High Performance Lithium Niobate Resonators for Passive Voltage Amplification in Radio Frequency Applications

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ABSTRACT

Internet of Things (IoT) applications are fostering the development of near-zero power consumption and event-driven Radio Frequency (RF) sensors that can outperform traditional networks based on scheduling algorithms. In case of infrequent events, asynchronous Wake-Up Radio Receivers (WuRx) are the desired solution to attain high sensitivity (<100 dBm) and ultra-low-power consumption (~tens of nW). The strict available power budget led to drastic reduction of active components in the WuRx, which relies more extensively on passive voltage amplification and filtering in the RF Front-End (RFFE).

High performance microelectromechanical (MEM) piezoelectric resonators are an ideal solution as Matching Network (MN) in the RFFE due to their compact footprint, high quality factor at resonance ($Q_s$) and range of operation (up to few GHz). Despite the availability of successful commercial products, such as Aluminum Nitride (AlN) Film Bulk Acoustic Resonators (FBARs), WuRx are driving the demand for devices which can simultaneously exhibit large quality factor and large $k_t^2$ or, overall, a large Figure of Merit (FoM, defined as $Q_s \cdot k_t^2$).

This dissertation focuses on the development of high performance X-cut Lithium Niobate (LN) MEMS Laterally Vibrating Resonators (LVRs) to be implemented as the matching network of novel WuRx architectures. The requirements of these devices in terms of operating frequency ($f_{RF}$), static capacitance ($C_0$), and Figure of Merit are investigated according to the sensitivity optimization of the Resonant Micromechanical Receiver (RMR) envisioned at Carnegie Mellon University. Different established and innovative piezoelectric MEMS technologies are compared to identify the solution that offers the highest FoM while satisfying the matching constraints provided by the RMR numerical optimization. Preliminary work on X-cut Lithium Niobate resonators operating in the $S_0$ mode exhibited incredibly high $k_t^2$ (>30%) combined with good $Q_s$ (~1,500), which translate to FoM greater than 400 (similar to commercial FBARs performance).
Since the operating frequency and the load condition of the matching network are usually fixed, the Figure of Merit, and in particularly the quality factor, is the only lever available to increase the sensitivity of the RMR. The resonators geometry is thus thoroughly investigated through Finite Element Analysis (FEA) to maximize $Q_s$ and consequently the achievable voltage amplification. Fabricated devices showed $Q_s$ as high as 8,000 in vacuum at 100 MHz, with $k_t^2 = 28\%$ and FoM > 2,500, the highest ever reported to date for piezoelectric MEMS operating in the frequency range of interest for WuRx. Devices at higher frequencies showed improved performance compared to the state-of-the-art, with $Q_s$ greater than 2,500 up to 550 MHz.

Damping mechanism in X-cut LN LVRS were investigated on fabricated devices. Air damping ($Q_{\text{air}}$) proven to be a significant source of quality factor degradation up to 200 MHz, with $Q_s$ reduction in the order of 20 – 30%, while becomes negligible above 550 MHz. Anchor losses were investigated on devices with different plate length ($L$) over width ($W$) aspect ratios. Shorter resonators exhibit lower $Q_s$ due to energy leakage through the substrate, over the entire frequency range of interest. As a consequence, the quality factor of devices with limited aperture ($L_e < 5 \lambda$) showcased a stronger dependency from the anchor dimensions. The electrical loading introduced by the Interdigitated Electrodes (IDE) and the interconnects were identified as the main damping mechanism in resonators with high $L/W$ ratios. Cryogenic measurements highlighted $Q_s$ as high as 26,000 at base temperature (10 K), hinting that anchor losses play a limited role in devices with a slender geometry. A tradeoff between electrical loading and anchor losses was also identified.

The limited static capacitance ($C_0$) achievable for the optimized resonator geometries required the investigation of alternative solutions to match those devices to the typical input capacitive load of an envelope detector (~1 pF). Arrays of identical, parallel resonators and alternative matching network configuration were investigated for this purpose. In the first case, arrays with $C_0$ of 1 pF and quality factor greater than 2,000, with $k_t^2 = 30\%$ and FoM = 600 were demonstrated at 50 MHz. Frequency mismatch between the elements was identified as the main damping mechanism for reduction in effective $Q_s$ and was modeled through a statistical Monte Carlo approach. An alternative matching network based on a combination of series and parallel resonators (L-network)
with identical $C_0$ was also investigated, showcasing a gain 30% higher than a simple series configuration (46 V/V vs. 35 V/V for a capacitive load of 1 pF) and a higher robustness in regards to frequency variations.
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CHAPTER 1: INTRODUCTION

The long-envisioned dream of an hyperconnected world, where an increasing quantity of information is available in real-time, is becoming a reality. The extension of the Internet connectivity to the Internet of Things (IoT) is a natural evolution towards a more advanced capability of remotely monitoring and controlling the environment through physical devices and everyday objects [1][2].

The number of devices connected to the Internet already grew from 1 million devices in 1992 to 42 billion in 2019 (Fig. 1.1a), with forecasts of 1 trillion devices connected by 2035. This exponential growth is largely motivated by the diffusion of smartphones in the 2000s and, more recently, by the development of IoT applications, such as wireless networks of sensors in consumer electronics. The larger number of connected devices comes with the necessity of higher data rate communication links. The global mobile data traffic increased from 7 to 24 exabytes per month between 2016 and 2019 and is expected to grow by 47% annually for the upcoming years (Fig. 1.1b). This growth directly translates into the demand of a higher number of LTE and 5G bands (Fig. 1.1c), and the need for more complex Radio Frequency Front-End (RFFE) modules in phones. For this reason, the demand for RF filters kept increasing over the past few years, and it is expected to further grow in the future (Fig. 1.1d). Ultimately, 5G and IoT emerging applications are driving the research on novel RFFE architectures and electronic components. For example, tunable and reconfigurable filters operating above few GHz would be a desirable solution to synthesize compact RFFE modules for mobile communications and allow faster data transfer. Research is currently active in identifying and developing the key hardware technologies for this purpose.
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1.1. The N-ZERO Project Challenge

Ultra-low-power (ULP) and high sensitivity receivers are another example of extremely interesting solutions for applications relying on large distributed networks of wireless nodes [3], such as Unattended Ground Sensors (UGS) for military infrastructure monitoring and border protection in active war zones (Fig. 1.2a). Often these sensors are deployed to monitor infrequent but critical event (~tens per day), with no previous knowledge of the event occurrence. The need for continuous, event-driven sensing poses serious challenges in terms of UGS battery lifetime.
Figure 1.2: (a) Example of distributed network relying on wireless nodes: Unattended Ground Sensors for war-zone monitoring [3]; and (b) goal of the N-ZERO program: extend battery life of Unattended Ground Sensors from month to years [2]. The flattening of battery lifetime for low event activity in synchronous WuRx is due to the fixed energy consumption of the scheduled wake-up.

In fact, state-of-the-art synchronous receivers show a limited efficiency due to the high power consumption of the wake-up circuitry compared to the sensor total power consumption if the event activity is lower than 10% of the total time (Fig. 1.2b). A simple model to estimate the lifetime of a battery is reported in [2] (Eq. 1.1). For a fixed battery capacity ($Q_{bat}$), the lifetime of the battery ($T_{bat}$) is a function of the event activity ($N$), the active time ($t_{on}$) and the current consumption in the active and sleep states ($I_{active}$ and $I_{sleep}$ respectively), plus the self-discharge rate of the battery ($I_{leakage}$).

If $N$ tends to zero, the battery duration is substantially limited by power consumption of the wake-up circuitry in the sleep state.

$$T_{bat} = \frac{Q_{bat}}{N t_{on} I_{active} + (1 - N t_{on}) I_{sleep} + I_{leakage}}$$ (1.1)

Despite the development of more efficient scheduling algorithms, the continuous sensing and processing of irrelevant data limits the duration of UGS batteries. Furthermore, the replacement of these nodes is often impractical and directly translates to larger maintenance costs that undermine the feasibility of military missions relying on them.
These considerations fostered the interest on novel sensors and receivers architectures capable of performing continuous, event-driven sensing with near zero power consumption (N-ZERO project) [3]. The goal of the N-ZERO project is to exploit the energy contained in the RF trigger signature itself to detect the event of interest while maintaining a low false alarm rate (FAR).

### 1.2. Wake-Up Radio Receivers (WuRx)

Always-on, asynchronous Wake-Up Radio Receivers (WuRx) attaining high sensitivity (-100 dBm) and ultra-low-power consumption (~tens of nW, comparable to the self-discharge rate of a battery) are an attractive technology to extend battery-life by several orders of magnitude (Fig. 1.2b). When an incoming RF signature is detected, the WuRx triggers the sensing electronics, which performs the measurement only when required.

In spite of investigations into ULP radios for the last two decades, all pre-2016 demonstrations showed a clear trade-off between power consumption and sensitivity (~1 decade in power for 20 dB of sensitivity, Fig. 1.3).

![Ultra-low-power Radio Survey 2005 – 2015](image)

**Figure 1.3: Ultra-low-power Radio Survey 2005 – 2015** [4].
In the last few years, novel RF WuRx architecture operating between hundreds of MHz and few GHz have been proposed to break this paradigm. The strict available power budget lead to a converge in WuRx topologies, which often consist in an RF Matching Network (MN), a mixing stage (such as an envelope detector or a demodulator) and a Baseband (BB) Amplifier. Fig. 1.4 reports different options for each building block constituting a typical ULP asynchronous WuRx [5][6].

![Figure 1.4: Typical architecture of an asynchronous WuRx in constitutive blocks (RFFE, mixing stage and baseband circuitry) and different options for its synthesis [5]. Ideally, active components are implemented only in the stage operating at baseband frequency to minimize power consumption.](image)

The ability to continuously screen the RF spectrum with near zero power consumption heavily relies on the optimization of the active components (ideally a sole BB amplifier operating in subthreshold regime) and passive voltage amplification techniques in the RF Front-End (RFFE). For this reason, active stages such as RF Low Noise Amplifier (LNA) are generally not implemented due to their large power consumption (~μW). As far as the RFFE is concerned, existing approaches rely on different types of passive matching network, including transformers, LC tanks, and piezoelectric MEMS resonators, which interface the 50 Ω source resistance of the antenna with the input impedance of the mixing stage. This input impedance is usually a complex
value, represented by a large resistor to ground in parallel with a capacitor. If the inductive component of the MN is properly selected, it is possible to resonate out this capacitance, cancelling it over a narrow frequency range while providing passive voltage amplification (or gain, $G$) of the RF signature. Both the operating 3dB-bandwidth and the gain presented to the input of the ED are ultimately determined by the loaded quality factor ($Q$) of the matching network.

For the mixing stage, several solutions have been investigated to perform the down-conversion of the signal from RF to baseband. The most common ones involve an Intermediate Frequency (IF) mixer [2], a CMOS-based Envelope Detector (ED) [7] or an electrostatic MEMS demodulator [8] (Fig. 1.4). IF mixers are widely implemented in RF systems, but the large power consumption associated with the presence of a Local Oscillator (LO) makes their implementation in ULP WuRx impractical. The near-zero power consumption of CMOS-based Envelope Detector (ED) and electrostatic MEMS demodulator makes those solution more feasible for the synthesis of ultra-low-power WuRx [2][5].

Ultimately, the signal detection can be performed by means of baseband correlator [9], comparators [10], active or passive rectifiers [5], or switches [11]. If not passive, all the components operating at baseband must operate in deep subthreshold regime to minimize power consumption.

### 1.3. MEMS Resonators as Matching Networks

Micromechanical (MEM) piezoelectric acoustic resonators are an appealing solution for impedance matching networks due to their wide range of operation (tens of MHz to few GHz), high quality factor at resonance ($Q_s$), compact footprint and low motional resistance ($R_m$). Surface Acoustic Wave (SAW) and Bulk Acoustic Wave (BAW) resonators have already proven to be key
building blocks for a wide range of electronic components, especially in RF filtering applications. The performance of these components can be exploited in different MN by operating them as inductors close to their resonance ($f_r$) or anti-resonance frequency ($f_p$). This will allow the replacement of large, low $Q$, discrete inductors and will provide larger signal voltage boost, leading to higher sensitivity and low power consumption. Despite the availability of successful commercial products, including Aluminum Nitride (AlN) Film Bulk Acoustic Resonators (FBARs), WuRx are driving the demand for devices with improved performance beyond the current state-of-the-art MEMS resonators.

Figure 1.5: Example of different piezoelectric MEMS resonators: (a) Scandium-doped Aluminum Nitride Film Bulk Acoustic Resonator (FBAR) fabricated at CMU Nanofab; (b) LiNbO$_3$ Surface Acoustic Wave (SAW) resonator developed for the acousto-optic gyroscope described in [12]; and (c) LiNbO$_3$ Laterally Vibrating Resonator (LVR) [13].

At the resonator level, this performance is controlled by three parameters: the quality factor ($Q_s$), the electromechanical coupling ($k_{t2}^2$), and the static capacitance ($C_0$). The quality factor represents the level of energy dissipation of the system and directly impacts the Insertion Losses (IL) and the bandwidth of the system (BW) in narrow band applications. The electromechanical coupling represents the capability of converting energy from the mechanical to the electrical domain and sets the bandwidth in filtering applications. The static capacitance defines the energy stored in the electrical domain and must be chosen according to the frequency of operation and the load characteristics for proper impedance matching. The product of the quality factor ($Q_s$) and the
electromechanical coupling \((k_t^2)\) defines the Figure of Merit (FoM), one of the most widely used metrics to define the performance of piezoelectric MEMS resonator.

As highlighted in [14][15], WuRx employing MEMS resonators in the RFFE require resonators capable of simultaneously exhibiting high quality factor and high electromechanical coupling or, overall, large Figure of Merit. The FoM contributes, with the frequency of operation and the static capacitance, in determining the motional resistance \(R_m\) of the device, and ultimately the losses introduced when implementing MEMS-based matching networks. Since \(C_0\) is usually selected to match the value of the capacitive input of the mixing stage, and the frequency is determined by the spectrum availability, the Figure of Merit is, with the MN architecture, the only lever available to decrease \(R_m\) and increase the passive voltage boost.

1.4. Thesis Outline

The dissertation is organized in the following chapters. In Chapter 2, the architecture of an asynchronous Ultra-low-power (ULP) Resonant Micromechanical Receiver (RMR) is described stage by stage and a first analytical description is provided for each block. Then, different matching networks based on discrete components and piezoelectric MEMS resonators are compared in terms of performance to identify the most suitable topology for the RMR. Ultimately, a numerical optimization of the RMR sensitivity is described to identify the key design parameters and the optimal operating point of each block of the WuRx.

In Chapter 3, piezoelectric MEMS Laterally Vibrating Resonators (LVRs) are introduced as elements to synthesize an RF matching network. The operating principles, the equivalent electrical model and the modeling of their static capacitance and series resistance are briefly described, such
as the impact of key parameters as $Q_s$, $k_t^2$ and $C_0$ on the voltage gain provided for a capacitive load representing the input impedance of an envelope detector. Different LVRs technologies are then compared in terms of Figure of Merit and fabrication process, with a focus on novel materials, such as Scandium-doped Aluminum Nitride alloys and Lithium Niobate crystalline cuts.

In Chapter 4, the design and performance of X-cut Lithium Niobate LVRs are investigated. Finite Elements Analysis is used to model the response of devices in the frequency range between 50 MHz and 550 MHz and define an experimental plan. Then, the response of fabricated devices is characterized and the impact of their geometry on $Q_s$, $k_t^2$, and $C_0$ investigated. Vacuum and cryogenic measurements are presented, and the main sources of losses identified by comparing experimental results and theoretical models. The performance of arrays of optimized resonators to match to large input load capacitances are investigated, and the losses introduced by frequency mismatch between elements characterized.

In Chapter 5, direct measurements of the voltage gain provided by a matching network based on 1-port X-cut LVRs are presented and the impact of non-idealities (such as spurious modes) on the gain is discussed. Alternative topologies based on more compact resonators are introduced and tested.

In Chapter 6, the entire body of the Ph.D. work is summarized, and some future research directions are presented.
CHAPTER 2: RESONANT MICROMECHANICAL RECEIVER (RMR)

Ultra-low-power (ULP) wake-up radio receivers (WuRx) are a desirable solution to extend the battery lifetime of wireless sensors deployed in large, unattended networks, in order to reduce the maintenance costs of such infrastructure. WuRx are usually composed by three basic building blocks: RF front-end (RFFE), mixing and baseband.

Several approaches have been recently demonstrated [6][7][16][17], implementing various solutions for each of the main building blocks. Due to the strict available power budget (~10 nW), the only approach to simultaneously attain high sensitivity (-100 dBm) while minimizing power consumption is to reduce the implementation of active stages in favour of passive solutions.

An interesting approach is the asynchronous Resonant Microelectromechanical Receiver (RMR) developed at Carnegie Mellon University and extensively described in [5]. The system relies on resonant micro-electromechanical systems: the RFFE is implemented with a piezoelectric MEMS resonator, which provides for passive voltage amplification, impedance matching and aggressive filtering, while the mixing stage is realized with an electrostatically actuated MEMS demodulator. This approach allows exploiting the passive voltage boost provided by the piezoelectric MEMS and the non-linearity introduced by the electrostatic MEMS for the demodulation utilizing the energy contained in the RF signature itself. The only active stage contemplated in the RMR architecture is the baseband circuitry, which could be either a voltage amplifier or a transimpedance amplifier.
(TIA). To keep the power consumption in the order of few nW, the CMOS circuitry is designed to operate in the subthreshold regime.

The carrier frequency \( (\omega_c) \) is set to match the resonant operating frequency of the piezoelectric resonator \( (\omega_{RF}) \), and is selected in the range between 50 MHz and 1 GHz. This range of frequency is of particular interest due to the availability of bands in the spectrum, and as a trade-off between performance and antenna dimensions. The baseband frequency \( (\omega_m) \) is selected to match the resonant frequency of the demodulator \( (\omega_r) \), which is usually set in the tens of kHz range as a trade-off between device footprint, performance and CMOS efficiency.

A schematic of the RMR is reported in Fig. 2.1. In this chapter, the main blocks will be analyzed and their mathematical model described to identify the main design parameters to attain ULP WuRx exhibiting a desired sensitivity of -100 dBm for a given power budget of 10 nW.

![Figure 2.1: Block diagram of the RMR envisioned at CMU [18].](image)

### 2.1. Matching Network (MN)

Matching Networks (MN) are a fundamental stage in each WuRx topology. Their main goal is to interface the impedance of the antenna (usually 50 Ω) with the high input impedance of the mixing stage, which is often constituted by a capacitor in parallel with a large resistor (Fig. 2.2).

The most important metrics to define the efficiency of a passive MN are the gain provided to the
input of the following stage, their loaded quality factor ($Q$) and their footprint, which becomes particularly relevant for applications such as small, easily deployable wireless nodes.

One of the most interesting features of passive matching networks is the capability of providing voltage boost to the input signal with no power consumption. This amplification is possible either by implementing a resonant circuit or through a transformer.

![Figure 2.2: Schematic representation of the RFFE of a WuRx implementing a matching network interfaced with the capacitive input of a mixing stage. The capacitor losses are represented by a parallel resistance to ground.](image)

In the first case, an inductive element is interfaced with a load capacitor to cancel its reactance over a narrow range of frequency, which is ultimately defined by the $Q$ of the matching network. Over this bandwidth, the input impedance seen by the voltage source (i.e., the RF signature) is minimized and mostly real, allowing a large current to flow across the MN and the load. Under these operating conditions, the reactive components of the system store and release energy at the same pace, building up a large peak voltage which can be sensed by a following stage [6]. The maximum attainable gain, and thus the signal boost, is given by the ratio of the input impedance of the mixing stage over the impedance seen by the voltage source at resonance.

In the second case, a transformer is introduced between the antenna and the input of the mixing stage to decrease the input impedance seen by the primary and increase the voltage at the secondary [6]. The voltage gain is, in this case, a function of the turn ratio ($N$) of the transformer and of the
ratio between the squared root of the output and input impedances, which defines the maximum attainable gain.

As mentioned before, the quality factor of the matching networks plays an important role in defining the maximum achievable gain, since it directly affects the resistance witnessed at resonance by the RF signature, but also impacts the 3-dB bandwidth of the system. For this reason, $Q$ must be selected accordingly to the desired modulation speed [18].

In this section, several matching network topologies will be compared in terms of passive voltage amplification, loaded $Q$ and footprint to identify the most suitable solution for the RMR under investigation.

2.1.1. Series Inductor (LC Tank)

The simplest matching network [6] is the LC tank and it is constituted by an inductor ($L$) in series with the input capacitive load of the ED (Fig. 2.3).

![Figure 2.3: Representation of a real inductor.](image)

The inductor must be properly sized according to the load capacitance ($C_{load}$) and the frequency of operation ($\omega$) as captured by Eq. 2.1. Since the inductor has a limited quality factor ($Q_L$), it is necessary to define the associated losses ($R_L$, Eq. 2.2). The maximum achievable gain is thus given by the ratio between the impedance of the load and the impedance seen at resonance, which is the sum of $R_{in}$ and $R_L$. 

13
\[ L = \frac{1}{\omega^2 C \text{load}} \] (2.1)

\[ R_L = \frac{\omega L}{Q_L} = \frac{1}{\omega C \text{load} Q_L} \] (2.2)

\[ G = \left| \frac{1}{\omega C \text{load}(R_{in} + R_L)} \right| \] (2.3)

The main limitations of this approach are the low quality factor of discrete inductors, usually lower than 100 [19], and their footprint.

### 2.1.2. Transformer with Parallel Inductor

Another approach successfully implemented in [20] synthesizes a matching network by interfacing the input impedance of the mixing stage with a parallel inductor \((L)\) at the secondary of a transformer (Fig. 2.4). A parallel resistor \((R_p)\) represents the losses introduced by the inductor, similarly to \(R_L\) described in Section 2.1.1.

![Figure 2.4: Representation of a matching network constituted by a transformer with a parallel inductor.](image)

The transformer turn ratio \((N, \text{Eq. } 2.4)\), is defined as the ratio between the windings at the primary \((N_1)\) and the windings at the secondary \((N_2)\). As reported in [21], the gain is maximized when \(R_p\) (\(R_p\) seen at the primary) is equal to the impedance of the source \((R_{in} = 50 \, \Omega)\), as reported by Eq. 2.5. This relationship is fundamental to define the turn ratio of the transformer when sizing.
the system. Voltages at the secondary can be expressed according to Eq. 2.6, and the maximum achievable gain by Eq. 2.7.

\[
N = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}
\]

(2.4)

\[
R'_p = R_pN^2 = R_{in} = 50 \, \Omega
\]

(2.5)

\[
V_2 = \frac{V_1}{N} = V_{in} \frac{R_pN^2}{R_{in} + R_pN^2} \frac{1}{N} = V_{in} \frac{\sqrt{\frac{R_p}{R_{in}}}}{2}
\]

(2.6)

\[
G = \frac{V_2}{V_{in}} = \frac{1}{2} \sqrt{\frac{R_p}{R_{in}}}
\]

(2.7)

This solution suffers of the same limitations of the LC tank. It is worth noticing that it is not necessary to resonate out the input capacitance of the mixing stage to attain passive voltage gain, especially if its impedance is much larger than \(R_{load}\). In this case, the same analysis is valid assuming \(R_p\) equal to \(R_{load}\).

### 2.1.3. 1-port MEMS Series Resonator

The typical behavior of a 1-port piezoelectric MEMS resonator is captured by the equivalent Butterworth-Van Dyke (BVD) electrical model (Fig. 2.5a). \(C_0\) represents the static capacitance of the device, while \(R_m\), \(C_m\), and \(L_m\) respectively the motional resistance, capacitance and inductance \[22\]. The motional parameters are function of various combinations of 4 parameters (Eq. 2.8 – 2.10): \(C_0\), the electromechanical coupling coefficient \(k_t^2\), the quality factor at resonance \(Q_s\), and the resonant frequency \(\omega_s = 2\pi f_s\). Different definitions can be adopted to define the \(k_t^2\) \[23\], and will be clearly stated throughout the dissertation.
Figure 2.5: (a) Butterworth-Van Dyke (BVD) model and its schematic representation; (b) admittance response of a 1-port resonator (blue), a capacitor (red), and 1-port resonator interfaced in series with a capacitor (yellow). The operating frequency shifts to higher frequency when the resonator is implemented as a matching network, and its motional resistance increases; and (c) operating frequency \( f_{RF} \) normalized over the natural frequency of a resonator \( f_s \) for different \( k_t^2 \) and \( C_{load}/C_0 \) ratios.

\[
R_m = \frac{\pi^2}{8} \frac{1}{k_t^2 C_0 Q_s \omega_s} \quad (2.8)
\]

\[
L_m = \frac{\pi^2}{8} \frac{1}{k_t^2 C_0 \omega_s^2} \quad (2.9)
\]

\[
C_m = \frac{8}{\pi^2} k_t^2 C_0 \quad (2.10)
\]

\[
Y = \frac{1}{Z_R} = j \omega C_0 + \frac{1}{R_m + j \omega L_m + \frac{1}{j \omega C_m}} \quad (2.11)
\]

An example of admittance response of a 1-port resonator (Eq. 2.11) is reported in Fig. 2.5b. The characteristic response presents a series resonance \( f_s \) and an anti-resonance frequency \( f_p \) due to the presence of \( C_0 \) in parallel with the motional branch.

