced by C for notational convenience)

$$\begin{aligned}
& \mathbb{E} \left[\bar{C}^{-\gamma} \frac{1}{\bar{E}} (\bar{R}_m - \bar{R}_4) - \gamma (\bar{R}_m - \bar{R}_4) \frac{1}{\bar{E}} \bar{C}^{-\gamma} \hat{C}_{t+1} + \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{R}_{m,t+1} - \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{R}_{4,t+1} \right. \\
& - \bar{C}^{-\gamma} (\bar{R}_m - \bar{R}_4) \frac{1}{\bar{E}} \hat{E}_{t+1} - \gamma \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{C}_{t+1} \hat{R}_{m,t+1} + \gamma \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{C}_{t+1} \hat{R}_{4,t+1} \\
& + (\bar{R}_m - \bar{R}_4) \gamma \bar{C}^{-\gamma} \frac{1}{\bar{E}} \hat{C}_{t+1} \hat{E}_{t+1} - \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{E}_{t+1} \hat{R}_{m,t+1} + \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{E}_{t+1} \hat{R}_{4,t+1} \\
& + \frac{1}{2} (\bar{R}_m - \bar{R}_4) \frac{1}{\bar{E}} \gamma^2 \bar{C}^{-\gamma} \hat{C}_{t+1}^2 + \frac{1}{2} (\bar{R}_m - \bar{R}_4) \frac{1}{\bar{E}} \bar{C}^{-\gamma} \hat{E}_{t+1}^2 + \frac{1}{2} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{R}_{m,t+1}^2 \\
& - \frac{1}{2} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{R}_{4,t+1}^2 - \frac{3}{6} (\bar{R}_m - \bar{R}_4) \gamma^2 \bar{C}^{-\gamma} \frac{1}{\bar{E}} \hat{C}_{t+1}^2 \hat{E}_{t+1} + \frac{3}{6} \frac{1}{\bar{E}} \gamma^2 \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1}^2 \hat{R}_{m,t+1} \\
& - \frac{3}{6} \frac{1}{\bar{E}} \gamma^2 \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1}^2 \hat{R}_{4,t+1} - \frac{3}{6} (\bar{R}_m - \bar{R}_4) \gamma \bar{C}^{-\gamma} \frac{1}{\bar{E}} \hat{C}_{t+1} \hat{E}_{t+1}^2 \\
& - \frac{3}{6} \frac{1}{\bar{E}} \gamma \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1} \hat{R}_{m,t+1}^2 + \frac{3}{6} \frac{1}{\bar{E}} \gamma \bar{C}^{-\gamma} \bar{R} \hat{C}_{t+1} \hat{R}_{4,t+1}^2 \\
& + \frac{3}{6} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{E}_{t+1}^2 \hat{R}_{m,t+1} - \frac{3}{6} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{E}_{t+1}^2 \hat{R}_{4,t+1} - \frac{3}{6} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{E}_{t+1} \hat{R}_{m,t+1}^2 \\
& + \frac{3}{6} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{E}_{t+1} \hat{R}_{4,t+1}^2 - \frac{1}{6} (\bar{R}_m - \bar{R}_4) \frac{1}{\bar{E}} \gamma^3 \bar{C}^{-\gamma} \hat{C}_{t+1}^3 - \frac{1}{6} (\bar{R}_m - \bar{R}_4) \bar{C}^{-\gamma} \frac{1}{\bar{E}} \hat{E}_{t+1}^3 \\
& + \frac{1}{6} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{R}_{m,t+1}^3 - \frac{1}{6} \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{R}_{4,t+1}^3 + \gamma \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{C}_{t+1} \hat{E}_{t+1} \hat{R}_{m,t+1} \\
& \left. - \gamma \bar{C}^{-\gamma} \frac{1}{\bar{E}} \bar{R} \hat{C}_{t+1} \hat{E}_{t+1} \hat{R}_{4,t+1} \right] = 0 + O(\epsilon^4)
\end{aligned}$$

Take the Difference of the portfolio choice equations to get

$$\begin{aligned}
& \mathbb{E} \left[\left(\hat{C}_{t+1} - \hat{C}_{t+1}^* - \hat{E}_{t+1}/\gamma \right) \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right. \\
& - \frac{1}{2} \gamma \left(\hat{C}_{t+1}^2 - \hat{C}_{t+1}^{2*} - \hat{E}_{t+1}^2/\gamma^2 \right) \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \\
& + \frac{1}{2} \left(\hat{C}_{t+1} - \hat{C}_{t+1}^* - \hat{E}_{t+1}/\gamma \right) \left(\hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) \\
& \left. \hat{C}_{t+1}^* \hat{E}_{t+1} \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] = 0 + O(\epsilon^4)
\end{aligned}$$

The first-order portfolio choice is the one that satisfies this equation.

