Corrigendum to "Endogenous market incompleteness without market frictions: Dynamic suboptimality of competitive equilibrium in multiperiod overlapping generations economies"

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27 March 2019

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Abstract

This note corrects an error in the original Henrisksen-Spear paper (published in *J. Econ. Theory*, Volume 147, Issue 2, March 2012, Pages 426-449) in the proof that the competitive equilibrium in a three-period lived stochastic overlapping generations model is not dynamically Pareto optimal.

The purpose of this note is to correct an error in the Henriksen-Spear paper stating that there is a dynamic Pareto improvement possible in the model via better risk-sharing between middle-aged and old agents. While the basic result in the paper - that the competitive equilibrium is dynamically suboptimal - remains true, the Pareto improvement needed to show the result requires a reallocation for improved risk-sharing between the old of any generation and the next period's as-yet unborn young. This, in turn, requires the existence of a central planner or government agency that can act on behalf of unborn generations.

The error in the original HS paper arose from a problem of incomplete convergence of the simulated time-series data in the model, and from our failure to look as deeply into the Euler equations as we should have. To show the

*We are grateful to Mauro Moretto for bringing this to our attention.

problem with the Euler equations, start with the first-order conditions for the young and middle-aged agents' utility maximizations. Using the same notation as HS, these require (omitting time subscripts)

$$\begin{aligned}
-u'(c_y^h)q^h + \pi^h u'(c_m^{hh}) + \pi^l u'(c_m^{hl}) &= 0 \\
-u'(c_y^h)p^h + \pi^h u'(c_m^{hh})[p^h + \delta^h] + \pi^l u'(c_m^{hl})[p^l + \delta^l] &= 0 \\
-u'(c_y^l)q^l + \pi^h u'(c_m^{lh}) + \pi^l u'(c_m^{ll}) &= 0 \\
-u'(c_y^l)p^l + \pi^h u'(c_m^{lh})[p^h + \delta^h] + \pi^l u'(c_m^{ll})[p^l + \delta^l] &= 0.
\end{aligned}$$

for young agents, and

$$\begin{aligned}
-u'(c_m^h)q^h + \pi^h u'(c_r^{hh}) + \pi^l u'(c_r^{hl}) &= 0 \\
-u'(c_m^h)p^h + \pi^h u'(c_r^{hh})[p^h + \delta^h] + \pi^l u'(c_r^{hl})[p^l + \delta^l] &= 0 \\
-u'(c_m^l)q^l + \pi^h u'(c_r^{lh}) + \pi^l u'(c_r^{ll}) &= 0 \\
-u'(c_m^l)p^l + \pi^h u'(c_r^{lh})[p^h + \delta^h] + \pi^l u'(c_r^{ll})[p^l + \delta^l] &= 0
\end{aligned}$$

for old agents. These first-order conditions can be re-written as

$$\begin{aligned}
u'(c_y^s) &= \frac{1}{q^s}\left[\pi^h u'(c_m^{sh}) + \pi^l u'(c_m^{sl})\right] \\
u'(c_y^s) &= \pi^h u'(c_m^{sh})\left[\frac{p^h + \delta^h}{p^s}\right] + \pi^l u'(c_m^{sl})\left[\frac{p^l + \delta^l}{p^s}\right]
\end{aligned}$$

with a similar set for old agents. Subtracting the first equation from the second we get

$$\pi^h u'(c_m^{sh})\left[\frac{p^h + \delta^h}{p^s} - \frac{1}{q^s}\right] + \pi^l u'(c_m^{sl})\left[\frac{p^l + \delta^l}{p^s} - \frac{1}{q^s}\right] = 0.$$

or, letting R^{sh} denote the (gross) return on the equity asset, and R_f the return on the bond,

$$\pi^h u'(c_m^{sh})\left[R^{sh} - R_f\right] + \pi^l u'(c_m^{sl})\left[R^{sl} - R_f\right] = 0$$

for young agents, and

$$\pi^h u'(c_r^{sh})\left[R^{sh} - R_f\right] + \pi^l u'(c_r^{sl})\left[R^{sl} - R_f\right] = 0$$

for middle-aged agents (each respectively looking forward one period). As long as the returns are different, then, we get that the state-contingent marginal rates of substitution will be equalized between middle-aged and old. This, in turn, implies that there is no possible risk-sharing improvement between young and middle-aged agents looking forward. It is also straight-forward to show that the rates of return will not equalize, since this implies that the equilibrium is strongly stationary (i.e. prices and allocations depend only on the current period's shock state). Since this is not possible, the returns must be different.

We know from the HS analysis that there must be a possible risk-sharing improvement between the middle-aged (looking forward one period) and next period's as-yet unborn young because if there were not, all agents' consumptions would be a constant share of the (shock-contingent) total resources, which then implies that the competitive equilibrium is strongly stationary, which we know is impossible. Given this, it is possible for a central planner to implement a Pareto improving reduction in the risk facing middle-aged agents in the next period, in exchange for increasing the average consumption of next period's young agents. Since this doesn't involve an change in the allocation of the existing old, it constitutes a short-run Pareto improvement.