The simplest approach to synthesize a matching network with a piezoelectric MEMS resonator is to interface it in series with a capacitive load, as reported in Fig. 2.2. In this configuration the device operates as a high-\( Q \) inductor effectively cancelling out the imaginary impedance of the load (assuming \( R_{load} >> (\omega C_{load})^{-1} \)). This allow a large current to flow in the motional branch to build up a large voltage across the load, which can be sensed by the mixing stage input. As highlighted in
Fig. 2.5b, the frequency of operation \((\omega_{RF} = 2\pi f_{RF})\) differs from the natural frequency of the resonator. The quality factor of the resonator is usually function of frequency [24], thus the \(Q\) measured at \(\omega_{RF}\) could be different from \(Q_s\). For simplicity, in the present dissertation the quality factor is assumed constant.

The pulling of the operating frequency is due to the need of the imaginary part of the resonator impedance \((Z_R)\) to exactly match the value of the complex impedance of the load \((Z_{load})\). At \(\omega_{RF}\) (Eq. 2.12), the passive voltage gain provided to the input load is a function of the impedance ratio between \(Z_{load}\) and the resistance of the antenna \((R_{in} = 50 \, \Omega)\) plus the losses introduced by the resonator operating at \(\omega_{RF}\) (Eq. 2.13). Such losses are a function of \(R_m\), modulated by a pulling factor \((k_p)\) greater than 1 and dependent on both \(C_0\) and \(C_{load}\) (Eq. 2.14).

\[
\omega_{RF} = \omega_s \sqrt{\frac{C_0 + C_{load} + C_m}{C_0 + C_{load}}} \tag{2.12}
\]

\[
G = \frac{Z_{load}}{R_{in} + Z_R + Z_{load}} = \sqrt{\frac{1}{\omega_{RF}C_{load}(R_{in} + \text{Re}[Z_R])}} \tag{2.13}
\]

\[
\text{Re}[Z_R] = k_p R_m = \left(\frac{C_{load} + C_0}{C_{load}}\right)^2 R_m \tag{2.14}
\]

Assuming constant \(C_{load}\), \(Q_s\), \(k^2\), and \(\omega_s\), the maximum achievable gain is merely a function of the static capacitance \(C_0\), which must be carefully selected to maximize the voltage boost [25]. It is possible to find an analytical solution to the problem by deriving Eq. 2.13 in respect to \(C_0\) (Eq. 2.15). Eq. 2.15 allows a single global maximum which can be found by numerically solving the quadratic Eq. 2.16, where \(\alpha\) represents the ratio between the capacitive load and the static capacitance (Eq. 2.17), \(K^2\) the piezoelectric coupling (Eq. 2.18) [26][23], and \(\Delta\) the ratio between the quality factor of the resonator and the equivalent quality factor of \(C_{load}\) evaluated at \(\omega_s\), including the losses deriving from the presence of \(R_{in}\) (Eq. 2.19).
\[
\max G = \min G \rightarrow \frac{d}{dC_0} \left[ \frac{1}{\omega_{RF}(R_{in} + k_p R_m)} \right] = 0 \tag{2.15}
\]

\[
\alpha^4 + \alpha^3(2 + K^2) + \frac{\alpha^2}{2}(K^2 - \Delta K^4) - 2\alpha(1 + K^2) - (3 + K^2) = 0 \tag{2.16}
\]

\[
\alpha = \frac{C_{load}}{C_0} \tag{2.17}
\]

\[
K^2 = \frac{8}{\pi^2 k^2_t} \tag{2.18}
\]

\[
\Delta = C_{load} R_{in} Q_s \omega_s = \frac{Q_s}{Q_{cl_{\omega=\omega_s}}} \tag{2.19}
\]

Fig. 2.6 reports the value of the optimal values of \(\alpha^{-1}\) and the pulling factor \((k_p)\) for different values of \(k^2_t\) and \(\Delta\). For typical values of a high-performance X-cut Lithium Niobate resonator [13], \(\alpha = 1.35, C_{0, opt} = 0.75 \cdot C_{load}\), and \(k_p = 3\). Interestingly, when \(k^2_t\) tends to 0, and consequently \(K^2\), Eq. 2.16 can be solved analytically. The solution is \(\alpha = 1, C_{0, opt} = C_{load}\), and \(k_p = 4\), meaning that in case of resonators exhibiting low \(k^2_t\), the gain is maximized when the static capacitance perfectly matches the load, and that the motional resistance at \(\omega_{RF}\) becomes larger by a factor of 4.

**Figure 2.6:** (a) Optimized ratio between the static capacitance and the load \((\alpha^{-1})\) as a function of \(k^2_t\) and \(\Delta\). Higher \(k^2_t, f_s, Q_s\) and \(C_{load}\) concur in decreasing the optimal static capacitance for a given load; and (b) pulling factor as a function of \(k^2_t\) for different levels of \(\Delta\). \(k_p\) follows the same trend as \(\alpha^{-1}\).
The main advantages of this solution over the ones introduced in Section 2.1.1 and 2.1.2 are the more compact footprint, which allows for a reduction of the area occupied by the MN, and larger $Q$ if high-performance MEMS resonators are employed.

### 2.1.4. 2-port MEMS Resonator

![Figure 2.7: BVD model of a 2-port resonator.](image)

The behavior of a 2-port MEMS resonator is captured by the Butterworth-Van Dyke (BVD) model reported in Fig. 2.7. Similarly to 1-port BVD model, $R_m$, $L_m$ and $C_m$ represents the motional resistance, inductance and capacitance, while $C_{0,in}$ and $C_{0,out}$ respectively the capacitances of the input and output ports. In the 2-port case, the motional parameters (Eq. 2.20 – 2.22) are both function of $C_{0,in}$ and $C_{0,out}$, and are usually reported at the primary or at the secondary of a transformer representing the transduction between the electrical and the mechanical domain. A turn ratio ($N$) is commonly used to represent the ratio between the input and the output capacitances (Eq. 2.23).

$$R_m = \frac{\pi^2}{8} \frac{1}{k_t Q_s \omega_s} \frac{N + 1}{N} \frac{1}{C_{0,in}}$$  \hspace{0.5cm} (2.20)

$$L_m = \frac{\pi^2}{8} \frac{1}{k_t \omega_s^2} \frac{N + 1}{N} \frac{1}{C_{0,in}}$$  \hspace{0.5cm} (2.21)

$$C_m = \frac{8}{\pi^2} k_t \frac{N}{N + 1} C_{0,in}$$  \hspace{0.5cm} (2.22)
\[ N = \frac{C_{0,\text{in}}}{C_{0,\text{out}}} \quad (2.23) \]

For a 2-port resonator, the output capacitance directly sums up to \( C_{\text{load}} \) to determine the load impedance (\( Z_{\text{load}} \)). The value of such impedance referred to the primary of the transformer and the voltage across it can be expressed respectively as Eq. 2.24 and 2.25. The total gain can be easily calculated as the ratio of the voltage measured across \( Z_{\text{load}} \) at the secondary (\( V_2 \)) and the voltage source (\( V_{\text{in}} \)) as captured by Eq. 2.26.

\[ Z'_{\text{load}} = \frac{1}{j\omega(C_{0,\text{out}} + C_{\text{load}})N^2} \quad (2.24) \]
\[ V'_{\text{load}} = \frac{Z'_{\text{load}}}{R_{\text{in}} + (R_m + j\omega L_m + \frac{1}{j\omega C_m} + Z'_{\text{load}}) || \frac{1}{j\omega C_{0,\text{in}}}} \quad (2.25) \]
\[ G = \left| \frac{V_2}{V_{\text{in}}} \right| = \left| \frac{Z_{\text{load}}}{R_{\text{in}} + (R_m + j\omega L_m + \frac{1}{j\omega C_m} + Z_{\text{load}}) || \frac{1}{j\omega C_{0,\text{in}}}} \right| \quad (2.26) \]

Similarly to the 1-port resonator described in Section 2.1.3, the operating frequency (\( \omega_{RF} \)) of a 2-port resonator implemented in a MN differs from the natural resonant frequency of the device due to the pulling operating by the series capacitive load. The frequency can be expressed according to Eq. 2.27, where \( \alpha \) represents the ratio between the capacitive load and the output capacitance (Eq. 2.28).

\[ \omega_{RF} = \omega_s \sqrt{\frac{(\alpha + 1)(N + 1) + K^2}{(\alpha + 1)(N + 1)}} \quad (2.27) \]

\[ \alpha = \frac{C_{\text{load}}}{C_{0,\text{out}}} = \frac{NC_{\text{load}}}{C_{0,\text{in}}} \quad (2.28) \]

To provide for passive voltage amplification, the imaginary part of the parallel between the \( C_{0,\text{in}} \) and the motional branch in series with \( Z_{\text{load}} \) must be equal to zero (Eq. 2.29). Differently from the
1-port resonator case, the losses introduced by the matching network are not amplified by the
frequency pulling and are equal to the motional resistance, \( R_m \) (Eq. 2.30). The overall gain can thus
be expressed as reported in Eq. 2.31.

\[
\text{Im} \left\{ \left( R_m + j\omega L_m + \frac{1}{j\omega C_m} + Z'_{\text{load}} \right) \left| \frac{1}{j\omega C_{o,\text{in}}} \right| \right\} = 0 \quad (2.29)
\]

\[
\text{Re} \left\{ \left( R_m + j\omega L_m + \frac{1}{j\omega C_m} + Z'_{\text{load}} \right) \left| \frac{1}{j\omega C_{o,\text{in}}} \right| \right\} = R_m \quad (2.30)
\]

\[
G = \left| \frac{1}{\omega (C_{o,\text{in}} + C_{\text{load}} N)(R_{\text{in}} + R_m)} \right| \quad (2.31)
\]

Eq. 2.31 shows that a MN implementing a 2-port resonator instead of a 1-port device does not
offer a significant advantage. Despite the absence of a pulling factor, 2-port devices exhibit a larger
motional resistance for a fixed static capacitance (Eq. 2.20) and increase the load that needs to be
resonated out, ultimately decreasing the gain. A decrease in \( N \) helps reducing the total load, at the
expenses of a higher motional resistance. For this reason, 2-port resonators exhibit performance
similar to 1-port resonators, with the cons of a more complicated topology and fabrication process.

### 2.1.5. Transformer with Parallel Resonator

![Figure 2.8: Representation of a matching network constituted by a transformer with a parallel resonator.](image)

A more advanced matching network architecture implements a combination of the transformer
approach described in Section 2.1.2 and the 1-port resonator approach described in Section 2.1.3.
The system is composed by a resonator interfaced in parallel with the input of the mixing stage at the secondary of a transformer (Fig. 2.8). The MEMS resonator is effectively employed as an inductor and the gain can still be expressed according to Eq. 2.7. In contrast to the series approach described in Section 2.1.3, the 1-port resonator is operated at its anti-resonance frequency ($\omega_p = 2\pi f_p$, Eq. 2.32) to synthesize a large resistance $R_p$, which can be expressed according to Eq. 2.33.

$$\omega_p = \omega_s \sqrt{\frac{C_0 + C_{load} + C_m}{C_0 + C_d}}$$

$$R_p = |Z_R|_{\omega = \omega_p} = \frac{Q_s \left(\frac{C_m}{C_0 + C_{load}}\right)}{\omega_s \left(C_0 + C_{load} + C_m\right)}$$

It is possible to calculate the value of $C_0$ that maximizes $R_p$ (and thus the gain) similarly to the process reported in Section 2.1.3, by solving Eq. 2.34. Results are captured by Eq. 2.35.

$$\max R_p \rightarrow \frac{d}{dC_0} \left[\frac{Q_s K^2 C_0}{\omega_s (C_0 + C_{load})(C_0 + C_{load} + K^2 C_0)}\right] = 0$$

$$C_0 = \frac{C_{load}}{\sqrt{1 + K^2}}$$

![Figure 2.9: Optimal $C_0$ over $C_{load}$ ratio as a function of the electromechanical coupling of the resonator.](image)
Assuming $k_t^2 = 0.3$, $C_{0,\text{opt}} = 0.9 C_{\text{load}}$ (Fig. 2.9). Differently from the optimal $C_0$ found for the series resonator approach, the optimal point is frequency independent.

### 2.1.6. Matching Networks Comparison

The different architectures described in the previous sections are analytically compared to identify the most suitable topology for the implementation of a matching network in the RFFE of a WuRx. The most important metrics are the gain provided to the load and the footprint of the MN.

![Comparison between different matching networks architectures](image)

**Figure 2.10**: Comparison between different matching networks architectures for a fixed $C_{\text{load}} = 1 \text{ pF}$.

For the analytical comparison, the $Q_L$ of discrete inductors was set to 100 [5], while the quality factors ($Q_c$ and $Q_p$) of piezoelectric resonators were varied between 1,000 and 5,000 according to existent literature [14], with a fixed $k_t^2$ equal to 30%. Transformers were assumed ideal and lossless for the current analysis. $C_{\text{load}}$ was set to 1 pF, which represents the typical load of an ED [20]. The
static capacitance of piezoelectric resonators was set to match their optimal value according to the
analysis reported in the previous sections.

As highlighted by Fig. 2.10, the LC tank and the transformer with a parallel inductor outperform
any resonator-based architecture for $Q_s$ and $Q_p$ in the order of 1,000. A direct comparison between
the series losses introduced by an LC tank and a series resonator approach (Eq. 2.36) shows how
the FoM of the resonator must be at least greater than $k_p Q_L$ to provide for a larger gain (Eq. 2.37).
For $Q_L = 100$ and $k_p \sim 3$ [14], the FoM of a 1-port resonator must be greater than 300.

$$R_L = k_p R_m$$

$$FoM = k_t^2 Q_s > k_p Q_L$$

A similar comparison can be performed between the approaches implementing a parallel
inductor and a parallel resonator. In this case, the parallel resistance of the parallel inductor must
be set equal to the resistance synthesized at the resonator anti-resonance (Eq. 2.38). The relationship
between $Q_p$ and $Q_L$ is expressed by Eq. 2.39. Assuming $k_t^2 = 30\%$, $Q_p$ must be roughly 15 times
higher than $Q_L$ to provide similar results.

$$R_p = \frac{Q_L}{\omega C_{load}} = \frac{Q_p K^2 C_0}{\omega_p (C_0 + C_{load})(C_0 + C_{load} + K^2 C_0)}$$

$$Q_p > \frac{4 + 2K^2}{K^2} Q_L$$

System based on piezoelectric resonators with high $Q_s$ and $Q_p (\sim 5,000)$ show higher gains than
other architectures especially at low frequency (<100 MHz).

The smaller footprint of MEMS devices, due to the absence of any large, external component
(such as a transformer), point out to the series approach as the most suitable MN for the RMR under
investigation. Despite higher theoretical performance, the presence of severe spurious at the anti-
resonance of demonstrated high performance devices [13] limits the implementation of architectures combining a transformer and a parallel resonator. Furthermore, as empathized in Fig. 2.10, this approach would equally benefit of the development of high Figure of Merit resonators, which is the main goal of the present dissertation.

### 2.2. Electrostatic MEMS demodulator

The mixing stage of the RMR is implemented with an electrostatic high-Q MEMS resonator operating at $\omega_t$ that takes the place of an Envelope Detector (ED) in more conventional WuRx designs. Different MEMS-based demodulator have been demonstrated in the past [28]–[30], but the RMR described in [5] offers the first implementation of such systems in the chain of a WuRx.

The non-linearity of an electrostatic parallel-plate capacitor is exploited to down-convert a modulated signal from RF to baseband without power consumption. The force ($F_{el}$) generated across the combs of a capacitive electrostatic drive ($C_d$) is, in fact, proportional to the square of the voltage ($V_d$) applied across $C_d$ [8]. For a signal modulated at $\omega_c \pm \omega_m$, such force presents a component at the baseband signal frequency (Eq. 2.40).
\[
F_{el} = \frac{1}{2} V^2(t) \frac{dC_d}{dx} \propto \left( \frac{1 + \cos(2\omega_m t)}{2} + \cos(\omega_m t) + 1 \right)
\] (2.40)

If \(\omega_m\) is set equal to the resonant frequency of the MEMS demodulator (\(\omega_r\)), \(F_{el}\) is maximized and can be exploited to generate a large motional current (\(i_x\)) at the output sense combs of the microelectromechanical structure (Eq. 2.41). This behavior is captured by the equivalent electrical circuit reported in Fig 2.11a, where \(i_x\) is represented as a voltage controlled current source. The magnitude of the output current is a function of operating frequency (\(\omega_r\)), the input drive (\(C_d\)) and output sense capacitances (\(C_s\)), their movable airgaps (\(g_d\) and \(g_s\)), the demodulator quality factor (\(Q_x\)), its total and equivalent mechanical stiffness in the direction of motion (\(k_{mech}\) and \(k_{eqx}\)), and the polarization voltage applied to the structure (\(V_p\)). The polarization voltage (Eq. 2.42) must be carefully selected to avoid instability, which is defined by the pull-in voltage (\(V_{pi}\)) [8]. For this reason, a safety factor \(\eta\) (<0.9) is generally included when defining the operational \(V_p\).

\[
i_x = \frac{\delta C_s \delta x}{\delta x \delta t} V_p = \frac{1}{2\pi} \frac{C_s C_d \omega_r Q_x}{g_s g_d k_{eqx}} V_p V_d^2
\] (2.41)

\[
V_p = \eta V_{pi} = \eta \sqrt{\frac{8k_{mech}g_s^2}{27C_s}}
\] (2.42)

Two different designs have been demonstrated for the mixing stage of the RMR described in [5]. In the first design [8], a CMOS-MEMS demodulator was fabricated utilizing the six-metal TowerJazz 0.18 \(\mu\)m CMOS process. The main advantage of this approach is the integration between MEMS and CMOS circuitry, which allows for an easier system integration and lower parasitics. The relatively low quality factor (\(Q_x\)) and capacitance per unit area attained [5] fostered the investigation of a demodulator implemented in thick Single-Crystal-Silicon (SCS). SCS resonators realized with the “Epi-Seal” process offer better impedance matching and larger capacitance [27] at the expenses of a three-chips WuRx solution [5].
2.3. CMOS TIA

Figure 2.12: (a) Equivalent model of a CMOS TIA. The static capacitance of a previous stage \( C_S \) is included; and (b) block diagram of the CMOS circuitry described in [5].

The last stage of the RMR is constituted by a CMOS Transimpedance Amplifier (TIA) followed by a Passive Latch Rectifier (PLR). The wake up signal is generated by passively rectifying the motional current generated at baseband by the MEMS resonant demodulator.

The aim of the TIA is to provide sufficient voltage boost to the signal \( V_{out} \) before presenting it to the input stage of the PLR. The TIA can be modeled to first approximation as a current controlled voltage source (Fig. 2.12). A portion \( (i_s) \) of the motional current \( (i_x) \) generated at the sensing comb of the demodulator flows into the TIA transimpedance \( (R_{TIA}) \), which is later converted into a voltage (Eq. 2.43) via the TIA output resistance \( (R_{out}, \text{Eq. 2.44}) \).

\[
V_{out} = i_s R_{out} \quad \text{(2.43)}
\]

\[
R_{out} = \frac{1}{\lambda I_{DC}} \quad \text{(2.44)}
\]

\[
i_s = i_x \frac{1}{R_{TIA} + j\omega_C (C_S + C_{TIA})} \quad \text{(2.45)}
\]

\( R_{out} \) represents the output impedance of the TIA and is a function of the CMOS biasing current \( (I_{DC}) \) and of the channel-length modulation parameter \( (\lambda) \). The value of \( R_{out} \) cannot be arbitrarily selected since the biasing current is strongly limited by the power budget of the system, which translates in an \( I_{DC} \sim 1 \text{ nA} \), while \( \lambda \) is a function of the adopted CMOS technology (\( \sim 0.7 \)).
design of $R_{\text{TIA}}$ is critical since a lower resistance allows for more transimpedance gain, but reduces the current ($i_s$) flowing into the TIA. If $R_{\text{TIA}}$ gets too close to the impedance of the sensing comb, more current will flow into $C_s$, ultimately reducing $V_{\text{out}}$ (Eq. 2.43). Therefore, a trade-off between performance and power consumption is necessary to identify an optimal design point. For the selected technology (TowerJazz 0.18 μm CMOS), the maximum $R_{\text{TIA}}$ and $R_{\text{out}}$ were set to 2.15 MΩ and 1.4 GΩ to comply with the power budget.

The TIA is followed by a Source Follower (SF) stage operating in deep subthreshold regime, which drives a multi-stage Passive Latch Recifier (PLR). The SF is necessary to decouple the TIA and the capacitive load introduced by the rectifier stages, which was set to 60 in a trade-off between latching time and triggering voltage (~1 V). According to [5], a $V_{\text{out}}$ of 40-45 mV is necessary at the input of the PLR to trigger the comparator (Schmitt trigger) at the end of the CMOS chain (Fig. 2.12b).

### 2.4. RMR Numerical Optimization

![Complete schematic of the RMR analyzed in this section.](image)

Fig. 2.13 reports the final simplified schematic of a Resonant Microelectromechanical Receiver composed by a series piezoelectric MEMS resonator implemented in the RFFE matching network, an electrostatic MEMS resonant demodulator and a CMOS circuitry. This last stage includes a ULP
TIA, a passive latch rectifier and a Schmitt trigger to provide a wake-up signal to the sensing electronic.

The relationships defining the passive voltage amplification \( G \) in the MN, the current generated at the output of the demodulator \( (i_x) \) and the voltage presented at the input of the PLR by the TIA \( (V_{out}) \) are captured by Eq. 2.46 – 2.48.

\[
G = \left| \frac{V_d}{V_{in}} \right| = \frac{1}{j\omega \text{RF} C_d \left( R_s + \frac{(C_d + C_0)^2 \pi^2}{8 \omega_0 C_0 k_x^2 Q_s} \right)}
\]

\[
i_x = \frac{1}{2\pi g_s g_d} \frac{C_s C_d \omega_r Q_x}{k_{eqx}} \eta \sqrt{\frac{8k_{mech}g_s^2}{27C_s}} \frac{V_d^2}{V_d^2}
\]

\[
V_{out} = \frac{i_x}{\lambda I_{DC} R_{TIA}} \frac{1}{\frac{1}{\lambda I_{DC} R_{TIA}} + \frac{1}{\frac{1}{\lambda I_{DC} R_{TIA}}}}
\]

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<tr>
<th>Parameters</th>
<th>RF MEMS resonator</th>
<th>MEMS demodulator</th>
<th>CMOS TIA</th>
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</thead>
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<tr>
<td>Target sensitivity (dB)</td>
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<td>Quality Factor ( (Q_s) )</td>
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</tr>
<tr>
<td>Operating frequency ( (\omega_{RF}) )</td>
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<td>Drive and sense airgaps ( (g_s/g_d) )</td>
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<td>Quality Factor ( (Q_s) )</td>
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<td>Resonant Frequency ( (\omega_r) )</td>
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<td>Equivalent stiffness ( (k_{eqx}) )</td>
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<td>Parasitics ( (k_t^2) )</td>
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<td>Mechanical stiffness ( (k_{mech}) )</td>
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<td>Parasitics</td>
<td>30 fF</td>
</tr>
</tbody>
</table>

In order to maximize the receiver sensitivity, the MEMS voltage amplifier operating frequency \( (\omega_{RF}) \), its static capacitance \( (C_0) \), the MEMS demodulator resonant frequency \( (\omega_r) \), and its drive \( (C_d) \) and sense static capacitances \( (C_s) \) must be optimized according to the system constraints. The
assumptions made for the other parameters and the ranges of investigation for the carrier and baseband frequencies are reported in Table 2.1 and are extrapolated from prior experimental results [31][8]. TIA parameters are set to comply with a power budget of 10 nW.