Next, sum the Euler Equations to get

$$\begin{aligned}
E_t \left[\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right] &= E_t \left[-\frac{1}{2} \left(\hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) - \frac{1}{6} \left(\hat{R}_{m,t+1}^3 - \hat{R}_{4,t+1}^3 \right) \right. \\
&\quad + \frac{\gamma}{2} \left(\hat{C}_{t+1} + \hat{C}_{t+1}^* \right) \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \\
&\quad - \frac{\gamma^2}{4} \left(\hat{C}_{t+1}^2 + \hat{C}_{t+1}^{2*} \right) \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) + \frac{\gamma}{4} \left(\hat{C}_{t+1} + \hat{C}_{t+1}^* \right) \left(\hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) \\
&\quad + \frac{1}{2} \hat{E}_{t+1} \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) - \frac{1}{4} \hat{E}_{t+1}^2 \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \\
&\quad \left. + \frac{1}{4} \hat{E}_{t+1} \left(\hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) - \frac{1}{2} \gamma \hat{C}_{t+1}^* \hat{E}_{t+1} \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] = 0 + O(\epsilon^4)
\end{aligned}$$

The state space solution for $(\hat{C}_{t+1} - \hat{C}_{t+1}^* - \hat{E}_{t+1}/\gamma)$ and $\hat{r}_{x,t+1}$ can be expressed as:

$$\begin{aligned}
(\hat{C} - \hat{C}^* - \hat{E}_{t+1}/\gamma) &= [\tilde{D}_0] + [\tilde{D}_1] \xi + [\tilde{D}_2]_i [\epsilon]^i + [\tilde{D}_3]_k \left([z^f]^k + [z^s]^k \right) + [\tilde{D}_4]_{ij} [\epsilon]^i [\epsilon]^j \\
&\quad + [\tilde{D}_5]_{ki} [\epsilon]^i [z^f]^k + [\tilde{D}_6]_{ij} [z^f]^i [z^f]^j + O(\epsilon^3)
\end{aligned}$$

$$\begin{aligned}
[\hat{r}_x]_m &= [\tilde{R}_0]_m + [\tilde{R}_1]_m \xi + [\tilde{R}_2]_{mi} [\epsilon]^i + [\tilde{R}_3]_{mk} \left([z^f]^k + [z^s]^k \right) + [\tilde{R}_4]_{mij} [\epsilon]^i [\epsilon]^j \\
&\quad + [\tilde{R}_5]_{mki} [\epsilon]^i [z^f]^k + [\tilde{R}_6]_{mij} [z^f]^i [z^f]^j + O(\epsilon^3)
\end{aligned}$$

Up to first-order accuracy, the expected excess return is zero and, up to second-order accuracy, it is a constant. This implies that $[\tilde{R}_3]_{mk} [z^f]^k = 0$ and the terms $[\tilde{R}_3]_{mk} [z^s]^k$ and $[\tilde{R}_6]_{mij} [z^f]^i [z^f]^j$ are constants. It also follows that

$$[\tilde{R}_0]_m = E[\hat{r}_x]_m - [\tilde{R}_3]_{mk} [z^s]^k - [\tilde{R}_4]_{mij} [\Sigma]^{ij} - [\tilde{R}_6]_{mij} [z^f]^i [z^f]^j$$

so

$$\begin{aligned}
[\hat{r}_x]_m &= E[\hat{r}_x]_m - [\tilde{R}_4]_{mij} [\Sigma]^{ij} + [\tilde{R}_1]_m \xi + [\tilde{R}_2]_{mi} [\epsilon]^i + [\tilde{R}_4]_{mij} [\epsilon]^i [\epsilon]^j \\
&\quad + [\tilde{R}_5]_{mki} [\epsilon]^i [z^f]^k + O(\epsilon^3)
\end{aligned}$$

Now recognize that ξ is endogenous and given by $\xi = [\gamma]_{mk} [z^f]^k [\hat{r}_x]^m$. This is a second-order term, so \hat{r}_x can be replaced by its first-order parts, i.e. by $[\tilde{R}_2]_{mi} [\epsilon]^i$. This implies that $\xi = [\tilde{R}_2]_i^m [\gamma]_{mk} [\epsilon]^i [z^f]^k$.