An important constraint derives from the need of having a demodulator frequency ($\omega_r$) smaller than the 3 dB bandwidth of the carrier frequency. This constraint is required to allow the modulation of the RF signature and sets a limit for the maximum loaded quality factor of the RF MEMS resonator that can be implemented in the RFFE (Eq. 2.49). As an example, $Q_s$ as high as 10,000 can be sustained for $R_m$ in the order of few $\Omega$s, assuming $k_p = 4$, $f_{RF} = 50$ MHz and $f_i = 20$ kHz.

$$Q_{\text{max}} = Q_s \frac{k_p R_m}{k_p R_m + R_{\text{in}}} = \frac{\omega_{RF}}{\omega_r}$$  \hspace{1cm} (2.49)

Fig. 2.14 shows the impact of the RF carrier frequency and the demodulator frequency on the expected WuRx sensitivity for $C_0 = C_d = C_s = 500$ fF, with $Q_{\text{max}}$ set by Eq. 2.49. Operating at lower $\omega_{RF}$ (~50 MHz) yields higher sensitivity, due to the larger impedance conversion that can be attained at RF between the 50 $\Omega$ input resistance and a fixed capacitive load.
Lower \( \omega_r \) (~20 kHz) improve the system sensitivity mostly because of the TIA bandwidth, as well as a slightly higher voltage gain attainable in the MN, at the sidebands are closer to the carrier frequency. Despite the analysis shows that operating at the lowest possible frequency is advantageous by a sensitivity standpoint, consideration related to the antenna size, the piezo MEMS resonator and MEMS demodulator footprints, and the spectrum availability must be taken into account when selecting the carrier frequency. For the purpose of this dissertation, the carrier and modulation frequencies were respectively set around 50 MHz and 20 kHz, to showcase the capabilities of the RMR architecture.

![Figure 2.15](image)

Figure 2.15: (a) Numerical optimization of the RMR sensitivity as a function of \( C_s \) and \( C_d \); and (b) simulated gain provided by the RF resonator to the input \( C_s \) of the demodulator. \( G \) is maximized for smaller \( C_s \) and is not affected by \( C_d \). The optimal \( C_d \) closely follows \( C_s \) as reported in Section 2.1.3.

The sensitivity of the RMR as a function of the sizing of the RF MEMS resonator (\( C_0 \)) and demodulator (\( C_d \) and \( C_s \)) is reported in Fig. 2.4a.

According to the numerical optimization reported in Fig. 2.15a, the optimal sensitivity is achieved for a demodulator drive capacitance (\( C_d \)) of 500 fF, as a trade-off between the gain which
can be provided by the MN (Fig. 2.15b) and the motional current generated at the output of the demodulator ($i_c$). The sense capacitance ($C_s$) should be increased up to the point its equivalent impedance matches the input impedance of the TIA (~3.7 pF). For the case under examination, a $C_d$ of 500 fF corresponds to an optimal RF MEMS static capacitance ($C_0$) of roughly 400 fF. More realistically, due to system integration parasitic capacitances, $C_d$ should be assumed close to 1 pF, which closely resemble the typical load of a WuRx ED [20].

The numerical optimization proves that our proposed RMR architecture is capable of sensitivities approaching -100 dBm for a large range of resonator and demodulator capacitances (Fig. 2.4a), proving its robustness to parasitics. More importantly for the purpose of this dissertation, it provides the design range in terms of $Q_0$, $k_t^2$ and $C_0$ for the resonators to be implemented in the MN of the RMR.
CHAPTER 3: PIEZOELECTRIC MEMS

RESONATORS TECHNOLOGIES

Piezoelectric-based acoustic MEMS resonators have been widely investigated and became commercially successful products, with devices deployed in RF filters, duplexers and oscillators. Quartz crystal oscillators for time-reference applications and Surface Acoustic Wave (SAW) resonators for RF filters were the most notable pioneering solution based on acoustic devices. In most recent times, Aluminum Nitride (AlN) Film Bulk Acoustic Resonators (FBARs) set the technological threshold for mobile filtering applications, which is projected to become the largest market for these devices in the years to come (Fig. 1.1d).

The challenge to enhance filter performance in terms of lower insertion losses (IL) and wider bandwidth (BW), and the development of matching networks based on piezoelectric MEMS for novel IoT applications is fostering the investigation of new classes of resonators based on different acoustic modes. Among those, the most notable are the fundamental symmetric mode ($S_0$) [31], fundamental shear horizontal wave ($SH_0$) [15], Bulk Acoustic Wave (BAW) [32], Incredible High Performance (IHP) SAW [33], and Thickness Shear Mode (TSM) [34]. The need for compact resonators with higher $Q$, and $k^2$ is also spurring interest for new classes of materials, such as doped-AlN alloys [35][36], thin films of Lithium Niobate (LiNbO$_3$, or LN) [37], Lead Zirconate Titanate (PZT) [38], and Gallium Nitride (GaN) [39]. Especially for Scandium-doped Aluminum Nitride (Sc$_x$Al$_{1-x}$N) alloys and LN resonators, research is currently focused on overcoming fabrication constraints and device performance, which are still far from the theoretical maximum.
This chapter focuses on Laterally Vibrating Resonators (LVRs), a promising class of devices which exhibits good $Q_s$ and $k^2$ for different piezoelectric material and fabrication versatility, making them an ideal solution for prototyping. The main excitation modes and the key geometrical features, and their effect on the devices static capacitance ($C_0$), are later introduced. Finally, resonators fabricated with different material technologies are compared to identify the solution that would maximize the gain in the RFFE of the WuRx described in Chapter 2.

3.1. Laterally Vibrating Resonators (LVRs)

Laterally Vibrating Resonators (LVRs) operating in the symmetric mode ($S_0$) are an appealing solution for filtering and novel IoT applications due compact footprint, high performance (FoM) in the frequency range of interest, and low motional resistance ($R_m$). Differently from FBAR resonators, LVRs frequency can be defined lithographically, allowing multi-frequency fabrication per die on substrates with various thicknesses. The promising results reported in preliminary studies [40][35][41][31] and their characteristics make LVRs ideal candidates for use in matching networks of WuRx receivers.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.1.png}
\caption{Laterally Vibrating Resonators made of different piezoelectric material: (a) X-cut Lithium Niobate LVRs. The edge reflectors are necessary to fully harness the maximum electromechanical coupling of the material; (b) Y-cut Lithium Niobate; and (c) 20% Scandium-doped Aluminum Nitride.}
\end{figure}
The typical embodiment for an LVR consists in a thin suspended plate of piezoelectric material, tethered to the substrate via longitudinal anchors (Fig. 3.2). The piezoelectric medium is excited via metallic Interdigitated Electrodes (IDEs) or Transducers (IDTs) which are patterned on the top surface. Similarly to SAW resonators, the resonant frequency is set to first approximation by the distance between the longitudinal symmetry axis of the metallized fingers, which is defined as pitch ($p$). The pitch must equal half of the acoustic wavelength ($\lambda$) of the desired mode, which can be estimated accordingly to Eq. 3.1, knowing the speed of propagation of wave in the medium ($v_p$).

$$\lambda = \frac{v_p}{f_s}$$

(3.1)

Several parameters, such as plate length ($L$), width ($W$), substrate thickness ($h$), anchors width ($W_a$) and length ($L_a$), and metal thickness ($t_m$) define the resonator mechanical stiffness, ultimately affecting its resonant frequency.

Figure 3.2: Top view of the typical embodiment of a Laterally Vibrating Resonator (LVR). A bottom electrode under the resonator plate can be present or not according to the desired excitation method.

The ratio between the width of the IDEs and the pitch (metalized vs. total area) is named electrode coverage ($c$). A patterned or unpatterned bottom electrode can be present or not, according to the desired excitation method and fabrication process.
This class of devices relies simultaneously on the direct and inverse piezoelectric effect for its functioning. When the frequency of the RF signature matches the mechanical resonance set by the IDT spacing, the signal is transduced in the acoustic domain at the input port as a standing wave. The acoustic wave propagates in the medium and is later re-converted in the electromagnetic domain at the output terminal.

In more details, five regions can be identified in a typical LVR geometry:

- Active region, or aperture ($L_e$);
- Finger-to-bus gap ($g$);
- Bus length ($L_b$);
- Anchors;
- Routing.

The active region is defined by the IDEs and is the area where piezoelectric transduction occurs. Ideally, all the energy is confined in this region to maximize the quality factor and electromechanical coupling of the resonator. The finger-to-bus gap represents the distance between the termination of the IDEs belonging to one port and the bus of the opposite port, which acts as an interconnect between all the fingers belonging to the same terminal. The sum of lengths of the aperture, the two gaps and bus defines the length of the plate ($L$), while the number of fingers (and edge reflectors if present, Fig 3.1a) sets the plate width. The width of a resonator can be expressed according to Eq. 3.2, where $N_p$ represents the number of finger pairs. Reflectors account for half a finger when defining the number of pairs.

$$W = \frac{\lambda}{2} (N_p + 1)$$  \hspace{1cm} (3.2)
Different anchor geometries can be implemented to connect the plate to the substrate [42][43]. The simplest solution is the adoption of two symmetrical anchors positioned perpendicularly to the wave propagation direction. Together with the gap and bus length, anchors’ length and width define the acoustic boundary conditions of the resonator. It was proven, for different technologies, that the sizing of the passive regions is fundamental in synthetizing a $\lambda/4$ transformer [44][45] to minimize the anchor losses, thus maximizing the resonator quality factor.

3.1.1. Static Capacitance ($C_0$) Modeling and Excitation Mode

![Diagram](image)

**Figure 3.3:** (a) Thickness Field Excitation mode and equivalent electrical circuit representing the static capacitance ($C_0$) of the device. Such capacitance is effectively the series of the capacitances of each terminal; and (b) Lateral Field Excitation mode and equivalent electrical circuit representing the static capacitance. In this case, $C_0$ is given by the parallel capacitance between the IDEs pairs.

The static capacitance of the resonator ($C_0$) plays a major role with $Q_s$ and $k_t^2$ in determining the performance of piezoelectric MEMS-based matching network, as described in Section 2.1.3. Its value must be properly selected according to the MN architecture and the mixing stage input load to maximize the impedance transformation, thus maximizing the gain. For this reason, an accurate model for $C_0$ is required to guide the design of devices matched to a given $C_{load}$. Two excitation methods are usually exploited in $S_0$ modes LVR (Fig. 3.3): Thickness Field Excitation (TFE) and
Lateral Field Excitation (LFE). The desired mode of vibration, and thus the excitation method, determines the exploited piezoelectric coefficient, and the IDT and plate geometries. In order to synthetize an LVR operating in pure $S_0$ mode, the $h/\lambda$ ratio should be kept below 0.05 [46].

In resonator relying on Thickness Field Excitation (Fig. 3.3a), the electric field is applied in the vertical direction (3) to generate a standing acoustic wave in the lateral direction (1). The piezoelectric coefficient exploited is $d_{31}$ [26]. Assuming perfect coupling into the piezoelectric film, $k_r^2$ can be estimated according to Eq. 3.3 [26], where $s_{11}$ is the compliance of the material in the lateral direction at constant electric field ($E$).

$$k_r^2 = \frac{d_{31}^2}{s_{11}E \varepsilon_r \varepsilon_0}$$ (3.3)

In this configuration, a bottom electrode is desirable to define the potential of the bottom surface and better drive the electric field in the desired direction. It is possible to exploit the $d_{31}$ coefficient even without a bottom electrode, at the expenses of a lower coupling coefficient [46].

For 1-port LVRs with bottom electrode, a simple $C_0$ model can be derived neglecting any fringe effect. The capacitance of each finger ($C_f$) can be modeled as parallel plate capacitor (Eq. 3.4), where $\varepsilon_0\varepsilon_r$ is the dielectric permittivity of the piezoelectric medium, $\lambda$ is the acoustic wavelength, $L_e$ is the aperture and $c$ is the finger coverage. The overall static capacitance, $C_0$, can be estimated as the series combination between the capacitance of the input and output IDTs (Eq. 3.5), considering the number of fingers at each terminal ($N_{f,in}$ and $N_{f,out}$) and their respective capacitance per finger ($C_{f,in}$ and $C_{f,out}$).

$$C_f = \frac{\varepsilon_0\varepsilon_r L_e \lambda c}{2h}$$ (3.4)

$$C_0 = \frac{N_{f,in}N_{f,out}C_{f,in}C_{f,out}}{N_{f,in}C_{f,in} + N_{f,out}C_{f,out}}$$ (3.5)
Differently from TFE resonators, devices operating with Lateral Field Excitation (LFE) exploit an electric field which is mainly directed in the horizontal direction (1). In this case, the piezoelectric coefficient of interest is \( d_{11} \) and \( k_t^2 \) can be estimated according to Eq. 3.6.

\[
k_t^2 = \frac{d_{11}^2}{s_{11}^E \varepsilon_r \varepsilon_0}
\]  

(3.6)

A vertical direction (3) component is usually present due to the direction of the electric field in the medium. Thus, \( C_0 \) is both a function of the dielectric constants in the 1 and 3 directions (\( \varepsilon_r^1 \) and \( \varepsilon_r^3 \), respectively). For \( h/\lambda < 0.05 \), it is possible to assume that the electric field is mainly constrained in the lateral direction, and that an approximated value for the capacitance of a pair of IDTs (\( C_p \)) can be estimated with a parallel plate capacitor model (Eq. 3.7).

\[
C_0 = C_p N_p = \frac{2 \varepsilon_0 \varepsilon_r L_e h}{\lambda (1 - c)} N_p
\]  

(3.7)

If the \( h/\lambda \) ratio is greater than 0.05, the parallel plates approximation is no more valid, and a different model is required [47]. In such case, \( C_0 \) can be more accurately described by Eq. 3.8, where \( \varepsilon_{eff} \) represents the effective permittivity (Eq. 3.9) and \( K(x) \) the elliptical integral of the parameter \( x \) [48]. The other parameters introduced to defined \( C_0 \) and \( \varepsilon_{eff} \) (\( q, k_0, k_1, k_0' \), and \( k_1' \)) are reported in Eq. 3.10 – 3.13.

\[
C_0 = \varepsilon_0 \varepsilon_{eff} \frac{K(k_0')}{K(k_0)} L_e N_p
\]  

(3.8)

\[
\varepsilon_{eff} = 1 + q (\varepsilon_r - 1)
\]  

(3.9)

\[
q = \frac{1}{2} \frac{K(k_1') K(k_0)}{K(k_1') K(k_0')}
\]  

(3.10)

\[
k_0 = \frac{1 - c}{1 + c}
\]  

(3.11)
\begin{align*}
k_1 &= \frac{\tanh\left[\frac{\pi}{8}\left(1-c\right)\frac{\lambda}{h}\right]}{\tanh\left[\frac{\pi}{8}\left(1+c\right)\frac{\lambda}{h}\right]} \quad (3.12) \\
k_i' &= \sqrt{1-k_i^2} \quad (3.13)
\end{align*}

Compared to TFE resonators, LFE suffer of lower capacitance per unit area (Fig. 3.4). This mean that in order to match the capacitance of demonstrated ED (~1 pF) and maximize the gain, larger structures are required. Differently from TFE, where a thinner piezoelectric film is required to increase the capacitance per unit area, thicker film help increasing LFE devices $C_0$. Unfortunately, such increment does not scale linearly with $h$, and thicker film proven to be harder to etch, leading to poorer sidewall angles and lower $Q_s$.

Figure 3.4: TFE vs LFE capacitance per unit area for different piezoelectric materials.

### 3.1.2. Impact of FoM and $C_0$ on Voltage Amplification

As highlighted in Section 2.1.3, the gain provided by a 1-port MEMS-based matching network to a fixed capacitive input load ($C_{load}$) is a function of the Figure of Merit (FoM), the resonant
frequency and its static capacitance \((C_0)\). An optimal value of \(C_0\) exists for a given \(C_{load}, f_s\) and FoM (Fig 2.6), such as a proper sizing of the resonator is necessary to maximize the MN performance. In order to identify the most suitable LVR technology, it is fundamental to understand the impact of the FoM and of \(C_0\) on the gain as a function of \(C_{load}\) and the frequency of operation \((f_{RF})\).

Fig. 3.5a reports the analytical simulation of the gain provided by a 1-port resonator to a fixed input load of 1 pF as a function of the operating frequency, for different values of FoM. A high FoM is desirable, especially at lower frequency, to minimize both \(R_m\) and \(k_p\) which significantly contribute in determining the insertion losses of the MN. The same conclusion can be derived considering the gain provided to variable load (Fig. 3.5b) for a fixed frequency \((f_s = 50 \text{ MHz})\). As mentioned beforehand, the FoM also impacts the \(C_0\) optimal point (Fig. 3.5c). Interestingly, the gain characteristic is relatively flat around the point of optimum \(C_0\), meaning that smaller resonator can be employed without significant reduction in gain if necessary.

![Figure 3.5](image)

Figure 3.5: (a) Voltage gain as a function of natural resonant frequency \((f_s)\) for fixed load \((C_{load} = 1 \text{ pF})\); (b) voltage gain as a function of the capacitive load for a fixed natural resonant frequency \((f_s = 50 \text{ MHz})\); and (c) voltage gain as a function of the static capacitance \((C_0)\) for fixed load \((C_{load} = 1 \text{ pF})\) and frequency \((f_s = 50 \text{ MHz})\), for different FoMs. Higher FoMs are desirable to reduce the optimal size of the resonators and to enhance the gain.
3.2. Comparison between LVRs Technologies

Numerical simulations point out that resonator with a FoM of at least 500 in the frequency range of interest (50 – 400 MHz), and with static capacitances in the order of $C_{load}$, are necessary to attain large passive voltage amplification in 1-port MEMS-based matching networks. In the present section, different demonstrated and experimental LVRs technologies will be compared in terms of FoM, static capacitance and fabrication process to identify the most promising solution to be implemented in the RFFE of novel WuRx.

3.2.1. Aluminum Nitride (AlN)

The advancements in reactive sputtering technology and the development of high-quality thin film of Aluminum Nitride (AlN) at the end of the 90s fostered the research on innovative RF MEMS devices to enable large scale mobile communication. Film Bulk Acoustic Resonators (FBARs) [49] are the brightest example of a successful, commercial product that revolutionized an existing market segment (in this case, the RF filtering niche). The constant need for lower Insertion Losses (IL) and higher fractional bandwidths led to the study of FBARs alternatives. In this framework, the first Contour Mode Resonators (CMRs), the class of resonators LVRs can be ascribed to [22], were demonstrated [50]. Despite significant improvements from preliminary results [40], both in terms of Figure of Merit and understanding of the device physics, AlN LVRs have not been able to attain performance similar to FBARs ($Q_s = 4,000$ and $k_t^2 = 7\%$ [5]). $Q_s$ in the order of 2-3,000 are generally reported in literature for 1 µm thick film operating in the hundreds of MHz range [44], with a maximum $k_t^2$ of 2% (FoM ~ 40 - 60). $Q_s$ up to 4,500 have been demonstrated for thicker films, but at the expenses of $k_t^2$ (0.7%) and capacitance per unit area [40]. More recently, research focused on
hybrid modes to enhance the electromechanical coupling, with promising results [51][52], but still far from the target set by cutting-edge applications, such as 5G filtering solutions or MEMS-based matching networks for IoT WuRx.

![Image of fabrication process](image.png)

Figure 3.6: Fabrication process of 1-port AlN LVRs: (1) Bottom electrode patterning via sputtering and lift-off; (2) AlN reactive sputtering; (3) AlN patterning via Cl-based RIE or ion milling; (4) top electrode patterning via sputtering and lift-off; and (5) XeF₂ isotropic etching release.

The typical fabrication process of an AlN LVRs is reported in Fig. 3.6. As first step, the platinum (Pt) bottom electrode is patterned on top of a high resistivity silicon (Si) wafer using DC sputtering and lift-off (step 1 in Fig. 3.6). The thickness of the bottom electrode is set as a fraction of the piezoelectric film thickness as a trade-off between the electrical loading introduced and the necessity of having a plane substrate for a correct growth of the AlN crystals. Pt thicknesses in the order of 100 nm are usually adopted for 1 μm thick films [53]. To ensure adhesion between the metallic layer and the silicon surface, 10 nm of Chromium (Cr) are pre-sputtered. Chromium is preferred over other adhesion layer materials, such as Titanium (Ti), due to its compatibility with hydrofluoric acid (HF), which is often used to strip silicon dioxide (SiO₂) masks. Successively, a uniform, thin film of AlN is deposited via reactive sputtering on top of the wafer (step 2 in Fig. 3.6). A negative photoresist (PR) soft mask is then patterned on top of the piezoelectric film for the definition of the release pit. A hard mask made of SiO₂ can be used instead of a PR soft one, at the
expenses of a more complicated fabrication process. The LVRs geometry is then patterned via chlorine (Cl) based Reactive Ion Etching (RIE, step 3 in Fig. 3.6) or ion milling. Successively, the aluminum-silicon-copper (AlSiCu) top electrode and the interconnects are patterned via DC sputtering and lift-off (step 4 in Fig. 3.6). Similarly to the bottom electrode, the optimal AlSiCu thickness is a trade-off between electrical and mass loading, and it is a function of the piezoelectric film thickness. For 1 μm AlN films, 200 to 300 nm of AlSiCu is an optimal choice to reduce series resistance without affecting the electromechanical coupling. The resonator is finally released via xenon difluoride (XeF$_2$) isotropic etch (step 5 in Fig. 3.6), which removes the Si underneath the plate from the release pit. A more detailed description of the fabrication process, including recipes and etch rates, is reported in Appendix A. An example of a fabricated device is reported in Fig. 3.7.

![Microscope image of a 500 MHz 1-port AlN LVR.](image)

**Figure 3.7:** Microscope image of a 500 MHz 1-port AlN LVR.

AlN 1-port LVRs fabricated as a testbench at CMU Nanofab Facility exhibited $Q_s$ as high as 3,910 and $k_t^2$ of 2%, for an overall FoM of 78 (Fig. 3.8). These results are consistent with the performance reported in literature, which are far from the specs required by the MEMS-based matching networks previously described, especially in terms of $k_t^2$. 
3.2.2. Scandium-doped Aluminum Nitride (ScAlN)

Doping is an appealing solution to increase the intrinsic $k_t^2$ of AlN films. Electromechanical coupling enhancement has been theoretically predicted [54], and different alloys investigated for this purpose [55][56]. Among all the possible dopants, Scandium (Sc) has proven to be the most attractive element due to its low inclusion energy [36] and theoretical $k_t^2$ improvement. In fact, the presence of Sc modifies the AlN Wurtzite structure, decreasing its Young’s modulus, while increasing its dielectric and piezoelectric constants [55]. This phenomenon is valid up to approximately 40% Sc-doping ($\text{Sc}_{0.4}\text{Al}_{0.6}\text{N}$), where the Wurtzite structure transitions into rock-salt structure, which does not exhibit piezoelectricity.

Sc-doped AlN films are interesting over other piezoelectric materials not only in terms of performance, but also for their CMOS compatibility. High-quality, sputterable thin film of ScAlN at different concentrations, which can be machined with similar technologies already developed for
AlN, are in fact the most ideal solution for mass production of consumer electronic components, since their implementation on production lines would require significantly lower investments.

The most promising results showed LVRs with $k_t^2$ of 4.5% and $Q_s$ of 1,300 - 1,600 between 200 and 500 MHz [35][36] for a doping around 20%, and $k_t^2$ up to 8% for 40% doping in the GHz range for Lamb Wave resonators [57]. These values of $k_t^2$ are respectively 2x and 4x higher than the typical performance of an AlN LVR.