Now, we can write (note, here the asset index that's summed over is q , the one that's held fixed is m):

$$\begin{aligned} (\hat{C} - \hat{C}^* - \hat{E}_{t+1}/\gamma) &= [\tilde{D}_0] + [\tilde{D}_2]_i [\epsilon]^i + [\tilde{D}_3]_k \left([z^f]^k + [z^s]^k \right) + [\tilde{D}_4]_{ij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left([\tilde{D}_5]_{ki} + [\tilde{D}_1] [\tilde{R}_2]_i^m [\gamma]_{mk} \right) [\epsilon]^i [z^f]^k + [\tilde{D}_6]_{ij} [z^f]^i [z^f]^j + O(\epsilon^3) \end{aligned}$$

$$\begin{aligned} [\hat{r}_x]_m &= E[\hat{r}_x]_m - [\tilde{R}_4]_{mij} [\Sigma]^{ij} + [\tilde{R}_2]_{mi} [\epsilon]^i + [\tilde{R}_4]_{mij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left([\tilde{R}_5]_{mki} + [\tilde{R}_1]_m [\tilde{R}_2]_i^q [\gamma]_{qk} \right) [\epsilon]^i [z^f]^k + O(\epsilon^3) \end{aligned}$$

Furthermore, we use the following expressions for consumption

$$\hat{C} = [\tilde{C}_2^H]_i [\epsilon]^i + [\tilde{C}_3^H]_k [z^f]^k + O(\epsilon^2)$$

$$\hat{C}^* = [\tilde{C}_2^F]_i [\epsilon]^i + [\tilde{C}_3^F]_k [z^f]^k + O(\epsilon^2),$$

return to asset m

$$[\hat{r}]_m = [\tilde{R}_2^m]_i [\epsilon]^i + [\tilde{R}_3^m]_k [z^f]^k + O(\epsilon^2),$$

and the exchange rate

$$\hat{E} = [\tilde{H}_2]_i [\epsilon]^i + [\tilde{H}_3]_k [z^f]^k + O(\epsilon^2).$$

Fixing asset m , we get

$$\begin{aligned} &[\tilde{D}_2]_i [\tilde{R}_2]_{mj} [\Sigma]^{ij} + [\tilde{D}_2]_i \left([\tilde{R}_5]_{mkj} + [\tilde{R}_1]_m [\tilde{R}_2]_j^q [\gamma]_{qk} \right) [\Sigma]^{ij} [z^f]^k \\ &+ \left(E[\hat{r}_x]_m - [\tilde{R}_4]_{mij} [\Sigma]^{ij} \right) [\tilde{D}_3]_k [z^f]^k + [\tilde{R}_2]_{mi} \left([\tilde{D}_5]_{kj} + [\tilde{D}_1] [\tilde{R}_2]_j^q [\gamma]_{qk} \right) [\Sigma]^{ij} [z^f]^k \\ &+ [\tilde{R}_4]_{mij} [\tilde{D}_3]_k [\Sigma]^{ij} [z^f]^k - \gamma [\tilde{R}_2]_{mi} \left([\tilde{C}_2^H]_j [\tilde{C}_3^H]_k - [\tilde{C}_2^F]_j [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} [z^f]^k \\ &+ \frac{1}{\gamma} [\tilde{R}_2]_{mi} [\tilde{H}_2]_j [\tilde{H}_3]_k [\Sigma]^{ij} [z^f]^k + \frac{1}{2} \left([\tilde{R}_2^m]_i [\tilde{R}_2^m]_j - [\tilde{R}_2^4]_i [\tilde{R}_2^4]_j \right) [\tilde{D}_3]_k [\Sigma]^{ij} [z^f]^k \\ &+ [\tilde{D}_2]_i [\tilde{R}_2]_{mj} [\tilde{R}_3^m]_k [\Sigma]^{ij} [z^f]^k \\ &+ \left([\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} [\tilde{H}_3]_k + [\tilde{C}_3^F]_k [\tilde{R}_2]_{mj} [\tilde{H}_2]_i \right) [\Sigma]^{ij} [z^f]^k = 0 + O(\epsilon^4) \end{aligned}$$

Since this is at the steady-state portfolio, $[\tilde{D}_2]_i [\tilde{R}_2]_{mj} [\Sigma]^{ij} = 0$ for all assets m and the

above equation is homogeneous in $[z^j]^k$ so that the following equation must be satisfied for all k and m :

$$\begin{aligned}
& [\tilde{D}_2]_i \left([\tilde{R}_5]_{mkj} + [\tilde{R}_1]_m [\tilde{R}_2]_j^q [\gamma]_{qk} \right) [\Sigma]^{ij} \\
& + \left(E[\hat{r}_x]_m - [\tilde{R}_4]_{mij} [\Sigma]^{ij} \right) [\tilde{D}_3]_k + [\tilde{R}_2]_{mi} \left([\tilde{D}_5]_{kj} + [\tilde{D}_1] [\tilde{R}_2]_j^q [\gamma]_{qk} \right) [\Sigma]^{ij} \\
& + [\tilde{R}_4]_{mij} [\tilde{D}_3]_k [\Sigma]^{ij} - \gamma [\tilde{R}_2]_{mi} \left([\tilde{C}_2^H]_j [\tilde{C}_3^H]_k - [\tilde{C}_2^F]_j [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} \\
& + \frac{1}{\gamma} [\tilde{R}_2]_{mi} [\tilde{H}_2]_j [\tilde{H}_3]_k [\Sigma]^{ij} + \frac{1}{2} \left([\tilde{R}_2^m]_i [\tilde{R}_2^m]_j - [\tilde{R}_2^4]_i [\tilde{R}_2^4]_j \right) [\tilde{D}_3]_k [\Sigma]^{ij} \\
& + [\tilde{D}_2]_i [\tilde{R}_2]_{mj} [\tilde{R}_3^m]_k [\Sigma]^{ij} \\
& + \left([\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} [\tilde{H}_3]_k + [\tilde{C}_3^F]_k [\tilde{R}_2]_{mj} [\tilde{H}_2]_i \right) [\Sigma]^{ij} = 0 + O(\epsilon^4)
\end{aligned}$$

Furthermore, we can express the second order of the expected excess return of asset m as

$$\begin{aligned}
E_t [\hat{R}_{m,t+1} - \hat{R}_{4,t+1}] &= E_t \left[-\frac{1}{2} \left(\hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2 \right) + \frac{\gamma}{2} \left(\hat{C}_{t+1} + \hat{C}_{t+1}^* \right) \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right. \\
&\quad \left. + \frac{1}{2} \hat{E}_{t+1} \left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1} \right) \right] + O(\epsilon^3)
\end{aligned}$$

Evaluating this using the first-order state-space solution for consumption, returns, and the exchange rate yields

$$\begin{aligned}
E_t [\hat{R}_{m,t+1} - \hat{R}_{4,t+1}] &= -\frac{1}{2} \left([\tilde{R}_2^m]_i [\tilde{R}_2^m]_j [\Sigma]^{ij} - [\tilde{R}_2^4]_i [\tilde{R}_2^4]_j [\Sigma]^{ij} \right) \\
&\quad + \frac{\gamma}{2} \left([\tilde{C}_2^H]_i + [\tilde{C}_2^F]_i \right) \left([\tilde{R}_2^m]_j - [\tilde{R}_2^4]_j \right) [\Sigma]^{ij} \\
&\quad + \frac{1}{2} [\tilde{H}_2]_i \left([\tilde{R}_2^m]_j - [\tilde{R}_2^4]_j \right) [\Sigma]^{ij} + O(\epsilon^3) \\
&= \frac{1}{2} \left([\tilde{R}_2^4]_i [\tilde{R}_2^4]_j - [\tilde{R}_2^m]_i [\tilde{R}_2^m]_j \right) \\
&\quad + \gamma [\tilde{C}_2^H]_i [\tilde{R}_2]_{mj} + \gamma [\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} \\
&\quad + [\tilde{H}_2]_i [\tilde{R}_2]_{mj} \left) [\Sigma]^{ij} + O(\epsilon^3)
\end{aligned}$$