The fabrication process for 20% Sc-doped AlN (Sc$_{0.2}$Al$_{0.8}$N) LVRs is virtually identical to the process flow described in Fig. 3.6 for an AlN resonator. The processed developed at CMU Nanofab starts with the sputtering and lift-off of 100 nm of Pt for the definition of the floating bottom electrodes of 1-port resonators (step 1 in Fig. 3.6). 10 nm of Cr are used as adhesion layer to allow compatibility with HF-based processes. The reactive sputtering (step 2 in Fig. 3.6) of the 1 μm piezoelectric thin film was performed externally by Advanced Modular Systems (AMS) [58]. The Sc-doped films proven to have a sensibly higher selectivity to standard Cl-based RIE (step 3 in Fig. 3.6) recipes commonly used, showing a reduced etch rate of 50 nm/min compared to the 200 nm/min registered for AlN resonator with identical geometry. A 2.5 μm SiO$_2$ hard mask is patterned in place of a thick (~5 μm) soft mask, to avoid PR overheating and issues during the mask removal process. The remaining oxide layer (~500 nm) is later removed with a buffered HF solution, which does not attack AlN nor ScAlN films. Both the top electrode patterning (4) and the resonator release (step 5 in Fig. 3.6) are identical to the steps reported for AlN devices. Microscope and SEM pictures of fabricated devices are reported in Fig. 3.9. Sidewall angle ($\phi$) were estimated around 70° (Fig. 3.10a). A value of $\phi$ as close as possible to 90° is associated with higher $Q_s$ and lower damping.
Figure 3.9: (a) Microscope image of a 2-port 20% Sc-doped AlN resonator operating around 500 MHz. VIAs are added to ground the bottom electrode; and (b) SEM image of a 1-port resonator operating around 500 MHz.

Visual inspection of fabricated devices highlighted two main fabrication issues: etch byproducts redeposition and out-of-plane bending. The residuals consist in thin flakes hanging from the resonator and release pit edges (Fig. 3.10b), which became visible only after the release process. Energy-dispersive X-Ray Spectroscopy (EDX) analysis found an elevated concentration of Boron (B) in the flakes, pointing out to the Cl-based RIE, which implements BCl₃ in the dry etching chemistry. The physical mechanism of flakes formations, which tend to disappear over time, has not been identified, nor their impact on the resonators’ performance. Devices fabricated adopting ion milling in place of RIE did not show byproducts after release [53].

Figure 3.10: (a) Detail of the sidewall angle of a 20% Sc-doped AlN LVR; (b) etch byproduct flakes hanging from the release trench; and (c) Failure of a 1-port LVR due to the elevate film stress gradient.
Stress gradient generated along the thickness direction caused out-of-plane bending in all fabricated devices (Fig. 3.10b) and, in some cases, catastrophic failure of the resonators (Fig. 3.10c). Such stress was estimated around 1 GPA/µm from test structures included in the tape-out (cfr. Appendix C).

Fabricated devices exhibited a maximum $Q_s$ of 1,240 and $k_t^2$ of 4.5% at 250 MHz, and a maximum $Q_s$ of 1,500 and $k_t^2$ of 3.6% at 500 MHz (Fig. 3.11). Further investigations [53] implementing a modified fabrication process did not show any significant improvement in terms of quality factor, and similar values of $k_t^2$. Despite improvements on the electromechanical coupling, such FoM (~60) are comparable to the performance achieved on AlN LVRs, which makes these devices not suitable for the application targeted in this dissertation.

Figure 3.11: Admittance responses of a 20% Sc-doped AlN resonator operating (a) around 250 MHz; and (b) around 500 MHz.

In conclusion, further improvements are necessary to fully harness the potential of Sc-doped AlN films. Recorded $Q_s$ are generally lower than the values recorded in identical AlN devices, meaning that losses mechanism have not been fully identified. Sputtering technology also needs to be improved to provide almost stress-free, high quality ScAlN films that can be reliably
implemented in production lines for mass production. Ultimately, the fabrication process needs to be further refined, with the development of reliable dry or wet etching procedures.

3.2.3. Y-cut Lithium Niobate (LN)

Different cuts and orientations of bulk Lithium Niobate (LiNbO₃ or LN) have been widely used for the fabrication of Surface Acoustic Wave (SAW) resonators in the past decades due to their high intrinsic $k_r^2$ [59]. In recent years, LN based piezoelectric thin films have been extensively explored for the development of high coupling ad high performance resonators [60][61]. Unfortunately, sputtering is not a viable solution since the c-axis oriented (Z-cut) LN is of limited interest, due to the low $k_r^2$ of lateral modes [62]. For this reason, researchers have focused on thin film transfer technologies to fabricate LN substrates varying from hundreds of nm to few μm. Two technologies have mostly been investigated to fabricate thin film of various LN orientation on Si or SiO₂ carriers: ion-slicing and Surface Activated Bonding (SAB). Ion-slicing [63] implements Helium (He) implanted ions to generate cracks at a controlled depth. The wafer is later bonded to a carrier wafer. Slicing is finally completed by heating up the stacking in a furnace, allowing the cracks generated by the implantation to propagate across the entire wafer diameter. This technique, although very promising for the making of thin, uniform films, resulted in limited transfer yields and material properties that did not match the film bulk values. Another appealing solution is Surface Activated Bonding. In this case, a LN wafer is bonded on a carrier wafer (usually Si or SiO₂) by means of plasma. Two thin layers (~10 nm) of SiO₂ and polysilicon are respectively sputtered on the back of
the LN wafer and the carrier to ensure adhesion. As last step, the desired thickness is achieved by Chemical Mechanical Polishing (CMP) and laser trimming to reduce the surface roughness to a few nm. Devices fabricated on SAB wafers showed better performance over ion-sliced ones [31], and have lately became the standard for research and commercial applications.

Among the different cuts, Y-cut Lithium Niobate is of particular interest due to its high \( d_{31} \) piezoelectric coefficient and dielectric constant \( \varepsilon_{r3} \), which makes it a perfect candidate for the making of TFE LVRs. Differently from sputtered AlN thin film, which exhibit an in-plane symmetry due to the c-axis oriented columnar structure, transfer Y-cut films show a high in-plane anisotropy. For this reason, the \( k_t^2 \) is a function of the wave propagation direction (\( \theta \)). Fig. 3.12 shows the dependency of \( k_t^2 \) and \( \varepsilon_r \) as a function of \( \theta \). Parameters are maximized for \( \theta = 0^\circ \) (0° XZ).

![Figure 3.12](image)

**Figure 3.12:** (a) EM coupling of TFE lateral mode (3-1) as a function of orientation for Y-cut Lithium Niobate. A global maximum exists at \( \theta = 0^\circ \); and (b) relative dielectric constant in the vertical direction as a function of orientation. The global maximum for \( \varepsilon_r \) coincides with the \( k_t^2 \) maximum (cfr. Appendix D for details on matrix rotations).

To allow the making of TFE LVRs, the SAB process developed for LN thin film was modified to integrate a Pt layer beneath the piezoelectric layer, as reported in [41]. Despite its theoretical
feasibility, patterned bottom electrodes have not been demonstrated with SAB technologies up to date, due to the surface planarity required by film transferring techniques.

The fabrication process is reported in Fig. 3.13. A thin film of Pt (100 nm) is sputtered on top of a high resistivity (HR) wafer (step 1 in Fig. 3.13). Successively, the LN thin film is transferred on top of the carrier (step 2 in Fig. 3.13) with the SAB process previously described. The process was performed externally by NGK Insulators, Ltd. The patterning of 1-port LVRs is performed with ion milling (step 3 in Fig. 3.13) using a 2 μm SiO₂ hard mask for 1 μm thick LN films. The remaining oxide is later stripped with fluorine based RIE.

Differently from AlN resonators, most of LN around the resonator plate is removed to reduce the amount of parasitic capacitance introduced. To define a rigid, outer frame, a 1 μm film of AlN is sputtered on top of the patterned plates. Such layer is later patterned and etched (4) with a wet
process using CD-26 heated up to 55°C and a soft mask of AZ4400. CD-26 contains 2.4% of tetramethyl-ammonium-hydroxide (TMAH), which is able to etch AlN at a rate of about 200 nm/min without attacking Pt or LN. Patterning of top IDEs (5) and resonator release (6) follows the same procedure described for AlN LVRs. A SEM picture of fabricated devices is shown in Fig. 3.14.

![SEM image of fabricated devices](image_url)

**Figure 3.14:** SEM image of a fabricated 1-port Y-cut LVR with zoom-in of the anchor, bus and finger-to-bus gap region. A 1 µm AlN patterned layer is deposited to reduce the parasitics introduced by the presence of an unpatterned bottom electrode.

As highlighted in Fig. 3.14, AlN is patterned on top of the LN plate up to the gap region. This design choice is implemented to reduce the impact of parasitic capacitance between the terminals of the device. The lack of a patterned bottom electrode, combined with the high relative dielectric constant of Y-cut LN (ε_r = 45), adds a feedthrough capacitance component (C_f, Fig. 3.15a) in parallel with the static capacitance of the resonator (C_0) which effectively decreases the measured k^2 (k^2_{lm}, Eq. 3.14). The addition of a layer of AlN, which presents a lower dielectric constant (ε_r = 9) helps decreasing the effective ε_r (ε_r_eq, Eq. 3.15) as reported in Fig. 3.15b, ultimately reducing C_f.
\[ k_t^2 = k_t^2 \frac{C_0}{C_0 + C_f} \tag{3.14} \]

\[ \varepsilon_{r,\text{eq}} = \frac{\varepsilon_{r,\text{LN}} \varepsilon_{r,\text{AlN}}}{t_{\text{LN}} \varepsilon_{r,\text{LN}} + t_{\text{AlN}} \varepsilon_{r,\text{AlN}}} (t_{\text{AlN}} + t_{\text{LN}}) \tag{3.15} \]

**a.**

![Diagram of a 1-port BVD model with parasitic capacitances](image)

**b.**

![Contour plot of equivalent dielectric relative constant](image)

Figure 3.15: (a) Representation of the feedthrough parasitic capacitance \( (C_f) \) in a 1-port BVD model; and (b) contour plot of the equivalent dielectric relative constant \( \varepsilon_{r,\text{eq}} \) of a stacking of Y-cut and AlN as a function of the material thicknesses \( (t_{\text{LN}} \text{ and } t_{\text{AlN}} \text{ respectively}) \).

Fabricated devices at 220 and 440 MHz exhibited \( Q_s \) around 1,100 – 1,200 and \( k_t^2 \) close to 5\%, lower than the theoretical prediction (Fig. 3.16). De-embedding was performed on measured data to identify the impact of parasitic feedthrough capacitances \( (C_i) \) and electrical loading on \( k_t^2 \) and \( Q_s \) respectively (Table 3.1). The static capacitance model captured by Eq. 3.5 was utilized to estimate \( C_0 \) for both the resonators. \( C_i \) was estimated subtracting the estimated value from the measured \( C_0 \). Estimation based on the overlapping area of the bottom electrode and the metallic traces, considering the effective dielectric constant \( (\varepsilon_{r,\text{eq}}) \) reported in Eq. 3.15, provided similar results. The series resistance \( (R_s) \) model introduced in Section 4.3.2 was similarly used to estimate the electrical loading introduced by the IDTs and the routing.
Figure 3.16: Admittance responses of Y-cut LN resonator operating (a) around 200 MHz; and (b) around 400 MHz. De-embedding of $C_0$, $k_t^2$ and $Q_s$ is reported in Table 3.1.

Table 3.1: Measured parameters, geometrical dimensions and estimated $R_m$, $C_0$ and $C_f$ for the devices reported in Fig. 3.16. De-embedded $k_t^2$ and $Q_s$ are also reported.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>220 MHz</th>
<th>440 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured static capacitance ($C_0$)</td>
<td>280 fF</td>
<td>200 fF</td>
</tr>
<tr>
<td>Measured motional resistance ($R_m$)</td>
<td>59.3 Ω</td>
<td>58.6 Ω</td>
</tr>
<tr>
<td>Measured EM coupling ($k_t^2$)</td>
<td>4.9%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Measured quality factor ($Q_s$)</td>
<td>1,182</td>
<td>1,100</td>
</tr>
<tr>
<td>Wavelength ($\lambda$)</td>
<td>27 μm</td>
<td>27 μm</td>
</tr>
<tr>
<td>Anchors width ($W_a$)</td>
<td>20.25 μm</td>
<td>9.75 μm</td>
</tr>
<tr>
<td>Anchors length ($L_a$)</td>
<td>20.75 μm</td>
<td>10.25 μm</td>
</tr>
<tr>
<td>Number of fingers ($N_f$)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Coverage ($c$)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Gap ($g$)</td>
<td>18.5 μm</td>
<td>7.55 μm</td>
</tr>
<tr>
<td>Aperture ($L_e$)</td>
<td>135.5 μm</td>
<td>104.5 μm</td>
</tr>
<tr>
<td>Metal thickness ($t_m$)</td>
<td>200 nm</td>
<td>300 nm</td>
</tr>
<tr>
<td>Modeled static capacitance</td>
<td>180 fF</td>
<td>90 fF</td>
</tr>
<tr>
<td>Modeled series resistance</td>
<td>10 Ω</td>
<td>5.7 Ω</td>
</tr>
<tr>
<td>Feedthrough capacitance</td>
<td>100 fF</td>
<td>110 fF</td>
</tr>
<tr>
<td>Spurious-free $k_t^2$</td>
<td>4.9%</td>
<td>3.4%</td>
</tr>
<tr>
<td>De-embedded $k_t^2$</td>
<td>7.6%</td>
<td>7.5%</td>
</tr>
<tr>
<td>De-embedded $Q_s$</td>
<td>1,426</td>
<td>1,220</td>
</tr>
<tr>
<td>Figure of Merit (FoM)</td>
<td>58</td>
<td>54</td>
</tr>
<tr>
<td>De-embedded FoM</td>
<td>110</td>
<td>92</td>
</tr>
</tbody>
</table>

The de-embedded $k_t^2$ for both resonators was estimated around 7.5%, close to the theoretical maximum of 8% reported in Fig. 3.12a. De-embedded $Q_s$ show that electrical loading for the geometry under examination has a limited impact on the device performance. Fabrication...
complexity and the overall low FoMs recorded (around 60 before de-embedding) show that the state-of-the-art Y-cut LN does not outperform AlN devices, despite promising improvements in terms of $k^2$.

3.2.4. X-cut Lithium Niobate (LN)

X-cut Lithium Niobate (LN) is another interesting material that has been widely investigated over the past few years for the fabrication of resonators in the hundreds of MHz and low GHz range operating in different modes [63][64].

Differently from Y-cut LN, X-cut LN is appealing due to its high $d_{11}$ piezoelectric coefficient, which can be exploited by LFE LVRs. Fig. 3.17 reports $k^2$ and $\varepsilon_r$ as a function of the wave propagation direction ($\theta$).

![Figure 3.17](image_url)

Figure 3.17: (a) EM coupling of LFE lateral mode (1-1) as a function of orientation for X-cut Lithium Niobate. A global maximum exists at $\theta = 30^\circ$; (b) Relative dielectric constant in the lateral direction as a function of orientation. The global maximum for $\varepsilon_r$ does not coincides with the $k^2$ maximum.
A global maximum for the electromechanical coupling exists for $\theta = 30^\circ$ (YZ 30°), as reported in [62]. The dielectric constant is maximized for $\theta = 0^\circ$, and its equal to 40 at the point of maximum $k_t^2$, 4x greater than AlN.

The simplest fabrication process for X-cut LFE LVRs is reported in Fig. 3.18 and consists of two masks, three steps process. Ion-slicing or SAB are used to fabricate a thin film of X-cut LN on top of a carrier, usually high resistivity Si (step 1 in Fig. 3.18). The resonator plates are then patterned with ion milling using a soft PR mask. For 1 μm thick films, 2.5 μm of AZ4210 are sufficient to ensure a good sidewall profile and the complete etch of the lithium niobate. The top electrode is patterned similarly to the procedure described for AlN and Y-cut, with the sputtering and lift-off of a film of AlSiCu (step 2 in Fig. 3.18). Ultimately, resonators are released with a XeF$_2$ isotropic etching (step 3 in Fig. 3.18).

![Figure 3.18](image_url)

**Figure 3.18**: Fabrication process of 1-port X-cut LVRs: (1) SAB of LN on HR Si and polishing; (2) LN patterning via ion milling; (3) top electrode patterning via sputtering and lift-off; and (4) XeF$_2$ isotropic etching release.

Preliminary investigation [65] performed prior to the work described in this thesis demonstrated X-cut LVRs operating in the $S_0$ mode with $k_t^2$ of 11.5% and $Q_s$ of 475 around 500 MHz (FoM = 55) on substrates fabricated with ion-slicing technology. Further improvements achieved $k_t^2$ of 30% and $Q_s$ of 1,200 around 400 MHz (FoM = 360) on SAB substrates.
3.3. Conclusions and Figure of Merit (FoM) Remarks

Table 3.2 summarizes the performance achieved by each technology described in this section in terms of achieved $k_t^2$, $Q_s$ and FoM. Among all the demonstrated LVRs, X-cut Lithium Niobate is the most promising in terms of FoM and, thus, passive voltage amplification in the RFFE of the WuRx described in Section 2.

Table 3.2: Recap of the performance demonstrated by the most notable LVRs technologies. Sc$_{0.4}$Al$_{0.6}$N and X-cut LN properties and performance are presented as reported in [57] and [66].

<table>
<thead>
<tr>
<th>Material</th>
<th>Theor. $k_t^2$</th>
<th>Meas. $k_t^2$</th>
<th>$Q_s$</th>
<th>Mode</th>
<th>$\varepsilon_r$</th>
<th>CMOS compatible</th>
<th>FoM</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN</td>
<td>2%</td>
<td>2%</td>
<td>3,900</td>
<td>TFE</td>
<td>9</td>
<td>Yes</td>
<td>78</td>
</tr>
<tr>
<td>Sc$<em>{0.2}$Al$</em>{0.8}$N</td>
<td>4%</td>
<td>4.5%</td>
<td>1,500</td>
<td>TFE</td>
<td>11</td>
<td>Yes</td>
<td>67</td>
</tr>
<tr>
<td>Sc$<em>{0.4}$Al$</em>{0.6}$N</td>
<td>10%</td>
<td>8%</td>
<td>-</td>
<td>TFE</td>
<td>13</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Y-cut LN (0° XZ)</td>
<td>8%</td>
<td>4.9%</td>
<td>1,200</td>
<td>TFE</td>
<td>45</td>
<td>Hybrid</td>
<td>58</td>
</tr>
<tr>
<td>X-cut LN (30° YZ)</td>
<td>30%</td>
<td>30%</td>
<td>1,200</td>
<td>LFE</td>
<td>40</td>
<td>Hybrid</td>
<td>360</td>
</tr>
</tbody>
</table>

The main disadvantages of such technology are the relatively low capacitance per unit area (Fig. 3.4), and the need for hybrid integration for CMOS interfacing. Both these limitations can be easily overcome if the FoM of X-cut LN LVRs is sensibly larger than any other technology. As highlighted in Fig. 3.5, capacitance matching between $C_0$ and $C_{\text{load}}$ can be traded off for suboptimal, high gains which outperform resonator with smaller FoMs. Similarly, hybrid integration can be tolerated despite the larger parasitics introduced.

In conclusion, preliminary investigation on X-cut Lithium Niobate $S_0$ mode devices outperformed any other demonstrated LVR technology, thanks to the simultaneously high $k_t^2$ and
$Q_s$, which contribute to the overall Figure of Merit. Since the maximum theoretical $k^2$ was demonstrated [31], $Q_s$ is the only remaining lever to further enhance the FoM and, overall, the performance achievable by these devices when implemented in MEMS-based matching networks. This topic is thoroughly discussed in Chapter 4.
CHAPTER 4: X-CUT LITHIUM NIOBATE LVRs

The first demonstration of X-cut Laterally Vibrating Resonators operating in the $S_0$ mode is reported in [63], where $k_t^2$ of 8.2% and $Q_s$ around 140 at 530 MHz were attained. Following investigations [67] reported similar electromechanical couplings (9%) at higher frequency (~720 MHz), with similar $Q_s$. The introduction of active reflectors helped improving $k_t^2$ up to 12% [65], with $Q_s$ greater than 400 at 500 MHz. Weighted electrodes and improvements in the machining process, in particular on the sidewall angle of LVR suspended plates, made possible the fabrication of devices which exhibited simultaneously high $k_t^2$ (21%) and high $Q_s$ (~1,300) in the same frequency range [60].

More recently, advancements in the SAB process for the production of thin films, combined with the previous knowledge developed for LN LVRs design and fabrication [68], led to the demonstration of resonators which exhibited $k_t^2$ close to the theoretical maximum (30%), combined with high $Q_s$ (>1,200) [31][66] around 400 MHz.

Despite the incredibly high electromechanical coupling exhibited, X-cut LVRs have suffered from relatively low $Q_s$ compared to other available technologies based on piezoelectric materials, including AlN FBARs and LVRs (~4,000) [40][69], IHP SAW [70], and crystalline quartz resonators [71]. The values measured for current state-of-the-art resonators are in fact far from the main sources of losses (electron-phonon dissipation, piezoelectric-semiconductor loss, air damping, Akhieser damping, and thermo-elastic damping) reported in [72], and generally from the expected performance of devices fabricated on single crystal substrate, rather than sputtered films.
Preliminary investigation performed with Finite Element Analysis (FEA) in COMSOL® using Perfectly Matched Layer (PML) boundary conditions, highlighted $Q_s$ greater than 20,000 for the anchor losses alone [72]. These facts hint that a thorough characterization of damping mechanism is necessary to fully understand the limitation of such technology and synthesize LVRs that can exhibit $Q_s$ greater than few thousands.

As highlighted in Section 3.3, for IoT applications, $Q_s$ must be maximized while preserving a high $k_t^2$ to maximize the overall Figure of Merit, while roughly matching the static capacitance of the device to the one of the input load ($C_{\text{load}}$). This last constrain is particularly severe for X-cut LVRs operating in the LFE mode due to their low capacitance per unit area. Fig. 4.1 reports the required area to match different loads, as a function of the operating frequency. Especially for large loads and low frequency of operation, the dimension of the resonators poses serious challenges in terms of design and fabrication and could severely impact their performance.

Figure 4.1: Active area required by X-cut LVRs to match different capacitive loads assuming a parallel plate capacitor, $C_0 = C_{\text{load}}$, $c = 0.3$, $v_p = 5,800$ m/s, $t_{LN} = 2$ µm, $N_p = 1$, and $\varepsilon_r = 40$. 
Following these considerations, it becomes clear that the design of an optimized X-cut LVRs to be implemented in the MN of a ULP WuRx must take into account the trade-off between the operating frequency, matching to $C_{\text{load}}$, size, fabrication complexity, and the need to maximize $k_t^2$ and $Q_s$ simultaneously.

In the first part of this chapter, preliminary studies on 50 MHz devices are reported. These studies are further expanded up to 550 MHz in the following section. A few devices operating around 800 MHz and 1 GHz are also demonstrated. In the third and last section, the main sources of damping are investigated according to the data collected on all the fabricated resonators.

4.1. Design and Fabrication of 50 MHz devices

According to the RMR optimization reported in Section 3.1.2, a frequency of 50 MHz was selected for the preliminary investigation of high performance X-cut LVRs. This frequency offers a sweet spot for the system level demo. In fact, lower frequencies offer a larger impedance conversion, thus larger gain, for a fixed $C_{\text{load}}$, which ultimately increases the system sensitivity. Furthermore, no restrictions were provided by the project funder on the design and sizing of the antenna, which would have otherwise limited the implementation of system operating below few hundreds MHz.

From a resonator stand-point, lower frequencies are generally associated with lower damping [73], thus larger $Q_s$ can be achieved to showcase the potential of a technology. Larger devices are also less sensitive to lithography misalignments and overetch, which have proven to impact the quality factor of LVRs [72]. Compared to higher frequency resonators, which have usually a far
more compact footprint, 50 MHz devices are more prone to breakage due to the stresses induced by the sputtered metal traces on top of the thin piezoelectric film. Therefore, the preliminary investigation on large devices also serves to testbench the fabrication process regarding this specific issue.

For the example under examination, a reference $C_{\text{lead}}$ of 1 pF was assumed for the sizing of the X-cut LVRs, to simulate the typical input capacitance of a MEMS demodulator or CMOS ED [20]. A static capacitance ($C_0$) of 1 pF was also targeted for the sake of simplicity. Different approaches can be implemented to increase the total capacitance of an X-cut LVR operating in the LFE mode: 1) Increase the electrode coverage ($c$); 2) increase the thickness of the piezoelectric film ($t_{\text{LN}}$); 3) increase the aperture ($L_e$) and thus the area of transduction; 4) Increase the number of pairs ($N_p$); and 5) arraying a finite number of individual resonators ($N_{\text{el}}$) in parallel.

Increasing the finger coverage proved to be a counter-productive choice since larger values of $c$ are associated with a severe $k_t^2$ reduction, as reported in [72]. Differently from TFE modes, where a thinner piezoelectric film is required to increase the capacitance per unit area, thicker films help increasing the $C_0$ for devices operating in the LFE mode. Unfortunately, the increase in $C_0$ does not scale linearly with $t_{\text{LN}}$ as reported in Section 3.1.1. Furthermore, thicker film machining renders plate patterning more complex and leads to poorer sidewall angles, which are a fundamental feature in LVRs to attain high $Q_s$. For this reason, a maximum thickness of 2 µm was set as a limit for the fabrication of 50 MHz devices. Similarly to $c$, the choice of $L_e$ and $N_p$ impacts the anchor losses and therefore the overall $Q_s$. The effects of geometry on $Q_s$ can be reasonably investigated via Finite Element Analysis (FEA) to identify an optimal geometry.
The investigation reported in this Section is based on the preliminary work reported in [31] and [66]. The results highlighted will be further expanded and extended to higher frequency in the following sections.

4.1.1. COMSOL® Simulations

The device active region, or aperture ($L_e$), and the number of fingers pairs ($N_p$) were identified as the most interesting parameters to vary to increase the static capacitance ($C_0$) of individual resonators, as mentioned in Chapter 3 and in the introduction of the present chapter. The impact of these geometry features on the quality factor ($Q_s$) was investigated via COMSOL® Finite Element Analysis (FEA) to provide significative insights on the optimal design of X-cut LVRs. The energy leakage through the anchors was modeled via Perfectly Matched Layers (PMLs) strategically positioned around the resonator frame, as reported in Fig. 4.2.

Two sets of simulations were designed to independently verify the impact of $L_e$ and $N_p$ on the quality factor ($Q_s$). A $\lambda$ of 116 µm was set to ensure the device operation around 50 MHz. The aperture was swept from $3 \lambda$ to $13 \lambda$ for a constant number of fingers pairs equal to 3, while the $N_p$ was swept from 3 to 11 for a constant $L_e$ of $11 \lambda$. The fingers-to-bus gap ($g$) and the bus length ($L_b$) were set to 0.75 $\lambda$ according to previous investigation on AlN LVRs [74] to maximize acoustic...
reflection. Anchors width \((W_a)\) and length \((L_a)\) were set to 0.25 \(\lambda\). The LN substrate thickness was fixed to 2 \(\mu m\), which represents an optimal tradeoff between ease of fabrication and maximum achievable capacitance per unit area. Ultimately, the top electrode metal was fixed to 200 nm to reduce the impact of any series resistance \((R_s)\) without impacting the main resonance [75].

The aperture simulation (Fig. 4.3a) highlighted that longer plates for fixed \(N_p\) showcase higher \(Q_s\). The maximum \(Q_s\) (~25,000) was recorded for \(L_e = 11 \lambda\). Plates with aperture larger than 13 \(\lambda\) were not investigated due to concern regarding electrical loading and fabrication yield rate. While increasing \(N_p\) theoretically helps increasing the static capacitance of the device (Eq. 3.7 – 3.8), COMSOL® simulations showed a drastic reduction in \(Q_s\) (Fig. 4.3b) for devices with larger number of fingers. Intuitively, slender geometries help better confining the vibrational energy in the central region of the resonator (Fig. 4.4a vs. Fig. 4.4c), reducing anchor leakage phenomena.

Interestingly, the simulation of the expected \(Q_s\) as a function of \(L_e\) is strongly not monotone, showing large performance dips at \(L_e = 10\) and 12 \(\lambda\). This phenomenon can be explained by closely looking at the admittance response of the simulated resonators and at the mode shape exhibited by the plates at resonance (Fig. 4.4b). The strong anisotropy exhibited by the lithium niobate crystal,
which is reflected by the density of the piezoelectric matrices \[76\], makes LN LVRs more prone in exhibiting spurious modes compared to other material technologies. Transversal modes have proven to be particularly severe in conventional plate geometries, causing mode conversion \[77\] and \(Q_s\) degradation when too close to the resonant frequency. This phenomenon is very well captured by the mode shape reported in Fig. 4.4b for an aperture \(L_e\) equal to 10 \(\lambda\).

![Figure 4.4: Zoom-in of resonance peaks of simulated X-cut LN LVRs for different values of aperture: (a) \(L_e = 3 \lambda\); (b) \(L_e = 10 \lambda\); and (b) \(L_e = 11 \lambda\). Shorter plates exhibit more anchor losses (a) than slender plates (c).](image)

### 4.1.2. Experimental Plan and Fabrication

According to the results provided by COMSOL® FEA simulations, an optimized geometry to maximize \(Q_s\) while maintaining high \(k_t^2\) and modest \(C_0\) is identified (Fig. 4.5). Throughout this section, this geometry is referred to as the optimized or standard geometry. The design parameters of such device are reported in Table 4.1.

Starting from the standard resonators, four different experimental plans were designed to investigate the impact of geometry on \(Q_s\), varying one-factor-at-a-time: aperture \((L_e)\), anchor width \((W_a)\), gap \((g)\), and coverage \((c)\). Table 4.1 reports the variations of each parameter from the standard geometry. Since the optimized geometry shows a static capacitance \((C_0)\) of about 110 fF, according
to FEA simulations, an array of 9 resonators was also included in the tape-out to attain a device with a total capacitance of 1 pF.

Figure 4.5: COMSOL® model (with reference system) of the standard resonator with PML boundary conditions. The resonator parameters are listed in Table 4.1.

Table 4.1: Standard geometry parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (λ)</td>
<td>116 μm</td>
<td>Nominal frequency (f_s)</td>
<td>50 MHz</td>
</tr>
<tr>
<td>Aperture (L_e)</td>
<td>11 λ</td>
<td>Plate length (L)</td>
<td>14 λ</td>
</tr>
<tr>
<td>Number of pairs (N_p)</td>
<td>3</td>
<td>Plate width (W)</td>
<td>2 λ</td>
</tr>
<tr>
<td>Bus length (L_b)</td>
<td>0.75 λ</td>
<td>Gap (g)</td>
<td>0.75 λ</td>
</tr>
<tr>
<td>Anchor width (W_a)</td>
<td>0.25 λ</td>
<td>Anchor length (L_a)</td>
<td>0.25 λ</td>
</tr>
<tr>
<td>Coverage (c)</td>
<td>0.3</td>
<td>Substrate thickness (t_{LN})</td>
<td>2 μm</td>
</tr>
<tr>
<td>Metal thickness (t_m)</td>
<td>200 nm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Experimental parametric investigation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchor width (W_a)</td>
<td>0.125 – 0.25 – 0.375 – 0.5 – 0.675 0 – 0.75 λ</td>
</tr>
<tr>
<td>Gap^x (g)</td>
<td>0.25 – 0.75 – 1.25 – 1.75 – 2.25 – 2.75 λ</td>
</tr>
<tr>
<td>Coverage (c)</td>
<td>0.3 – 0.5 – 0.8</td>
</tr>
<tr>
<td>Aperture (L_e)</td>
<td>8 – 11 – 14 λ</td>
</tr>
</tbody>
</table>

^xFor the gap study, the length of the plate was fixed to L = 16 λ instead of 14 λ.
Devices were fabricated with the process described in Section 3.2.4 on a 2 µm X-cut LN substrate provided by NGK. Details of the fabrication process are reported in Appendix A.

SEM pictures of individual and arrayed fabricated resonators are reported in Fig. 4.6. Due to the dimensions of the resonators, the overetch (~0.5 µm) can be considered negligible (Fig. 4.7). The measured sidewall angle of fabricated devices (φ) is greater than 80° (Fig. 4.7a), a value which is commonly considered optimal for the minimization of losses and spurious modes generated by the backscattering of acoustic waves [78].

Figure 4.6: SEM pictures of: (a) individual X-cut LVRs. The depicted devices are part of the anchor variation test. Grounds were wire-bonded before measurements to provide a common reference to the VNA ports; and (b) a device constituted by 9 arrayed standard resonators. A large metal bus is implemented to connect the terminals of each device.

Figure 4.7: (a) Detail of the anchor region of a resonator; and (b) angled SEM image picturing the edge of a resonator. The measured sidewall angle (φ) is greater than 80°. Devices fabricated via ion milling experience irregular patterning in the corner region due to sharp-tip effect. Lift-off residuals are also noticeable, but their impact on performance was considered negligible given their small size with respect to the operating wavelength.
4.1.3. Device Characterization

Fabricated devices were wafer-probed in laboratory conditions \((T = 293 \text{ K} \text{ and } P = 760 \text{ mTorr})\), in vacuum \((T = 293 \text{ K}, P < 10^{-3} \text{ Torr})\) and at different temperatures (as low as 130 K) with GSG150 probes in a 2-port configuration to extract their characteristic response. S-parameters were recorded with an Agilent N5230A VNA and later converted into Y-parameters via software (MATLAB®). 95% Confidence Intervals (95% C.I., or \(2\sigma\)) are reported when more than 5 data points were collected.

The impact of device aperture \((L_e)\) is reported in Fig. 4.8a. A global optimum exists for \(L_e = 11 \lambda\) and confirms the intuition of the existence of a trade-off between anchor losses and the electrical loading introduced by the IDTs. This point of trade-off is expected to be a different value of \(L_e\) for different frequencies for a fixed geometry in terms of \(\lambda\), since \(R_s\) does not scale as \(R_m\) over frequency.

Anchors losses have been widely investigated as damping mechanism in AlN LVRs [44]. According to COMSOL® simulations, slender plates help attaining higher \(Q_s\) due to a better acoustic energy confinement. Under this hypothesis, the effect of anchors should be negligible for relatively long resonators \((L_e > 7 \lambda\) at 50 MHz). This effect was never shown in AlN LVRs due to the difficulty of fabricating devices with a high \(L/W\) ratio, mainly related to the intrinsic stress gradient introduced by the reactive sputtering techniques necessary to fabricate thin piezoelectric films. According to experimental data (Fig. 4.8b), \(W_a\) does not impact \(Q_s\) for devices with an \(L_e\) of 11 \(\lambda\).

As for the anchor width, the gap size \((g)\) was previously identified as a key parameter affecting the amount of energy leaking through the substrate [45], and ultimately \(Q_s\). Fabricated devices with fixed plate length \((L = 16 \lambda\) show a clear relationship between quality factor and gap, as reported
in Fig. 4.8c. Similarly to $L_e$, a larger gap is believed to facilitate the energy confinement in the resonator active region, hence enhancing $Q_s$ while reducing the energy leakage through the anchors. Interestingly, larger gaps do not impact the $k_t^2$, up to $g = 2.75 \lambda$.

![Figure 4.8](image1.png)

**Figure 4.8:** Impact of different geometrical variables on the $Q_s$ of fabricated 50 MHz devices: (a) aperture ($L_a$); (b) anchors width ($W_a$); and (c) finger-to-bus gap ($g$).

![Figure 4.9](image2.png)

**Figure 4.9:** Impact of electrode coverage ($c$) of fabricated 50 MHz devices on: (a) quality factor ($Q_s$); (b) electromechanical coupling ($k_t^2$); and (c) Figure of Merit.

Lastly, the effects of coverage on $Q_s$ were investigated. A larger coverage is desirable since it increase the achievable capacitance per unit area, both for resonators operating in TFE and LFE mode. Unfortunately, the optimal coverage that maximizes $k_t^2$ for $S_0$ mode X-cut LVRs is achieved for $c = 0.3$ as reported in previous publications [65]. As reported in Fig. 4.9a, there is no advantage
in operating with larger coverage because the quality factor closely follows the reduction in $k_t^2$ (Fig. 4.9b). This effect directly translates to a reduction of the Figure of Merit as $c$ increases (Fig. 4.9c).

Quality factors in the order of 4,500 - 5,000 were consistently recorded on devices with different geometries measured under vacuum, with an average increase of 20% from atmosphere condition (Fig. 4.10). Air damping proved to be a significant source of losses only above few Torrs, making commercially available vacuum package a viable solution to fully harness the performance of these devices (Fig. 4.10b).

Electromechanical couplings as high as 28% were recorded on devices with a coverage ($c$) equal to 0.3. The $k_t^2$ was estimated with the arctangent method (Eq. 4.1), which provides the most conservative results and complies with the BVD model reported in Section 2.1.3.

$$k_t^2 = \frac{\pi f_s}{2 f_p} \arctan \left( \frac{\pi f_s}{2 f_p} \right)$$  \hspace{1cm} (4.1)

As reported in [23], for low intrinsic $k_t^2$, methods tend to converge to a unique value. This approximation is no more valid when the electromechanical coupling surpasses 10%. Previous results on X-cut LVRs [66] showing $k_t^2$ as high as 30% utilized a different definition, reported by
Eq. 4.2. Adopting this definition for the devices under examination would provide values of $k_t^2$ as high as 31.5%, which has been reported in literature [79].

$$k_t^2 = \frac{\pi^2}{4} \left(1 - \frac{f_s}{f_p}\right)$$  \hspace{1cm} (4.2)

Measurements in vacuum of optimized devices highlighted resonators exhibiting both high $Q_s$ and $k_t^2$. The device showing the highest recorded Figure of Merit at 50 MHz, defined by Eq. 4.3 [26], is reported in Fig. 4.11. $k_{eff}^2$ represents the effective coupling coefficients [23] and is defined by Eq. 4.4. All the measured devices experience severe in-band spurious modes, which have been accurately predicted by COMSOL® FEA preliminary investigations. Interestingly, the quality factors of such spurious are reduced for lower gap values and maximized for larger $g$, as the $Q_s$ of the $S_0$ mode (Fig. 4.11b).

$$FoM = Q_s \left(\frac{k_{eff}^2}{1 - k_{eff}^2}\right)$$  \hspace{1cm} (4.3)

$$k_{eff}^2 = \frac{f_p^2 - f_s^2}{f_p^2}$$  \hspace{1cm} (4.4)

![Figure 4.11](image_url)

Figure 4.11: (a) Admittance response of the optimized X-cut LVR (measured in vacuum) showing the highest Figure of Merit recorded during the testing phase. COMSOL® simulation of the device under investigation is reported, showing good agreements between numerical prediction and experimental measurements. A feedthrough capacitance of 20 fF and a frequency shift of 0.2 MHz were added to consider the effects of non-idealities neglected by the FEA model; and (b) admittance response of X-cut LVRs with different $g$.  

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Cryogenic measurements performed between 130 K and 293 K on devices with different geometries found a strong linear dependency between the quality factor and the temperature (Fig. 4.12a), especially in the 130 K – 250 K range. AlSiCu resistivity was extracted from de-embedding structures included in the tape-out (Fig. 4.12b) as a function of temperature in the range 180 – 293 K. The high $Q_s$ recorded at 130 K (up to 10,000) and the linear behavior hints that the electrical loading introduced by the IDEs plays a major role as a damping factor in determining the measured $Q_s$. No measurements were performed below 130 K due to the sudden breakage of devices at the interface of the release region, likely due to stresses induced by thermal contraction (Fig. 4.13).

Starting from the cryogenic measurement of an optimized device, the linear and quadratic Temperature Coefficients of Frequency ($TCF_1$ and $TCF_2$, respectively) were extracted (Fig. 4.12c). The model adopted to describe the change in resonant frequency as a function of the temperature is captured by Eq. 4.5, where $f_{s0}$ represents the series resonant frequency measured at $T_0 = 293$ K.

Results are in line with the values reported in literature of X-cut Lithium Niobate resonators [72], with measured $TCF_1$ and $TCF_2$ of -75 ppm/K and -121 ppb/K², respectively.

$$f_s(T) = f_{s0}[1 + TCF_1(T - T_0) + TCF_2(T - T_0)^2]$$  \hspace{1cm} (4.5)

Figure 4.12: (a) Quality factor as a function of temperature for different investigated devices; (b) AlSiCu Resistivity ($\rho$) as a function of temperature; and (c) resonant frequency as a function of temperature.
Figure 4.13: SEM picture of tested X-cut LVRs failed during cryogenic tests due to stresses induced by thermal contraction. Note that in this set of devices, ground pads on opposite sides of the resonator were connected via wirebonds.

A wide-span measurement was also performed on the fabricated device exhibiting the best performance in terms of Figure of Merit (Fig. 4.14). Weak SH$_0$ mode and its first harmonic were recorded around 33 and 103 MHz, respectively. Odds harmonic of the S$_0$ mode were recorded at integer multiples of the anti-resonant frequency ($f_p$), as the 2$^{nd}$ and 4$^{th}$ harmonics. These modes proven to be very weak and highly damped and are likely present due to fabrication non-idealities.

Figure 4.14: Wide-span admittance response of an optimized X-cut LVR measured under vacuum. Identified modes and resonant frequencies ($f_r$) are reported. Harmonics of the S$_0$ mode are present at multiple integers of the anti-resonance frequency of the main mode ($f_p$).
Table 4.3 recaps $f_s$, $Q_s$, $k_t^2$ and the $f_s \cdot Q_s$ product for all the identified modes. $Q_s$ were recorded only for modes showing clear responses at resonance to allow 3-dB extraction. Harmonics of the $S_0$ mode, and in particular the 3$^{rd}$, 5$^{th}$, 9$^{th}$ and 11$^{th}$ showed particularly high $f_s \cdot Q_s$ products, comparable to values reported in literature [80], which could be exploited for the fabrication of low noise oscillators.

Table 4.3: Identified modes exhibited by an optimized X-cut LVR and main extracted parameters. Modes at 33.72, 102.99, 115.99, 240.35, and 406.1 MHz showed noisy or split peaks, which prevented the extraction of $Q_s$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Harmonic</th>
<th>$f_s$ [MHz]</th>
<th>$Q_s$</th>
<th>$k_t^2$</th>
<th>$f_s \cdot Q_s$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SH_0$</td>
<td>1</td>
<td>33.72</td>
<td>-</td>
<td>2.18%</td>
<td>-</td>
<td>Noisy peak</td>
</tr>
<tr>
<td>$S_0$</td>
<td>1</td>
<td>50.88</td>
<td>5,329</td>
<td>27.8%</td>
<td>2.7e11</td>
<td>Highest FoM recorded</td>
</tr>
<tr>
<td>$SH_0$</td>
<td>3</td>
<td>102.99</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Noisy peak</td>
</tr>
<tr>
<td>$S_0$</td>
<td>2</td>
<td>115.99</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Noisy peak</td>
</tr>
<tr>
<td>$S_0$</td>
<td>3</td>
<td>174.46</td>
<td>22,630</td>
<td>0.27%</td>
<td>4e12</td>
<td>-</td>
</tr>
<tr>
<td>$S_0$</td>
<td>4</td>
<td>240.35</td>
<td>-</td>
<td>0.13%</td>
<td>-</td>
<td>Noisy peak</td>
</tr>
<tr>
<td>$S_0$</td>
<td>5</td>
<td>285.7</td>
<td>5,300</td>
<td>1.06%</td>
<td>1.5e12</td>
<td>-</td>
</tr>
<tr>
<td>$S_0$</td>
<td>7</td>
<td>406.81</td>
<td>-</td>
<td>0.73%</td>
<td>-</td>
<td>Noisy peak</td>
</tr>
<tr>
<td>$S_0$</td>
<td>9</td>
<td>526.25</td>
<td>23,600</td>
<td>0.07%</td>
<td>1.3e13</td>
<td>-</td>
</tr>
<tr>
<td>$S_0$</td>
<td>11</td>
<td>640.18</td>
<td>16,410</td>
<td>0.15%</td>
<td>1e13</td>
<td>-</td>
</tr>
<tr>
<td>$S_0$</td>
<td>13</td>
<td>758.81</td>
<td>2,200</td>
<td>0.26%</td>
<td>1.7e12</td>
<td>-</td>
</tr>
</tbody>
</table>

4.1.4. Arrays Performance

The admittance response of an array measured in air constituted by 9 optimized elements is reported in Fig. 4.15a. Compared to an individual resonator, arrays showcase a sensibly lower $Q_s$ (~1,800 vs. 4,000) and thus a reduced Figure of Merit. Data collected from identical copies of the optimized resonator, not arrayed in a single structure (Fig. 4.15b) pointed out that frequency mismatch between elements is likely the cause of quality factor degradation experienced by arrays.

Table 4.4 recaps the statistical variations recorded for $f_s$, $Q_s$, $k_t^2$ and $C_0$ in 8 identical, adjacent resonators, and the resonant frequency recorded for each device. While measured $Q_s$, $k_t^2$ and $C_0$
variations can be considered modest, the mismatch in \( f_s \) is considered severe enough to impact the array performance.

![Admittance response of an array constituted by 9 optimized parallel LVRs, measured in air; and SEM image of identical standard resonators measured to characterize the statistical distribution of \( f_s, Q_s, k^2 \) and \( C_0 \).](image)

**Figure 4.15:** (a) Admittance response of an array constituted by 9 optimized parallel LVRs, measured in air; and (b) SEM image of identical standard resonators measured to characterize the statistical distribution of \( f_s, Q_s, k^2 \) and \( C_0 \).

**Table 4.4:** Statistical distribution of \( f_s, Q_s, k^2 \) and \( C_0 \) measured on 8 identical standard resonators. Resonant frequency as a function of the position are also reported.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Average (( \mu ))</th>
<th>Standard deviation (( \sigma ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant frequency (( f_s ))</td>
<td>50.844 MHz</td>
<td>0.004 MHz</td>
</tr>
<tr>
<td>Quality factor (( Q_s ))</td>
<td>3.900</td>
<td>50</td>
</tr>
<tr>
<td>EM coupling (( k^2 ))</td>
<td>0.27</td>
<td>0.003</td>
</tr>
<tr>
<td>Static capacitance (( C_0 ))</td>
<td>115.7 fF</td>
<td>0.89 fF</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Device</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_s ) [MHz]</td>
<td>50.8430</td>
<td>50.8412</td>
<td>50.84555</td>
<td>50.8410</td>
<td>50.8409</td>
<td>50.8444</td>
<td>50.8466</td>
<td>50.8486</td>
</tr>
</tbody>
</table>

To prove this hypothesis, an in-air break-test was performed on the resonator array. Devices were manually broken starting from the outer elements towards the inner ones. The admittance was measured after each iteration, for a total of 9 distinct measurements. As reported by Fig. 4.16a, \( Q_s \) sharply increases as the number of elements decreases, while \( k^2 \) remains overall constant (Fig. 4.16b). As expected, \( C_0 \) linearly varies with the number of elements (Fig. 4.16c).

The test pointed out that a relationship between the number of elements (\( N_e \)) and \( Q_s \) exists, likely due to frequency mismatch between parallel devices. Such source of degradation will be further investigated, among other losses mechanisms, in Section 4.3.5.
4.2. Design and Fabrication of Higher Frequency Devices

4.2.1. Experimental Plan and Fabrication

The incredibly high performance attained with 50 MHz X-cut LVRs, which exhibit Figure of Merits more than four times higher compared to commercially available resonators [13], fostered the investigation of devices at higher frequency, which can be exploited for IoT, filtering and time referencing applications. The experimental plan developed for 50 MHz devices was extended up to 1 GHz to validate the design and the results attained at low frequencies. Table 4.5 reports the investigated frequencies, including the selected wavelengths. COMSOL® FEA simulations were performed on the optimized geometry reported in Table 4.1 to identify the $\lambda$ that approached the desired operating frequencies. For each $f_s$, the relative pitch ($p$), finger width and reflector width (assuming $c = 0.3$), which also represents the minimum feature size of the device, are also reported.
Table 4.5: Investigated frequencies and rounded wavelengths for S₀ mode resonators.

<table>
<thead>
<tr>
<th>Frequency (f₀)</th>
<th>Wavelength (λ)</th>
<th>Pitch (p)</th>
<th>Fingers width (ε = 0.3)</th>
<th>Reflectors width (ε = 0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 MHz</td>
<td>116 µm</td>
<td>58 µm</td>
<td>17.4 µm</td>
<td>8.7 µm</td>
</tr>
<tr>
<td>100 MHz</td>
<td>56 µm</td>
<td>28 µm</td>
<td>8.4 µm</td>
<td>4.2 µm</td>
</tr>
<tr>
<td>200 MHz</td>
<td>30 µm</td>
<td>15 µm</td>
<td>4.5 µm</td>
<td>2.25 µm</td>
</tr>
<tr>
<td>400 MHz</td>
<td>15 µm</td>
<td>7.5 µm</td>
<td>2.25 µm</td>
<td>1.13 µm</td>
</tr>
<tr>
<td>550 MHz</td>
<td>11 µm</td>
<td>5.5 µm</td>
<td>1.65 µm</td>
<td>0.83 µm</td>
</tr>
<tr>
<td>800 MHz</td>
<td>8 µm</td>
<td>4 µm</td>
<td>1.2 µm</td>
<td>0.6 µm</td>
</tr>
<tr>
<td>1 GHz</td>
<td>6 µm</td>
<td>3 µm</td>
<td>0.9 µm</td>
<td>0.45 µm</td>
</tr>
</tbody>
</table>

Since preliminary results at 50 MHz pointed out that the electrical loading coming from the interconnects and the IDEs is one of the main sources of $Q_s$ degradation, a study on the variation of top electrode metal thickness ($t_m$) was performed via FEA. Investigations on AlN LVRs demonstrated the impact of different metals and thicknesses on damping (especially TED [81]) and electromechanical coupling. COMSOL® simulations (assuming $t_LN = 2 \mu$m, aluminum as top electrode metal and the optimal geometry reported in Table 4.1) highlighted a dependency between spurious modes and metal thickness ($t_m$). The analysis shows that thicker electrodes are associated with more severe in band spurious modes (Fig. 4.17) across the entire frequency range of interest.