Using this, we get

$$\begin{aligned}
\mathbb{E} [\hat{r}_x]_m [\tilde{D}_3]_k &= \mathbb{E} [\hat{r}_x]_m \left([\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k - \frac{1}{\gamma} [\tilde{H}_3]_k \right) \\
&= \frac{1}{2} \left([\tilde{R}_2^A]_i [\tilde{R}_2^A]_j - [\tilde{R}_2^m]_i [\tilde{R}_2^m]_j \right) \left([\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} \\
&\quad - \frac{1}{2} \frac{1}{\gamma} \left([\tilde{R}_2^A]_i [\tilde{R}_2^A]_j - [\tilde{R}_2^m]_i [\tilde{R}_2^m]_j \right) [\tilde{H}_3]_k [\Sigma]^{ij} \\
&\quad + \frac{\gamma}{2} \left([\tilde{C}_2^H]_i [\tilde{R}_2]_{mj} + [\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} \right) \left([\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} \\
&\quad - \frac{1}{2} \left([\tilde{C}_2^H]_i [\tilde{R}_2]_{mj} + [\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} \right) [\tilde{H}_3]_k [\Sigma]^{ij} \\
&\quad + \frac{1}{2} [\tilde{H}_2]_i [\tilde{R}_2]_{mj} \left([\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k - \frac{1}{\gamma} [\tilde{H}_3]_k \right) [\Sigma]^{ij}
\end{aligned}$$

It follows that

$$\begin{aligned}
& [\tilde{D}_2]_i \left([\tilde{R}_5]_{mkj} + [\tilde{R}_1]_m [\tilde{R}_2]_j^q [\gamma]_{qk} \right) [\Sigma]^{ij} \\
&+ [\tilde{R}_2]_{mi} \left([\tilde{D}_5]_{kj} + [\tilde{D}_1] [\tilde{R}_2]_j^m [\gamma]_{mk} \right) [\Sigma]^{ij} \\
&+ \left([\tilde{C}_2^H]_i - [\tilde{C}_2^F]_i \right) [\tilde{R}_2]_{mj} [\tilde{R}_3^m]_k [\Sigma]^{ij} \\
&- \frac{\gamma}{2} [\tilde{R}_2]_{mj} \left([\tilde{C}_2^H]_i - [\tilde{C}_2^F]_i \right) \left([\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} \\
&+ \frac{1}{\gamma} [\tilde{R}_2]_{mi} [\tilde{H}_2]_j [\tilde{H}_3]_k [\Sigma]^{ij} - \frac{1}{2} \left([\tilde{C}_2^H]_i [\tilde{R}_2]_{mj} + [\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} \right) [\tilde{H}_3]_k [\Sigma]^{ij} \\
&- \frac{1}{\gamma} \left([\tilde{H}_2]_i [\tilde{R}_2]_{mj} [\tilde{R}_3^m]_k - [\tilde{H}_2]_i [\tilde{R}_2]_{4j} [\tilde{R}_3^A]_k \right) [\Sigma]^{ij} \\
&+ \left([\tilde{C}_2^F]_i [\tilde{R}_2]_{mj} [\tilde{H}_3]_k + [\tilde{C}_3^F]_k [\tilde{R}_2]_{mj} [\tilde{H}_2]_i \right) [\Sigma]^{ij} \\
&+ \frac{1}{2} [\tilde{H}_2]_i [\tilde{R}_2]_{mj} \left([\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k - \frac{1}{\gamma} [\tilde{H}_3]_k \right) [\Sigma]^{ij} = 0
\end{aligned}$$

Next, using the fact that $[\tilde{D}_2]_i [\tilde{R}_2]_{mj} [\Sigma]^{ij} = 0$ and that $[\tilde{R}_2]_{4j} = 0$, this simplifies to

$$\begin{aligned}
& [\tilde{D}_2]_i [\tilde{R}_5]_{mkj} [\Sigma]^{ij} \\
& + [\tilde{R}_2]_{mi} \left([\tilde{D}_5]_{kj} + [\tilde{D}_1] [\tilde{R}_2]_j^m [\gamma]_{mk} \right) [\Sigma]^{ij} \\
& - \frac{\gamma}{2} [\tilde{R}_2]_{mj} \left([\tilde{C}_2^H]_i - [\tilde{C}_2^F]_i \right) \left([\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} \\
& + \left([\tilde{C}_3^F]_k [\tilde{R}_2]_{mj} [\tilde{H}_2]_i \right) [\Sigma]^{ij} \\
& + \frac{1}{2} [\tilde{H}_2]_i [\tilde{R}_2]_{mj} \left([\tilde{C}_3^H]_k - [\tilde{C}_3^F]_k \right) [\Sigma]^{ij} = 0,
\end{aligned}$$

which can be solved for $[\gamma]_{mk}$.

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