**Figure 4.17:** COMSOL® simulated admittance response capturing the impact of Al thickness ($t_m$) on the spurious modes at (a) around 200 MHz; and (b) around 1 GHz. The out-of-band mode showcased at around 1.2 GHz for $t_m = 300 \text{ nm}$ is a thickness mode and it is not caused by the metal thickness.
Furthermore, the maximum tolerable thickness before the insurgence of unwanted modes is a function of the resonant frequency as reported in Fig. 4.18a, meaning that low $h/\lambda$ ratios can handle thicker electrodes, and further reduce $R$. In order to minimize the fabrication complexity, a thickness of 300 nm was selected. Interestingly, the $k_t^2$ is not affected by the metal thickness (Fig. 4.18b), likely due to the positioning of the electrodes at the anti-nodes of the generated standing wave [82].

Similarly to the experimental plan implemented for 50 MHz devices, a standard geometry was identified (Table 4.6) and variations were designed around such geometry. Few 50 MHz devices with different apertures ($L_0$) and gaps ($g$) were included in the tape-out (Table 4.7) to verify the results attained in previous runs. The main set of variations for each frequency under investigation is reported in Table 4.8 and includes more extended aperture, anchor width and gap tests compared to the preliminary tests reported in Section 4.1. The anchor width ($W_a$) test is performed in fact with three different levels of aperture, to quantify the impact of anchor losses on plates with different aspect ratios. The gap variation test is performed on plates with two different lengths (10 and 14 $\lambda$).
for similar reasons. Due to the compactness of devices operating at higher frequency, more data points were included both for $L_e$ and $g$ tests.

Table 4.6: Standard geometry constant parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pairs ($N_p$)</td>
<td>3</td>
<td>Plate width ($W$)</td>
<td>$2\lambda$</td>
</tr>
<tr>
<td>Bus length ($L_b$)</td>
<td>0.75 $\lambda$</td>
<td>Gap ($g$)</td>
<td>0.75 $\lambda$</td>
</tr>
<tr>
<td>Anchor width ($W_a$)</td>
<td>0.25 $\lambda$</td>
<td>0.75 $\lambda$ (&gt;400 MHz)</td>
<td>0.25 $\lambda$</td>
</tr>
<tr>
<td>Coverage ($c$)</td>
<td>0.3</td>
<td>Substrate thickness ($t_{LN}$)</td>
<td>1 - 2 $\mu$m</td>
</tr>
<tr>
<td>Metal thickness ($t_m$)</td>
<td>300 nm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Experimental plan for 50 MHz devices.

50 MHz variations experimental plan

<table>
<thead>
<tr>
<th>Variable (primary)</th>
<th>Variable (secondary)</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture ($L_e$)</td>
<td></td>
<td>$3 - 7 - 11\lambda$</td>
</tr>
<tr>
<td>Gap ($g$)</td>
<td></td>
<td>$0.25 - 0.75 - 2.25 - 3.75\lambda$</td>
</tr>
</tbody>
</table>

Table 4.8: Experimental plan for 100, 200, 550 and 800 MHz devices.

100, 200, 550, and 800 MHz variations experimental plan

<table>
<thead>
<tr>
<th>Variable (primary)</th>
<th>Value</th>
<th>Variable (secondary)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture ($L_e$)</td>
<td>$3 - 5 - 7 - 9 - 11 - 13\lambda$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Anchor width ($W_a$)</td>
<td>$0.25 - 0.5 - 0.75 - 1 - 1.25 - 1.5 - 1.75 - 2\lambda$</td>
<td>Aperture ($L_e$)</td>
<td>$3 - 7 - 11\lambda$</td>
</tr>
<tr>
<td>Gap$^I$ ($g$)</td>
<td>$0.25 - 0.75 - 1.25 - 1.75 - 2.25 - 2.75 - 3.25 - 3.75\lambda$</td>
<td>Plate length ($L$)</td>
<td>$10\lambda$</td>
</tr>
<tr>
<td>Gap$^{II}$ ($g$)</td>
<td>$0.25 - 0.75 - 1.25 - 1.75 - 2.25 - 2.75 - 3.25 - 3.75 - 4.25 - 4.75 - 5.25 - 5.75\lambda$</td>
<td>Plate length ($L$)</td>
<td>$14\lambda$</td>
</tr>
</tbody>
</table>

Experimental test on devices operating at 400 MHz and 1 GHz was further expanded, with the inclusion of coverage ($c$) variations in addition to a denser aperture ($L_e$) test (Table 4.9). Arrays with different numbers of elements ($N_{el}$) and repetition of devices were included for statistical variation analysis.
Table 4.9: Experimental plan for 400 MHz and 1 GHz devices.

<table>
<thead>
<tr>
<th>Variable (primary)</th>
<th>Value</th>
<th>Variable (secondary)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture ($L_e$)</td>
<td>$3 - 4 - 5 - 6 - 7 - 8 - 9$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$- 10 - 11 - 12 - 13 - 14$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anchor width ($W_a$)</td>
<td>$0.25 - 0.5 - 0.75 - 1 -$</td>
<td>Aperture ($L_e$)</td>
<td>$3 - 7 - 11 \lambda$</td>
</tr>
<tr>
<td></td>
<td>$1.25 - 1.5 - 1.75 - 2 \lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gap$^I$ (g)</td>
<td>$0.25 - 0.75 - 1.25 -$</td>
<td>Plate length ($L$)</td>
<td>$10 \lambda$</td>
</tr>
<tr>
<td></td>
<td>$1.75 - 2.25 - 2.75 -$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3.25 - 3.75 \lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gap$^{II}$ (g)</td>
<td>$0.25 - 0.75 - 1.25 -$</td>
<td>Plate length ($L$)</td>
<td>$14 \lambda$</td>
</tr>
<tr>
<td></td>
<td>$1.75 - 2.25 - 2.75 -$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3.25 - 3.75 - 4.25 -$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4.75 - 5.25 - 5.75 \lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coverage (c)</td>
<td>$0.1 - 0.2 - 0.3 - 0.4 -$</td>
<td>Aperture ($L_e$)</td>
<td>$3 - 7 - 11 \lambda$</td>
</tr>
<tr>
<td></td>
<td>$0.5 - 0.8$</td>
<td></td>
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</tr>
</tbody>
</table>

By a fabrication stand-point, the investigation of X-cut LN LVRs poses serious challenges. The width of the active reflectors, necessary to exploit the substrate high piezoelectric coupling, approaches sub-micron dimensions slightly above 400 MHz, assuming $c = 0.3$ (Table 4.5). This fact, combined with the need of increasing the top electrode thickness to minimize $R_s$, makes the lift-off of the IDTs an impractical solution. To accommodate for smaller feature sizes, the process flow described in Section 4.1.2 was modified, as depicted in Fig. 4.19.

Figure 4.19: Modified fabrication process for X-cut LVRs: (1) SAB substrate fabrication; (2) top electrode sputtering and patterning via RIE; (3) ion milling patterning; and (4) XeF$_2$ isotropic release.
In this improved fabrication process, a thin film (300 nm) of AlSiCu is first sputtered on top of the LN surface. The IDTs and interconnects are then patterned via Cl-based Reactive Ion Etching (RIE) using a dark field mask and a positive, thin resist (S1805) to resolve minimum feature sizes up to 500 nm (step 2 in Fig. 4.19). The fabrication of the IDTs before the LN etching process also minimizes any contamination of the top surface, which can impact the adhesion of the metallic film and thus the resonator performance. The top electrode is then masked by the thick PR layer used to pattern the resonator plate, to protect it during the ion milling step (step 3 in Fig. 4.9). Devices are ultimately released via XeF$_2$ isotropic etch (step 4 in Fig. 4.9). A more accurate description of the fabrication process is reported in Appendix A.

Despite fabrication process improvements, reflectors patterning above 400 MHz still posed severe challenges due to the proximity of the minimum feature size to the maximum resolution allowed by the i-line stepper currently available at the CMU Nanofab Facility (0.5 µm). To bypass this issue, devices at 550 MHz, 800 MHz and 1 GHz were taped-out with reflectors having the same width as the central fingers. In this configuration, named “self-aligned reflectors”, the correct dimensioning of the outer IDTs relies on the patterning of the resonator plates via ion milling, which effectively serves to etch the unnecessary portion of the reflectors. A comparison between the regular reflector layout and the self-aligned version is reported in Fig. 4.20. This approach heavily relies on the alignment accuracy of the lithographic tool, and on the accurate prediction of the lateral overetch (Fig. 4.20) during ion milling. An overetch of 0.5 µm was added on each edge of the resonator anchors and plate according to previous experiments. A simplified analytical model for the calculation of the overetch is reported in Appendix E.
Fig. 4.21 reports various examples of fabricated devices on 1 µm and 2 µm thick X-cut LN films, at different frequencies. Devices in the range 50 – 550 MHz proven to be working, validating both the approaches for the electrode patterning. Unfortunately, due to misalignment during the lithographic definition of the resonator plates, devices operating above 550 MHz showcase a partially etch reflector. Such misalignment was quantified to be 200 nm and is visible both in devices operating at relatively low frequency (200 MHz, Fig. 4.22a-b) and high frequency (1 GHz, Fig. 4.22c). The impact of the misalignment is particularly severe in the latter case, since the damage sensibly narrows one of the edge active reflectors, disconnecting it from the bus. This effect causes a change in the electrical boundary condition, which results in the excitation of different modes with lower coupling coefficients and the insurgence of split peaks.
Figure 4.21: SEM of fabricated X-cut LVRs operating around: (a) 100 MHz; (b) 200 MHz; (c) 400 MHz; and (d) 550 MHz.

Figure 4.22: Effect of fabrication process non-idealities on 200 MHz: (a) right reflector; (b) left reflector. The 200 nm misalignment between the etch mask and the top electrode mask is clearly visible. (c) Effects of fabrication non-idealities on a 1 GHz device. The etching of the right electrode caused low yield rate and failure on most of the devices.
4.2.2. Device Characterization

Fabricated devices were characterized with the approach described in Section 4.1.3. The investigation focused on devices fabricated on 1 µm, which showed a better yield and more consistent results after preliminary measurements. Measurements in atmosphere, vacuum and at cryogenic temperature (50 K) were performed on most devices. Compared to previous cryogenic measurements on 50 MHz resonators, devices at higher frequencies did not suffer from breakage due to thermally induced stresses. A subset of devices, consisting in anchor variations (with an aperture of 3 and 7 λ) and aperture variations were measured from laboratory conditions (293 K) to cryogenic temperature (50 K) in step of 40 K to evaluate the change in $Q_s$ as a function of temperature ($T$). Few devices were measured at a base temperature of 10 K.

![Figure 4.23: (a) Measured electromechanical coupling ($k^2$) as a function of $h/\lambda$ ratio; and (b) dispersion curve measured on X-cut LN (YZ 30°) LVRs.](image)

The measured average electromechanical coupling ($k^2$) and the average speed of propagation ($v_p$) evaluated during preliminary measurements are reported in Fig. 4.23. The results are consistent with the values reported in literature [62] and the COMSOL® simulations summarized in Fig. 4.18. Fig. 4.24 reports four scatterplots showing the measured quality factor in air and in vacuum for
every tested device, divided by resonant frequency. As expected, air damping has a larger impact on devices operating at lower frequency. Air damping will be analyzed in detail in Section 4.3.1.

The effect of the aperture \( L_e \) on the quality factor is reported in Fig. 4.25. The measurements closely follow the trends previously reported for 50 MHz resonators. In general, a global maximum can be identified for each different frequency. As discussed in Section 4.1.3, the point of maximum \( Q_s \) represents a tradeoff between anchor losses and electrical loading.

\[ a. \]

\[ b. \]

Figure 4.25: Effect of the aperture \( L_e \) on \( Q_s \) for devices measured: (a) in air; (b) in vacuum.
For a fixed geometry, electrical losses do not scale with frequency, being dependent as first approximation by the number of squares and the metal thickness. Vice versa, $R_m$ does scale inversely with frequency if $C_0$ and the FoM are assumed constant, meaning that higher frequency devices experience larger resistive losses. For this reason, devices at lower frequencies showcase lower $Q_s$ for smaller values of $L_e$.

**Figure 4.26:** Effect of the anchors width ($W_a$) on the quality factor measured in air for devices with an aperture of: (a) $L_e = 3 \lambda$; (b) $L_e = 7 \lambda$; and (c) $L_e = 11 \lambda$.

Fig. 4.26 and 4.27 report the results of anchor width ($W_a$) variations in air and vacuum, respectively. $W_a$ was swept from $0.25 \lambda$ to $2 \lambda$ (fully anchored configuration) for three different levels of aperture (3, 7 and 11 $\lambda$). As highlighted in Fig. 4.26a and 4.27a, shorter devices are more prone in exhibiting lower $Q_s$ for larger values of $W_a$, while longer devices seem less affected ($L_e = 7 \lambda$, Fig. 4.26b and 4.27b) or show the opposite trend ($L_e = 11 \lambda$, Fig. 4.26c and 4.27c). The measurements confirm the intuition reported in Section 4.1 that anchor losses are a strong function of the ratio between the length ($L$) and the width ($W$) of the plate. Intuitively, smaller values of $W_a$ prevent energy leaking via the substrate in shorter resonators, thus increasing $Q_s$. In larger resonator,
shorter anchors introduce more electrical loading, which partially explains the trends reported in Fig. 4.26b-c and 4.27b-c.

![Figure 4.27](image1.png)

Figure 4.27: Effect of the anchors width ($W_a$) on the quality factor measured in vacuum for devices with an aperture of: (a) $L_e = 3\lambda$; (b) $L_e = 7\lambda$; and (c) $L_e = 11\lambda$.

Measurements in air and vacuum of gap ($g$) variations on plates with a total length ($L$) of 10\,$\lambda$ and 14\,$\lambda$ are reported in Fig. 4.28 and 4.29, respectively.

![Figure 4.28](image2.png)

Figure 4.28: Effect of the fingers-to-bus gap ($g$) for resonator with a plate length ($L$) of 10\,$\lambda$ on: (a) $Q_s$, measured in air; (b) $Q_s$, measured in vacuum; and (c) electromechanical coupling ($k_t^2$).
Trends closely follows the results measured on 50 MHz devices. For \( L = 10 \lambda \), \( Q_s \) is maximized for \( g = 2.75 \lambda \), and abruptly drops for large fingers-to-bus distances (Fig. 4.28a-b). The electromechanical coupling \( (k_t^2) \) follows a similar trend (Fig. 4.28c). Measurements of gap variations on resonators with \( L = 14 \lambda \) (Fig. 4.29), reported maximum \( Q_s \) around \( 3.25 - 3.75 \lambda \) (except for \( f_s = 200 \text{ MHz} \)), as well as \( k_t^2 \) reduction beyond this optimal point. Since the plate length is fixed, the aperture decreases as the gap increases according to Eq. 4.6. Since \( L_b = 0.75 \lambda \), an optimal \( L_e \) of 3 \( \lambda \) and 5 \( \lambda \) can be calculated for \( L = 10 \lambda \) and 14 \( \lambda \), respectively. This means that an aperture of at least 0.3 – 0.35 \( L \) is necessary to ensure an optimal transduction of the piezoelectric material.

\[
L_e = L - 2(g + L_b)
\]

(4.6)

Figure 4.29: Effect of the fingers-to-bus gap (g) for resonator with a plate length (L) of 14 \( \lambda \) on: (a) \( Q_s \), measured in air; (b) \( Q_s \), measured in vacuum; and (c) electromechanical coupling \( (k_t^2) \).

Coverage variations on 400 MHz devices highlighted similar trends to the subset investigated at 50 MHz (Fig. 4.30). The electromechanical coupling \( (k_t^2) \) is maximized for \( c = 0.3 \), as widely reported in literature [72]. \( Q_s \) is maximized between \( c = 0.3 \) and \( c = 0.5 \), hinting that slightly larger
coverages can be implemented without Figure of Merit degradation, increasing the capacitance per unit area.

![Figure 4.30](image) Effects of coverage (c) on X-cut LVRs operating around 400 MHz on: (a) $Q_s$; and (b) $k_t^2$.

The best measured devices (in vacuum) for each frequency are reported in Fig. 4.31 - 4.33. Figure of Merits of 2,688, 1,741, 964 and 751 were recorded for devices operating at 100, 200, 400 and 550 MHz, respectively. At the time of this thesis, these values represent the highest FoM ever recorded for devices operating at Very High Frequency (VHF) and Ultra High Frequency (UHF). Good $Q_s$ (> 1,000) and FoMs (> 240) were also recorded on few devices operating at 800 MHz and 1 GHz, despite the poor active reflectors patterning (Fig. 4.22)

![Figure 4.31](image) Fabricated X-cut LVRs operating around: (a) 100 MHz; and (b) 200 MHz.
A subset of devices (aperture test and anchor tests with $L_e = 3 \lambda$ and $7 \lambda$) at different frequencies (100, 200, and 400 MHz) was monitored throughout the entire cooling process to 50 K, in steps of 40 K, to characterize the $Q_s$ variation as a function of frequency. Fig. 4.34 shows the results of the aperture variations at different frequencies and temperatures. On average, devices operating at lower frequencies and with larger apertures show more consistent $Q_s$ increments. Trends appear to be roughly linear, considering that a normalized standard deviation ($\sigma/\mu$) of 8% was measured on high $Q_s$ (> 15,000) resonators.
This phenomenon hints that electrical loading plays a fundamental role in determining the quality factor of devices at room temperature, when anchor losses are minimized (i.e., when \( L_e > 3 \lambda \)). Measurements of anchor variations (Fig. 4.35 and 4.36) strongly support this hypothesis, showing trends that are consistent with the aperture test reported in Fig. 4.34.
Figure 4.36: Quality factor of anchors variations (with fixed $L_e = 7 \lambda$) measured at different temperatures for devices operating: (a) around 100 MHz; (b) around 200 MHz; and (c) around 400 MHz.

The best cryogenic measurements recorded for devices operating around 100 MHz is reported in Fig. 4.37, with a zoom-in of the resonance peak. The highest $Q_s$ recorded ($> 26,000$) is comparable with the anchor losses estimated by COMSOL models (Fig. 4.3).

Figure 4.37: Admittance response of the resonator exhibiting the recorded highest $Q_s$ at base temperature (10 K).
4.3. Characterization of Damping Mechanism

4.3.1. Air Damping

Air damping was estimated according to the air and vacuum measurements reported in the previous section (scatterplots of Fig. 4.24). The air damping \( Q_{\text{air}} \) was estimated according to Eq. 4.7, where \( Q_{\text{atm}} \) and \( Q_{\text{vac}} \) represent the resonator quality factors measured in air and in vacuum, respectively.

The measured median \( Q_{\text{air}} \) is reported in Fig. 4.38. The median is adopted instead of a simple mean to minimize the impact of outliers and measurement uncertainty. The measured \( Q_{\text{air}} \) was later fit to a square root function (Eq. 4.8). Trends are consistent with simple viscous damping measurements reported in literature [73].

\[
Q_{\text{air}} = \frac{1}{\left(\frac{1}{Q_{\text{atm}}} - \frac{1}{Q_{\text{vac}}}\right)}
\]  

(4.7)

\[
Q_{\text{air}}(f) = a\sqrt{f} + b
\]  

(4.8)

Figure 4.38: Extracted quality factor \( (Q_{\text{air}}) \) due to air damping.
4.3.2. Series Resistance Modeling

Devices which exhibit a large Figure of Merit (FoM) combined with relatively large capacitance ($C_0$) are extremely susceptible to any series resistance ($R_s$) introduced by interconnects and IDEs due to their low motional resistance ($R_m$, Eq 2.8). If $R_m$ is comparable or lower than $R_s$, the measured or loaded quality factor ($Q_s$) is mainly limited by the electrical loading rather than any other damping mechanism. It is possible to estimate the unloaded quality factor ($Q_u$) according to Eq. 4.9 [74], where $R_m$ represents the measured motional resistance, while the difference between $R_m$ and $R_s$ determine the unloaded motional resistance.

$$Q_u = Q_s \frac{R_m}{R_m - R_s}$$  \hspace{1cm} (4.9)

In order to properly estimate $Q_u$, a series resistance ($R_s$) model was developed by dividing the distributed loading of the resonator into smaller lumped components. Fig. 4.39 shows a direct comparison between the resonator topology and the equivalent resistive network.

![Figure 4.39: Equivalent resistive network of a LFE LVR.](image)
All the components beside the loading introduced by the IDTs were estimated according to Ohm’s law (Eq. 4.10), where $L$, $W$ and $t$ represent respectively the length, width and thickness of a metallic trace. The modeled resistance ($R_s$, Eq. 4.11) accounts for the resistance introduced by the pads ($R_{pad}$), pads-anchor routing ($R_{route}$), anchors ($R_{anc}$), bus ($R_{bus}$), fingers extending in the gap region ($R_{gap}$, Eq. 4.12), and by the IDEs ($R_{IDE}$, Eq. 4.13). The model of the interdigitated transducers is based on the work reported in [83].

$$R = \frac{\rho L}{W t} \quad (4.10)$$

$$R_s = 2 \left( R_{pad} + R_{route} + R_{anc} + \frac{R_{bus}}{4} + R_{gap} \right) + R_{IDE} \quad (4.11)$$

$$R_{gap} = \frac{2}{\sum \frac{1}{R_{f_i}}} = \frac{4 \rho g}{\lambda c t_m (N_p + 1)} \quad (4.12)$$

$$R_{IDE} = \frac{4 \rho L_e}{3 \lambda c t_m N_p} \quad (4.13)$$

Table 4.10 reports the parameters adopted for $L$ and $W$ for each lumped resistance considered in the model. $L_{pad}$, $W_{pad}$ and $L_{route}$ respectively represent the length and width of the probing pads and the length of the metallic trace connecting the pads to the anchors (Fig. 4.21a). An example of the value of each resistance of the described mode is also reported, along with the geometry of the analyzed resonator. For the case under consideration, most of the series resistance is introduced by the IDEs, which contributes for roughly 60% of the overall electrical loading. While the resistances introduced by traces can be minimized up to the bus region by increasing the metal thickness, the gap and the transduction region are particularly sensitive to mass loading effects, which can ultimately lead to FoM degradation and the insurgence of spurious modes.
Table 4.10: Lengths and widths of each element of the resistive network depicted in Fig. 4.39. For the estimation of the resistance of each component, the following geometry was investigated: $\lambda = 56 \, \mu m$, $L_{\text{route}} = 76 \, \mu m$, $L_{\text{pad}} = 80 \, \mu m$, $L_a = W_a = g = L_b = 0.75\lambda$, $L_e = 13\lambda$, $N_p = 3$, $c = 0.3$, $t_m = 300 \, \text{nm}$, and $\rho = 5.7 \times 10^{-8} \, \Omega \cdot \text{m}$.

<table>
<thead>
<tr>
<th>Element</th>
<th>Length ($L$)</th>
<th>Width ($W$)</th>
<th>Resistance ($R$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{pad}}$</td>
<td>$0.5L_{\text{pad}}$</td>
<td>$W_{\text{pad}}$</td>
<td>0.10 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{route}}$</td>
<td>$L_{\text{route}}$</td>
<td>$0.5(W_{\text{pad}} + W_a)$</td>
<td>0.30 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{anc}}$</td>
<td>$L_a$</td>
<td>$W_a$</td>
<td>0.19 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{bus}}$</td>
<td>$W$</td>
<td>$L_b$</td>
<td>0.13 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{f}}$</td>
<td>$g$</td>
<td>$0.5\lambda c$</td>
<td>0.95 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{refl}}$</td>
<td>$g$</td>
<td>$0.25\lambda c$</td>
<td>1.9 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{gap}}$</td>
<td>-</td>
<td>-</td>
<td>0.48 $\Omega$</td>
</tr>
<tr>
<td>$R_{\text{IDE}}$</td>
<td>$L_e$</td>
<td>$0.5\lambda c$</td>
<td>3.67 $\Omega$</td>
</tr>
<tr>
<td>$R_{s}$</td>
<td>-</td>
<td>-</td>
<td>6.04 $\Omega$</td>
</tr>
</tbody>
</table>

An equivalent resistive model for an array of resonators was also developed starting from the $R_s$ estimated for the individual device (Fig. 4.15b). Compared to a single resonator, the array topology reported in Fig. 4.40 is constituted by a fixed number of elements ($N_{\text{el}}$) connected via a long metallic bus. The distributed resistance introduced by the interconnects can be modeled via smaller lumped resistances ($R_i$) connecting the anchors of each parallel resonators. In this case, the resistance of the single resonator ($R_{\text{res}}$) is slightly lower than $R_s$ (Eq. 4.11) since $R_{\text{pad}}$ and $R_{\text{route}}$ are accounted for the entire array, and not for each parallel resonator.

Figure 4.40: Equivalent resistive network of an array constituted by parallel LFE LVRs.
4.3.3. Impact of Electrical Loading

The series resistance model described in Section 4.3.2 was applied to understand the results of the aperture variations tests for which various resonator quality factors were measured in vacuum ($Q_{\text{vac}}$) and at base temperature (10 K, $Q_{10\text{K}}$). A contact resistance ($R_{c0}$) of 2 Ω, extrapolated from de-embedding structures with different lengths, was added to the $R_s$ reported in Eq. 4.11. The unloaded quality factor ($Q_u$) was later calculated for devices operating at 100, 200, 400, and 550 MHz according to Eq. 4.14.

$$Q_u = Q_{\text{vac}} \frac{R_{\text{mes}}}{(R_{\text{mes}} - R_s - R_{c0})}$$

(4.14)

Results are reported in Fig. 4.41 and 4.42. $Q_u$ closely follows the measured data at cryogenic temperature ($Q_{10\text{K}}$) for all the frequencies under examination. Discrepancies between the unloaded quality factor ($Q_u$) and the quality factor measured at base temperature ($Q_{10\text{K}}$) are likely due to measurement errors in vacuum (e.g., see $L_e = 11 \lambda$ at 100 MHz, Fig. 4.41a). As expected, longer devices are more vulnerable to electrical loading than shorter resonators.

Figure 4.41: Quality factor measured in vacuum ($Q_{\text{vac}}$) and at base temperature ($Q_{10\text{K}}$), and unloaded quality factor ($Q_u$) for different apertures at: (a) 100 MHz; and (b) 200 MHz.
4.3.4. Anchor Losses

Ultimately, the impact of anchor geometry on the quality factor is investigated. Fig. 4.43 reports the measured quality factor at base temperature (10 K) on anchor width variations for different frequencies and apertures. At this temperature, any thermally-related damping mechanism (including electrical loading and TED) are considered negligible compared to the acoustic losses due to energy leaking in the substrate (Eq. 4.15).

\[ Q_{s|10K} = \frac{1}{\sum \frac{1}{Q_i}} \sim Q_{anch} \]  \hspace{1cm} (4.15)

Fig. 4.43 reports the measured quality factors. The trends closely follow the results recorded in atmosphere and vacuum, meaning that anchor losses are the most limiting mechanism for shorter plates, while they do not play a significant role for slender resonators. Quality factors recorded on devices exhibiting \( L_e = 7 \lambda \) and 11 \( \lambda \) (Fig. 4.43b and 4.43c) do not show significant trends, likely due to the impact of fabrication non-idealities and mode conversion phenomena.
Figure 4.43: Quality factor as a function of anchor width ($W_a$) measured at base temperature (10 K) for different frequencies and values of aperture: (a) $L_e = 3 \lambda$; (b) $L_e = 7 \lambda$; (c) $L_e = 11 \lambda$.

Fully anchored devices were proven to exhibit less aggressive spurious responses, likely due to the suppression of transversal modes at the expenses of maximum attainable quality factor (especially for smaller values of $L_e$). Fig. 4.44 reports the characteristic response of three resonators with different anchors ($W_a = 0.25 \lambda$ and $2 \lambda$)

Figure 4.44: Admittance response of devices with different anchor width ($W_a = 0.25$ and $2 \lambda$) and an aperture of: (a) $L_e = 3 \lambda$; (b) $L_e = 7 \lambda$; and (c) $L_e = 11 \lambda$.

Fig. 4.45 reports the comparison between the vacuum measurements ($Q_{vac}$) and cryogenic measurements ($Q_{10K}$) performed on two subsets of devices operating around 400 MHz with different
apertures and anchors widths. As reported before, shorter anchors width \( (W_a = 0.25, \text{Fig. 4.45a}) \) devices with aperture \( L_e < 5\lambda \) exhibit larger anchor losses, due to the larger amount of energy leaking via the substrate. As the aperture increases, the quality factor measured at room temperature tends to decrease, while the \( Q_s \) measured at base temperature roughly flattens between 4,000 and 5,000, meaning that the electrical loading is the dominant damping mechanism. Different from devices with \( W_a = 0.25\lambda \), fully anchored resonators show a monotone increase in their \( Q_s \) at cryogenic temperature, meaning that the transition between the anchor losses dominated region and the electrical loading dominated region is smoother, due to the larger amount of energy leaking through the larger tethering width. For large values of aperture \( (L_e > 11\lambda) \), the measured \( Q_s \) is substantially independent from the anchor geometry, meaning that almost no energy escapes the central portion of the resonator.

Interestingly, fully anchored devices do not show sharp drops in the quality factor as \( L_e \) increases, hinting that spurious modes play a fundamental role in determining the \( Q_s \) of some geometries, as previously suspected.

![Figure 4.45: Quality factor measured in vacuum \( (Q_{vac}) \) and at base temperature \( (Q_{10K}) \), and unloaded quality factor \( (Q_u) \) for different apertures and anchor width dimensions: (a) \( W_a = 0.25\lambda \); and (b) \( W_a = 2\lambda \).](image)
4.3.5. Frequency Mismatch in Arrays of Resonators

Measurements reported in Section 4.1.4 highlighted the impact of the number of elements \((N_{el})\) of an array of resonators on the measured quality factor \((Q_a)\). Resonant frequency \((f_s)\) mismatch between elements is suspected to be the primary source of \(Q_a\) degradation. To prove this hypothesis, the effect of statistically distributed parameters \((f_s, Q_s, k_t^2,\) and \(C_0)\) on the quality factor of an array constituted by \(N_{el}\) elements operating around 50 MHz is investigated with a Monte Carlo approach. \(N_{el}\) is varied from 1 to 9 to provide a direct comparison with the results reported in Section 4.1.4. A number of Monte Carlo simulations \((N_{MC})\) equal to 1,000 is performed for each \(N_{el}\), for a total of 9,000 tests.

For each \(N_{el}\), individual resonators are randomly generated according to the statistical distribution of \(f_s, k_t^2, Q_s\) and \(C_0\) reported in Table 4.4, which are then used to calculate randomly distributed motional parameters \((R_m, L_m\) and \(C_m)\). The admittance of each array \((Y_{array})\) is then estimated by superimposing the response of the \(N_{el}\) randomly generated resonator responses (Eq. 4.16). The quality factor of the simulated array \((Q_{as})\) is later extracted with the 3 dB method from the admittance response.

\[
Y_{array} = \sum_{i=1}^{N_{el}} Y_i = \sum_{i=1}^{N_{el}} \left( \frac{1}{f_\omega C_0 + \frac{1}{R_m + j_\omega L_m + \frac{1}{f_\omega C_m}}} \right)
\]  

(4.16)

The average array quality factor \((\mu_{Q_{as}})\) and its standard deviation \((\sigma_{Q_{as}})\) are then calculated from the \(N_{MC}\) test replicates for each \(N_{el}\). Fig. 4.46 reports the results of the Monte Carlo simulation (in form of box plots) compared to the \(Q_{as}\) measured in the break-test reported in Section 4.1.4. For low number of elements, the measured quality factor closely follows the value predicted by the
Monte Carlo simulation. Experimental data diverge from the theoretical model for larger number of elements due to the impact of interconnects as $N_{el}$ increases (as explained in next paragraph).

Figure 4.46: Monte Carlo simulations of an array of resonators generated from the randomly distributed parameters listed in Table 4.4 and recorded $Q_{sa}$ as a function of the number of elements.

One-factor-at-time (OFAT) Monte Carlo simulations, in which a single parameter is varied while the standard deviations of the others are set equal to zero, confirm that the mismatch in resonant frequency is the predominant $Q_{sa}$ degradation mechanism (Fig. 4.47).

Figure 4.47: One-factor-at-time Monte Carlo simulations showing the impact of $Q_s$, $k^2$, $f_s$, and $C_0$ variability on the $Q_{sa}$: (a) average quality factor ($\mu_{Q_{sa}}$); and (b) standard deviation ($\sigma_{Q_{sa}}$).
The Monte Carlo model previously described is further refined by including the impact of the interconnects connecting the array elements. The model closely follows the resistive electrical model developed for arrays of resonators described in Section 4.3.2. In the case under examination, the resistance of the resonator \( R_{res} \) is replaced by the randomly generated \( R_m \). This approximation is valid since the randomly generated \( R_m \) is based on measurements performed in air and is not de-embedded of the series resistance \( R_s \) component. In this case, for each \( N_{el} \), the array admittance \( Y_{array} \) is calculated according to Eq. 4.17, where \( Y_{N_{el}} \) represents the last randomly generated resonator response \( Y_i \), while \( Y_{brx} \) and \( Y_{blx} \) respectively represents the admittances of the right and left branches of a symmetric device as the one reported in Fig. 4.1.2. The generic expression of the admittance of a branch \( Y_b \) is reported by Eq. 4.18, where \( N_b \) represents the number of elements in a branch \( i.e., \, 4 \) for an arrays of 9 elements), and \( R_t \) the trace resistance, estimated from the distance between the individual resonators anchors and the interconnects width and thickness. For the arrays reported in Section 4.1.4, \( R_t \) is estimated in 1.25 Ω, given an anchor-anchor distance of 3 \( \lambda \) (348 \( \mu m \)), an interconnect width of 100 \( \mu m \) and an AlSiCu thickness of 192 nm.

\[
Y_{array} = Y_{N_{el}} + Y_{brx} + Y_{blx} \quad (4.17)
\]

\[
Y_b = \sum_{i=1}^{N_b} \left[ \left( \frac{1}{Y_i} + \frac{1}{Y_{i-1}} + 2R_t \right) + Y_i \right] \quad (4.18)
\]

Fig. 4.48 reports the comparison between the measured data and the Monte Carlo simulation performed considering the impact of the metallic bus. Compared to Fig. 4.46, the experimental data almost coincide with the expected value of the estimated \( Q_a \) distribution, confirming that resistive losses due to interconnects play an important role in limiting the quality factor of arrays of resonators.
Figure 4.48: Modified Monte Carlo simulations of an array of resonators generated from the randomly distributed parameters listed in Table 4.4 and recorded $Q_{as}$ as a function of the number of elements, including the effects of the interconnects.

Arrays operating around 400 MHz devices, fabricated on 1 μm X-cut Lithium Niobate, are also measured to validate the hypothesis that frequency mismatch is direct cause of $Q_{as}$ degradation. As reported in Fig. 4.49, devices with different number of elements ($N_0$) show multiple peaks at resonance, confirming that frequency mismatch between elements is the fundamental reason limiting the quality factor of those devices. Resonators operating at higher frequencies are more prone to exhibit larger frequency variations due to their smaller features size, which make them more sensitive to substrate and metal thickness variations and fabrication non-idealities. Table 4.11 reports the statistical characterization of identical devices operating around 400 MHz with the optimized geometry reported in Table 4.6 and an aperture of $7\,\lambda$. The resonant frequency standard deviation ($\sigma_{f_0}$) is estimated to be 357 kHz, or 900 ppm, more than 10 times the value recorded for 50 MHz devices [13].

Table 4.11: Statistical characterization of devices operating at 400 MHz, with the geometry described in Table 4.6 and $L_e = 7\,\lambda$.

<table>
<thead>
<tr>
<th>Device</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$\mu_{f_0}$</th>
<th>$\sigma_{f_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
<td>MHz</td>
</tr>
<tr>
<td>397.735</td>
<td>398.483</td>
<td>397.996</td>
<td>398.198</td>
<td>398.245</td>
<td>397.853</td>
<td>397.414</td>
<td>397.989</td>
<td>0.3574</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.49: (a) Admittance responses of arrays of resonators operating around 400 MHz with different number of elements in parallel; and (b) zoom-in on the resonance peaks.

A hybrid analytical-experimental model is later developed to understand whether lithium niobate and metal thickness variations can be considered one of the major sources of these frequency variations. Devices with the optimized geometry reported in Table 4.6 are simulated via COMSOL® FEA operating around 50 MHz ($\lambda = 116$ µm) and 400 MHz ($\lambda = 15$ µm) and their resonant frequency ($f_r$) extracted for different LN thickness (0.5, 1, 2, and 3 µm) and top electrode metal thicknesses (100, 200, 300, and 400 nm of Al). Simulated data are later fitted with a spline surface ad reported in Fig. 4.50.

Figure 4.50: Spilt surface fitting of the resonant frequencies extracted from COMSOL® simulations for devices operating: (a) around 50 MHz; and (b) around 400 MHz.
The substrate thickness is characterized with a similar approach from the data provided by the vendor (Table 4.12), by fitting the data point with a spline surface. Fig. 4.51 reports the comparison between the fitting and actual X-cut LN wafer.

Table 4.12: Lithium Niobate thickness distribution from NGK.

<table>
<thead>
<tr>
<th>Point</th>
<th>LN thickness [nm]</th>
<th>Point</th>
<th>LN thickness [nm]</th>
<th>Point</th>
<th>LN thickness [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1540.3</td>
<td>8</td>
<td>1029.4</td>
<td>15</td>
<td>1009.8</td>
</tr>
<tr>
<td>2</td>
<td>1044.1</td>
<td>9</td>
<td>1135.0</td>
<td>16</td>
<td>903.9</td>
</tr>
<tr>
<td>3</td>
<td>1114.0</td>
<td>10</td>
<td>1378.0</td>
<td>17</td>
<td>1379.5</td>
</tr>
<tr>
<td>4</td>
<td>1351.5</td>
<td>11</td>
<td>1027.2</td>
<td>A</td>
<td>1403.4</td>
</tr>
<tr>
<td>5</td>
<td>1567.0</td>
<td>12</td>
<td>1149.1</td>
<td>B</td>
<td>1456.5</td>
</tr>
<tr>
<td>6</td>
<td>1445.0</td>
<td>13</td>
<td>1432.5</td>
<td>C</td>
<td>1247.6</td>
</tr>
<tr>
<td>7</td>
<td>1154.1</td>
<td>14</td>
<td>1324.1</td>
<td>D</td>
<td>1273.4</td>
</tr>
</tbody>
</table>

Figure 4.51: Comparison between: (a) the spline surface fitting of the data reported in Table 4.12; and (b) the LN wafer used during the fabrication process.

Ultimately, the metal thickness ($t_m$) variations across the wafer are characterized via direct measurement, assuming a parabolic 3D distribution over a wafer of radius $R$. Eq. 4.19 captures the dependency of $t_m$ as a function of the cartesian coordinates $x$ and $y$ for a given wafer radius ($R$), metal thickness measured at center of the wafer ($t_{max}$), and metal thickness variation between the center and the wafer edge ($\Delta t_m$). Fig. 4.52 reports an example of simulated $t_m$ distribution assuming $t_{max} = 300$ nm, $\Delta t_m = 40$ nm and $R = 2\"$. 
\[ t_m(x, y) = \left( -\frac{\Delta t_m}{R^2} \right) (x^2 + y^2) + t_{m,\text{ax}} \] (4.19)

Figure 4.52: Simulated parabolic 3D distribution of sputtered metal thickness.

An algorithm is later developed to estimate the standard deviation of the resonant frequency as a function of the LN and top electrode metal thickness distributions. A random couple of \(x\) and \(y\) is selected within a circumference with a diameter of 3”, to exclude the outer region of the wafer. Starting from this point, a number of points (\(N_p\)) set equal to 8 is sampled in the horizontal direction and the respective \(x_i\) and \(y_i\) recorded. These coordinates are used to extract the LN and metal thicknesses for each of the \(N_p\) points (according to the thickness profiles reported in Fig. 4.51 and 4.52) and estimate a statistical distribution for the \(t_{LN}\) and \(t_m\). Starting from random values of \(t_{LN}\) and \(t_m\) extracted from these statistical distributions, the correspondent value of \(f_s\), derived from the spline surface fitting reported in Fig. 4.50, are extracted. The test is repeated a number of times (\(N_f\)) equal to 1,000 to estimate the statistical distribution of \(f_s\). For 50 MHz devices, a distance between resonators of 3 \(\lambda\) is set for the sampling, while a \(t_{\text{max}}\) of 192 nm and a center-edge thickness variation of 40 nm is assumed according to experimental data. For 400 MHz devices, a distance equal to 4 \(\lambda\) is set for the sampling, and a \(t_{\text{max}}\) of 290 nm with a center-edge variation of 60 nm is adopted for the metal distribution estimation. LN thickness distributions are fitted according to the data.
provided by the manufacturer. Fig. 4.53 reports the results of frequency simulations performed on 50 MHz and 400 MHz devices as a function of the number of tests ($N_f$). The standard deviation ($\sigma_b$) for 50 MHz devices predicted by the model is equal to 2.704 kHz, versus a measured $\sigma$ of 4 kHz. 400 MHz simulated devices showed a predicted $\sigma_b$ of 89.383 kHz versus a recorded standard deviation of 397 kHz. While 50 MHz simulations closely follow the experimental data, 400 MHz predicted value is sensibly lower than experimental records.

The simulations confirm that lithium niobate and electrode metal thickness variations play a critical role in determining the resonant frequency of X-cut LN LVRs. These problematics can be addressed by an improvement of the thin film transfer technology, which is already implementing polishing and trimming to maintain the thickness variations in the order of few tens of nm. The discrepancies between measured and predicted frequency distributions can attributed to the variability induced by lithographic and etching processes on the resonator geometrical dimensions. These fabrication non-idealities have a larger impact on higher frequency devices, as reported in Fig. 4.22, and could be partially addressed by utilizing more advanced lithographic equipment.

![Resonant Frequency vs. Number of Tests](image1.png)  
Figure 4.53: Simulated resonant frequencies as a function of the number of tests for: (a) 50 MHz devices; and (b) 400 MHz devices.
4.3.6. Conclusions on Damping Mechanism

In the present chapter, the main damping mechanism impacting X-cut LN LVRs operating between 50 and 550 MHz were investigated. Compared to traditional AlN LVRs, lithium niobate resonators experience larger losses due to electrical loading. The intrinsic quality factor of the piezoelectric thin film (> 26,000 at 100 MHz) and the large $k_t^2$ available for the mode of interest, minimize the motional resistance ($R_m$), making these devices more prone to experience $Q_s$ degradation due to parasitic series resistances. No noticeable effects due to TED were recorded during the cryogenic measurements. The quality factor degradation over frequency, which is strongly highlighted in Fig. 4.35 and 4.36, hints that other losses mechanisms are playing a fundamental role in determining the $Q_s$ of higher frequency devices.
CHAPTER 5: X-cut LVRs as Matching Networks

5.1. 1-port LVRs Gain

In Chapter 4, X-cut LVRs with record-breaking Figure of Merits are reported, making them an extremely appealing technology for the implementation of MEMS-based matching network. To assess the performance of fabricated devices, the gain provided by resonators implemented in the RFFE matching network are simulated from the gathered experimental data. To properly simulate the expected gain, it is necessary to fully characterize any parasitics between the path signal and the ground. A more accurate and widely used model for the matching network implementing a series 1-port resonator is depicted in Fig. 5.1. The model introduces a parasitic parallel resistance ($R_p$) and capacitance ($C_p$) to ground on both the input and the output port. These are the two main parasitic components that will have a direct impact on the RFFE gain.

![Figure 5.1: Modified model of the MEMS-based matching network described in Section 2.1.3 including parasitic resistances ($R_p$) and capacitances ($C_p$) to ground.](image)

While $C_p$ has been estimated in few tens of fF [66] and can be easily lumped into the design of $C_{load}$, the estimation of $R_p$ results more challenging due to the limited dynamic range of most commonly used measurement instruments, including the VNA adopted for the device characterization. VNA measurements using an Intermediate Frequency (IF) bandwidth of 100 Hz
to reduce noise effects showed no differences between the $R_p$ measured on an on-chip open test structure and the $R_p$ measured with the probes lifted from the substrate, effectively limiting the maximum measurable value of $R_p$ to be between 50 and 100 kΩ. Such value is clearly not compatible with experimental measurements of gains provided by devices fabricated on similar substrates and with identical pads configuration [5], since an $R_p$ of 50 kΩ would shunt most of the current provided by the resonator to ground. For this reason, the extracted Y-parameters matrix cannot be readily used to estimate the passive voltage gain provided by the resonator to a given $C_{\text{load}}$. The model used in this section is captured by Eq. 5.1, where $Z_{\text{load}}$ represents the impedance to ground at the output node (Eq. 5.2) and $Z_R$ the impedance of the resonator, estimated from the measured $Y_{12}$ data (Eq. 5.3).

\[
G \approx \frac{Z_{\text{load}}}{Z_{\text{load}} + Z_R + R_{\text{in}}} \tag{5.1}
\]

\[
Z_{\text{load}} = \frac{1}{(j\omega C_{\text{load}} + j\omega C_p + \frac{1}{R_p})} \tag{5.2}
\]

\[
Z_R = -\frac{1}{Y_{12}} \tag{5.3}
\]

Both $C_p$ and $R_p$ can be to first approximation modeled as the parasitics between the signal and the ground pads, as reported in Fig. 5.2. Simulations performed on ADS Momentum, assuming a LN thickness ($t_{\text{LN}}$) of 2 µm, a Si thickness of 500 µm and a dissipation factor (tan$\delta$) of 0.001 and 0.004 respectively for LN and Si, are reported in Fig. 5.3.

At 100 MHz, the resistance to ground extracted from the simulation is close to 140 dB, equal to 10 MΩ, while the parasitic capacitance was estimated to be about 20 fF, as confirmed by direct experiment. Assuming the validity of the reported simulation, $R_p$ and $C_p$ can both be neglected for
the gain simulation due to their high impedance compared to the variable of interest ($C_{load}$) at the frequencies under investigation.

Figure 5.2: Simplified model of the parasitics ($R_p$ and $C_p$) between the signal (S) and ground (G) pads.

Figure 5.3: ADS Momentum simulations of: (a) parasitics resistance to ground ($R_p$); and (b) parasitic capacitance to ground ($C_p$) as a function of frequency.

5.1.1. 100 MHz LVRs Gain Simulation

The admittance response of the best measured device (in vacuum) operating around 100 MHz is reported in Fig. 5.4. With a $Q_s$ greater than 8,000 and a $k_t^2$ of nearly 30%, it represents the resonator exhibiting the highest Figure of Merit (2,688) reported in literature [79] for devices operating in the Very High Frequency (VHF) range. The same reported $Y_{12}$ data is used to simulate
the gain provided to a capacitive load when implementing the device in the matching network topology described in Section 2.1.3.

![Figure 5.4: Admittance response (measured in vacuum) of the resonator exhibiting the highest Figure of Merit (FoM) ever recorded for a device in the VHF range.](image)

\[
\begin{align*}
Q_s &= 8.279 \\
R_m &= 13.5 \ \Omega \\
L_m &= 141.1 \ \mu\text{H} \\
C_m &= 16.3 \ \text{fF} \\
FoM &= 2.688 \\
\end{align*}
\]

Fig. 5.5 reports the results of the voltage amplification simulations, which are performed in MATLAB®. Four different capacitive loads (300, 500, 700 fF, and 1 pF) are selected to track the gain as the operating frequency \(f_{\text{RF}}\) moves from the anti-resonance \(f_p\) to the resonant frequency \(f_s\). Numerical simulation based on the device fitting reported in Fig. 5.5 are represented by dashed lines to highlight the differences from the theoretical solution.

Interestingly, the simulated gain attained for smaller capacitive loads is sensibly lower than the expected theoretical one (i.e., 300 fF), while closely match the expected value for larger load values. This phenomenon can be explained by the presence of in-band spurious modes, which are known to cause gain degradation and peak splitting by interfering with the main mode. For a \(C_{\text{load}}\) of 300 fF and is in fact possible to distinguish two different peaks, one around 107 MHz, and another one closer to 108.5 MHz. Peak splitting and spurious modes impact on gain will be investigated more
in depth in Section 5.2. The discrepancy between the operating frequencies of simulated and theoretical peaks, which is estimated in 0.1 MHz (<0.1%) is due to approximations in the estimations of the BVD motional parameters ($L_m$ and $C_m$).

![Graph of gain vs frequency showing simulated and theoretical peaks]

**Figure 5.5:** MATLAB® simulations of the expected gain as a function of the capacitive load ($C_{load}$) according to $Y_{12}$ parameters (solid line) and BVD model fitting (dashed line).

![Graphs showing gain as a function of parasitic resistance for different loads]

**Figure 5.6:** Simulated gain as a function of different parasitic resistances to ground ($R_p$) for a load of: (a) 300 fF; (b) 700 fF; and (c) 1 pF.
Ultimately, the impact of $R_p$ on the simulated gain is investigated. Four different values of the parasitic impedances (50, 250, and 500 kΩ, and 5 MΩ) are simulated for three different capacitive loads (300 and 500 fF, and 1 pF). Results are reported in Fig. 5.6. As expected, low values of $R_p$ impacts smaller loads by limiting the maximum achievable gain, which is ultimately set by the ratio between the load impedance and the source. For loads in the order of 1 pF, an $R_p$ greater than 1 MΩ is considered sufficient to avoid significant gain reduction for large $C_{load}$ (> 300 fF).

5.1.2. Considerations on Arrays

As thoroughly described in Section 4.3.5, arrays of resonators show a strong dependency between the quality factor ($Q_{as}$) and the number of elements ($N_{el}$) due to frequency mismatch between the individual devices. Fig. 5.7a shows the simulated gain of an array of devices exhibiting the $Q_{as}$ reported in Fig. 5.7b and the $k^2$ and $C_0$ reported in Fig. 4.16, interfaced with a $C_{load}$ of 1 pF. The voltage amplification is maximized for a number of arrayed resonators varying between 3 and 5, while sensibly drops for larger values of $N_{el}$ and for individual devices. As discussed in Section 2.1.3, there is a suboptimal region in which the resonator is still able to provide large passive voltage amplification to a capacitive load, without being perfectly matched. The optimization reported in Section 2.1.3 is based on the hypothesis that $Q_s$ is independent of $C_0$. Since it is impossible to arbitrarily synthetize resonators with the desired static capacitance while attaining maximum $Q_s$, a trade-off between $C_0$ and $Q_s$ exists. For this reason, operating with a smaller number of arrayed elements is often advantageous by a gain standpoint, since operating in the suboptimal region with a higher FoM overcomes the $Q_s$ degradation introduced by a larger $N_{el}$. 

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5.2. Effects of Spurious Modes on Gain

Spurious modes are defined as any undesired mode exhibited by microelectromechanical resonators. In-band spurious modes are especially detrimental in filtering applications, since they increase the transmission losses in the pass band, causing signal attenuation and distortion. For this reason, the origin of such modes has been widely investigated [84][85], such as techniques to mitigate their impact [86] or predict their emergence [87]. As highlighted in Section 5.1, unwanted modes are harmful also for resonators implemented as narrow band matching networks, since they can cause peak splitting, frequency shift and gain attenuation compared to spurious-free devices.

Unfortunately, the large intrinsic $k_t^2$ and the strong asymmetricity of the LN lattice makes X-cut LVRs particularly prone to exhibiting in and out-of-band spurious modes, as widely reported in literature [82]. Recent results showed devices fabricated on thin film of LN with spurious-free responses [15][88], but with limited $Q_s$ compared to the devices operating at similar frequencies presented in Section 4.
The simplest approach to model the behavior of a resonator exhibiting spurious modes is via a Multi-BVD model (Fig. 5.8), in which each unwanted mode is represented by a parallel resonant branch [23]. A comparison between a spurious-free response (BVD model) and a Multi-BVD approach is reported in Fig. 5.9. The simulations are performed according to the measured data of a 50 MHz device (Table 5.1).

![Multi-BVD model of a 1-port LVR.](image)

Figure 5.8: Multi-BVD model of a 1-port LVR. $R_{ms}$, $L_{ms}$, and $C_{ms}$ represent the motional parameters of the spurious modes.

![Simulated admittance response and phases of the simulated resonators.](image)

Figure 5.9: (a) Simulated admittance response of 1-port LVR operating around 50 MHz considering only the main $S_0$ mode (blue) and including 7 in-band spurious modes (red); and (b) phases of the simulated resonators.

<table>
<thead>
<tr>
<th>$i$-th Mode</th>
<th>Frequency, $f_i$ [MHz]</th>
<th>Frequency, $f_p$ [MHz]</th>
<th>Quality Factor, $Q_s$</th>
<th>EM coupling, $k^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ($S_0$)</td>
<td>51.63</td>
<td>59.28</td>
<td>5.831</td>
<td>0.3</td>
</tr>
<tr>
<td>1</td>
<td>52.1677</td>
<td>52.2204</td>
<td>58.186</td>
<td>0.0025</td>
</tr>
<tr>
<td>2</td>
<td>53.215</td>
<td>53.244</td>
<td>7.067</td>
<td>0.0013</td>
</tr>
<tr>
<td>3</td>
<td>53.305</td>
<td>53.3235</td>
<td>28.490</td>
<td>0.0009</td>
</tr>
<tr>
<td>4</td>
<td>54.55</td>
<td>54.690</td>
<td>490</td>
<td>0.0063</td>
</tr>
<tr>
<td>5</td>
<td>55.965</td>
<td>55.98</td>
<td>2.294</td>
<td>0.0007</td>
</tr>
<tr>
<td>6</td>
<td>56.32</td>
<td>56.352</td>
<td>1.206</td>
<td>0.0014</td>
</tr>
<tr>
<td>7</td>
<td>57.75</td>
<td>57.785</td>
<td>439</td>
<td>0.0015</td>
</tr>
</tbody>
</table>
The impact of spurious modes on the admittance of a resonator when interfaced with a capacitive load can be illustrated by using a simplified model which accounts for a single spurious mode located closely to the operating frequency \( f_{RF} \). A capacitive load of 100 fF is selected for a resonator exhibiting \( Q_s, k_i^2 \) and \( f_s \) for the main mode as reported in Table 5.1. In case of spurious free response, the operating frequency is purely set by the capacitive load and it is close to 56.68 MHz. If a spurious mode with \( f_{ss} = 55.6 \text{ MHz}, \ Q_{ss} = 5,000, \) and \( k_{ss}^2 = 0.2\% \) is introduced in the model as an additional resonant branch, the proximity between the operating frequency of the main mode and the spurious causes a split peak (Fig. 5.10a), which directly translates into a pulling of the operating frequency and reduction of the maximum achievable gain (Fig. 5.10b). As a secondary effect, the presence of a spurious mode close to the anti-resonance causes a pulling of \( f_p \), as shown in Fig. 5.9a.

**Figure 5.10:** (a) Simulated admittance responses of a resonator without spurious modes (blue), with spurious modes (red), and after pulling in presence of spurious modes (orange); and (b) split peak due to the presence of an in-band spurious mode.
In this section, an analytical modeling of the pulled anti-resonant and operating frequency is presented to provide a physical explanation of the phenomenon. The effect of spurious modes key parameters \(Q_{ss}, k_{ts}^2, \text{ and } f_{ss}\) on the gain provided by a 1-port LVR when implemented in the matching network described in Section 2.1.3 is also investigated. A high \(Q\) resonator, exhibiting the modes reported in Table 5.1, is used as an example throughout the entire section.

5.2.1. Frequency Pulling Modeling

As highlighted in Fig. 5.94, the presence of spurious modes close to the anti-resonance causes the pulling of \(f_p\), which complicates the extraction of \(k_t^2\) [23]. Assuming \(R_m = R_{ms} = 0\), the admittance of a two-branch Multi-BVD equivalent circuit can be modeled as in Eq. 5.4, where \(\omega_{ss}\) represents the resonant frequency of the spurious mode.

\[
Y = j\omega C_0\left(1 - \frac{\omega^2}{\omega_s^2}\right)\left(1 - \frac{\omega^2}{\omega_{ss}^2}\right) + j\omega C_m\left(1 - \frac{\omega^2}{\omega_{ss}^2}\right) + j\omega C_{ms}\left(1 - \frac{\omega^2}{\omega_{ss}^2}\right)
\]

\[
\left(1 - \frac{\omega^2}{\omega_s^2}\right)\left(1 - \frac{\omega^2}{\omega_{ss}^2}\right)
\]

\(5.4\)

Intuitively, the series resonances of the main and the spurious modes remain unaltered since they are represented by two conjugated couples of zeros. To calculate the anti-resonance, it is necessary to set the numerator equal to zero. Such equation can be rewritten as in Eq. 5.5, where \(\omega_p\) and \(\omega_{pa}\) (Eq. 5.6 and 5.7) represent the unpulled anti-resonant frequencies of the main and of the spurious modes, respectively. A more detailed derivation of the model is reported in Appendix B.

\[
\omega^4 - \omega^2(\omega_p^2 + \omega_{ps}^2) + \omega_s^2 \omega_p^2 + \omega_s^2 \omega_{ps}^2 - \omega_s^2 \omega_{ss}^2 = 0
\]

\(5.5\)

\[
\omega_p = \omega_s \sqrt{\frac{C_0 + C_m}{C_0}}
\]

\(5.6\)
\[ \omega_{ps} = \omega_{ss} \sqrt{\frac{C_0 + C_{ms}}{C_0}} \]  

(5.7)

An analytical expression can be found for the roots of Eq. 5.5, which respectively represent the pulled anti-resonance frequency of the main mode (\(\omega'_p\), Eq. 5.8) and of the spurious mode (\(\omega_{ps}'\), Eq. 5.9). Interestingly, the two pulled anti-resonances are equidistant from a central frequency (\(\omega_{pc}\)), which is function of \(\omega_p\) and \(\omega_{ps}\) (Eq. 5.10), proportionally to a distance factor (\(\delta_p\), Eq. 5.11).

\[ \omega'_p = \sqrt{\frac{\omega_{pc}^2}{2} \left( \omega_p^2 + \omega_{ps}^2 \right) - 4 \left( \omega_{ss}^2 \omega_p^2 + \omega_p^2 \omega_{ps}^2 - \omega_{ps}^2 \omega_{ss}^2 \right)} \]  

(5.8)

\[ \omega'_{ps} = \sqrt{\frac{\omega_{pc}^2}{2} - \left( \omega_p^2 + \omega_{ps}^2 \right) - 4 \left( \omega_{ss}^2 \omega_p^2 + \omega_p^2 \omega_{ps}^2 - \omega_{ps}^2 \omega_{ss}^2 \right)} \]  

(5.9)

\[ \omega_{pc} = \sqrt{\frac{\omega_p^2 + \omega_{ps}^2}{2}} \]  

(5.10)

\[ \delta_p^2 = \frac{1}{2} \sqrt{\left( \omega_p^2 + \omega_{ps}^2 \right) - 4 \left( \omega_{ss}^2 \omega_p^2 + \omega_p^2 \omega_{ps}^2 - \omega_{ps}^2 \omega_{ss}^2 \right)} \]  

(5.11)

Assuming constant \(\omega_s\), \(\omega_p\), and \(k_{ts}^2\), a variation of \(\omega_{ss}\) (and consequently of \(\omega_{ps}\)), generates an increment of \(\delta_p^2\) when \(\omega_{ps}\) approaches \(\omega_p\), causing the divergence of the roots.

This phenomenon is captured by Fig. 5.11a, which represents the contour plot of the admittance response of a simulated resonator as a function of frequency (\(f\)), when varying the spurious mode anti-resonant frequency (\(f_{ps}\)). The numerical simulation is performed assuming a \(f_s\) of 51.63 MHz, a \(Q_s\) of 5,800, a \(k^2\) of 30%, and a \(C_0\) of 115.9 pF for the main mode (as in Table 5.1), while the spurious quality factor (\(Q_{ss}\)) is set to 20,000, and its \(k_{ts}^2\) to 0.63%. The resonant frequency of the spurious mode is calculated from the anti-resonance according to Eq. 4.1. As reported in Fig. 5.11a, the spurious mode has no influence on the anti-resonance of the main mode up to roughly 57 MHz,
when both $f'_p$ and $f'_{ps}$ start diverging. The analytical derivation of the pulled anti-resonant frequencies (Eq. 5.8 and 5.9) closely follow the numerical simulation (Fig. 5.11b).

**Figure 5.11:** (a) Contour plot of the admittance of a resonator as a function of the anti-resonance frequency of a spurious mode ($f_s = 50$ MHz, $Q_s = 5,000$, $k_t^2 = 0.3$ and $C_0 = 70$ fF); and (b) analytical simulation of the frequency pulling.

![Contour plot and analytical simulation](image)

If a capacitive load is interfaced in series with the 1-port resonator (Fig. 5.12), as in the matching network series configuration described in Section 2.1.3, the resonant frequency of the main mode ($f_s$) and of the spurious mode ($f_{ss}$) are no longer decoupled and influence the operating frequency of the system. A mathematical expression similar to Eq. 5.4 can be found by setting the denominator of the admittance of the circuit represented in Fig. 5.12 equal to zero (Eq. 5.12). An equation similar
to Eq. 5.5 (Eq. 5.13) can be then derived, where \( \omega_{RF} \) (Eq. 5.14) and \( \omega_{RFs} \) (Eq. 5.15) represents the operating resonant frequency of the main and spurious modes, respectively, as described in Section 2.1.3.

\[
j\omega \left[ (C_{load} + C_0) \left( 1 - \frac{\omega_s^2}{\omega_{ss}^2} \right) + C_m \left( 1 - \frac{\omega_s^2}{\omega_{ss}^2} \right) + C_{ms} \left( 1 - \frac{\omega_s^2}{\omega_{ss}^2} \right) \right] = 0
\]

(5.12)

\[
\omega^4 - \omega^2(\omega_{RF}^2 + \omega_{RFs}^2) + \omega_{RF}^2 \omega_{ss}^2 + \omega_{RFs}^2 \omega_s^2 - \omega_s^2 \omega_{ss}^2 = 0
\]

(5.13)

\[
\omega_{RF} = \omega_s \sqrt{\frac{C_{load} + C_0 + C_m}{C_{load} + C_0}}
\]

(5.14)

\[
\omega_{RFs} = \omega_{ss} \sqrt{\frac{C_{load} + C_0 + C_{ms}}{C_{load} + C_0}}
\]

(5.15)

The mathematical derivation of the pulled resonance frequency of the main (\( \omega_{RF}' \), Eq. 5.16) and spurious (\( \omega_{RFs}' \), Eq. 5.17) modes follows the same steps described to find \( \omega_p' \) and \( \omega_{ps}' \). Even in this case, the two pulled resonant frequencies are equidistant from a central pulled resonant frequency (\( \omega_{RFc} \)) defined as in Eq. 5.18, proportionally to a distance factor (\( \delta_{RF} \), Eq. 5.19).

\[
\omega_{RF}' = \sqrt{\omega_{RFc}^2 - \delta_{RF}^2}
\]

(5.16)

\[
\omega_{RFs}' = \sqrt{\omega_{RFc}^2 + \delta_{RF}^2}
\]

(5.17)

\[
\omega_{RFc} = \sqrt{\frac{\omega_{RF}^2 + \omega_{RFs}^2}{2}}
\]

(5.18)

\[
\delta_{RF}^2 = \frac{1}{2} \left( \frac{(\omega_{RF}^2 + \omega_{RFs}^2) - 4(\omega_{ss}^2 \omega_{RF}^2 + \omega_s^2 \omega_{RF}^2 - \omega_s^2 \omega_{ss}^2)}{2} \right)
\]

(5.19)

The effect of a moving spurious mode on the pulled operating frequencies (\( f_{RF}' \) and \( f_{RFs}' \)) was investigated using the same approach described for the pulling anti-resonant frequencies. A load of 150 fF was introduced in the simulation to pull the main mode around 54 MHz. Similarly to the
results described by Fig. 5.11a, the operating frequency of the main mode \( f_{RF} \) gets pulled to lower frequencies until the anti-resonance is far enough (~ 1 MHz) from the peak of the main mode (Fig. 5.13a). As in Fig. 5.11b, the analytical model captured by Eq. 5.16 and 5.17 for \( f_{RF}' \) and \( f_{RFs}' \) matches the numerical simulation (Fig. 13b).

**Figure 5.13**: (a) Contour plot of the admittance of a pulled resonator as a function of the anti-resonance frequency of a spurious mode \( (f = 50 \text{ MHz}, Q = 5,000, k_r^2 = 0.3 \text{ and } C_0 = 70 \text{ fF}) \); and (b) analytical simulation of the frequency pulling.

The analytical and numerical models developed in this section hint that while a complete suppression of spurious modes would be desirable by an application stand-point, their presence could be tolerated if such modes are positioned far enough from the operating frequency of the loaded resonator (or from its anti-resonance if implemented in a parallel configuration, as in the transformer described in Section 2.1.5).
5.2.2. Impact of Spurious Modes Key Parameters on Gain

The impact of the spurious mode key parameters \( f_{ss}, Q_{ss} \) and \( k_{ts}^2 \) is numerically investigated assuming a spurious-free device operating around 50 MHz with the characteristics described in Table 5.1. A series capacitance of 100 fF is added in series with such resonator to set its operating frequency \( \omega_{RF} \) to 55.68 MHz, as described in the introduction of the present chapter.

Fig. 5.14a reports the effect of the spurious resonant frequency \( f_{ss} \) on the gain peak due to the main mode (blue line) and the unwanted mode (red line). \( Q_{ss} \) and \( k_{ts}^2 \) are respectively fixed to 5,000 and 0.2%, while \( f_{ss} \) is expressed as a frequency variation \( \Delta \) from the undisturbed operating frequency \( f_{RF} \) according to Eq. 5.20. As the spurious mode approaches \( f_{RF} \), the gain of the main mode starts dropping, while the gain due to the unwanted mode increases. The worst case scenario is represented by a perfectly split peak, in which both the gains have the same magnitude.

![Figure 5.14: (a) Magnitude of the voltage amplifications generated by the main mode peak (blue line) and spurious mode peak (red line) as a function of the frequency distance between modes; and (b) frequency pulling as a function of the frequency distance between a main and a spurious mode.](image-url)
As highlighted in Fig. 5.14a, a frequency shift of 2% is enough to keep the peaks separated (Fig. 5.14b) and limit the gain degradation introduced by the spurious mode.

\[
\Delta = 1 - \frac{f_{ss}}{f_{RF}} \tag{5.20}
\]

A variation in the quality factor of the spurious mode \(Q_{ss}\) impacts both the maximum achievable gain and the bandwidth over which the voltage amplification is reduced. Low \(Q_{ss}\) spurious modes are more deleterious than high \(Q_{ss}\) ones, since they cause a larger gain reduction of the main peak and increase in the bandwidth over which the gain is degraded (Fig. 5.15).

**Figure 5.15:** Magnitude of the voltage amplifications generated by the main mode peak and the spurious mode peak as a function of spurious mode quality factor \(Q_{ss}\).

**Figure 5.16:** Magnitude of the voltage amplifications generated by the main mode peak and the spurious mode peak as a function of spurious mode EM coupling \(k_{ts}^2\)
The spurious modes electromechanical coupling \((k_o^2)\) has a similar effect on the bandwidth, while does not sensibly affect the maximum attainable gain (Fig. 5.16).

In conclusion, if a spurious mode cannot be fully suppressed, high \(Q_{ss}\) and low \(k_o^2\) spurious modes which are at least shifted by 2% from the spurious-free operating frequency are more tolerable by the matching network described in Section 2.1.3. For this reason, apodization techniques may not be an optimal solution since they extensively rely on the reduction of spurious modes quality factors to limit their impact [86]. While this approach is desirable for filtering applications, it worsen the impact of unwanted modes if they are not fully suppressed for the application targeted by this dissertation. One possible solution is to vary the geometrical dimensions affecting the spurious mode locations (i.e., finger-to-bus gap) once a \(C_{load}\), and thus a \(f_{RF}\), is set.

![Graphs showing simulated gain and ratio between gain with spurious and spurious-free gain](image)

**Figure 5.17:** (a) Simulated gain implementing the spurious-free BVD model (blue line) and the Multi-BVD model (red line) reported in Fig. 5.9 for a load of 1 pF; and (b) ratio between the gain in presence of spurious \((G_s)\) vs. spurious-free gain \((G_{sf})\).

These considerations can be extended to a full Multi-BVD model, as the one described in the introduction of the present chapter. In this case, the capacitive load \((C_{load})\) is swept from 10 fF to 2 pF. As reported in Fig. 5.17a, the spurious-free gain \((G_{sf}\) which accounts only for the main mode)
is maximized around a $C_{\text{load}}$ of 150 fF, which represents the optimal load for a resonator with a $C_0$ of 115.9 fF. As expected, the gain drops as the load increases, due to the lower impedance transformation ratio. If spurious modes are present, the voltage gain ($G_s$) shows a large degradation for capacitive loads up to 500 fF. This phenomenon occurs since spurious modes close to the anti-resonance have experimentally proven to exhibit lower $Q_s$ than unwanted modes close to $f_s$. Fig. 5.17b reports the ratio between the gain attained in presence of spurious versus the spurious-free maximum attainable gain. These consideration hints that a trade-off between spurious modes and resonator size exists, and that it is convenient to operate in the suboptimal gain region (i.e., with smaller resonators) for large loads to increase $f_{RF}$, moving the operating frequency away from the anti-resonance frequency. Two numerical examples for the resonator under examination are reported in Fig. 5.18, for 100 fF and 1 pF capacitive loads.

Figure 5.18: Simulated gain for the models reported in Fig. 9 for two loads: (a) 100 fF; and (b) 1 pF.
5.3. L-network Topology

As presented in the previous sections, there are limitations on the maximum achievable gains when attempting to properly match the static capacitance ($C_0$) of the resonator to large values of $C_{\text{load}}$. High quality factors, and consequently high FoMs, are in fact attained only for relatively small resonators with limited aperture ($L_e$) and number of finger pairs ($N_p$), due to the need of keeping a large $L/W$ ratio while minimizing the impact of electrical loading. This problem worsens at higher frequencies, since the motional resistance ($R_m$) is inversely proportional to $\omega$, while the series resistance ($R_s$) is only dictated by the device geometry and the top electrode metal thickness, which cannot be incremented arbitrarily (Fig. 4.18a). Fabrication non-idealities also impact devices operating at higher frequency more aggressively, limiting their performance. As reported in Sections 4.1.4 and 5.1.2, arrays of parallel resonators offer only minor advantages since their quality factor ($Q_{sa}$) is degraded by variations of the resonant frequency of the elements composing the array. Ultimately, resonators matched to the static capacitance of the load tends to operate closer to the anti-resonance region (Fig. 2.5), where spurious mode show lower $Q_{sa}$ and are more likely to degrade the voltage boost provided by the matching network.

For all these reasons, the development of more complex matching networks topologies [38], which can implement compact resonators exhibiting limited $C_0$ and incredibly high FoMs, is of
particular interest for the applications targeted by this dissertation. One possible solution consists in arranging a pair of resonators in an innovative series-parallel network configuration, or L-network, as reported in Fig. 5.19.

The L-network works similarly to the series approach described in Section 2.1.3, providing for impedance matching between the antenna \( (R_{\text{in}} = 50 \, \Omega) \) and a capacitive load \( (C_{\text{load}}) \). The ratio between the load and source impedances defines the maximum attainable gain, assuming the absence of insertion losses. In the L-network topology, a resonator \( (R_2, \text{Fig. 5.19a}) \) operating in its inductive region is implemented in parallel with \( C_{\text{load}} \) to increase its impedance over a narrow bandwidth. The series resonator \( (R_1, \text{Fig. 5.19a}) \) resonates out the parallel between \( R_2 \) and \( C_{\text{load}} \), allowing for a large current to flow and charge the effective load \( (C_{\text{load,eff}}) \). The \( C_{\text{load,eff}} \) synthetized by a L-network is captured by Eq. 5.21, and ideally must be lower than \( C_{\text{load}} \) to provide any advantage by a voltage gain standpoint.

\[
C_{\text{load,eff}} = C_2 + C_{\text{load}} + \frac{1}{-\omega L_{m2} + j \omega R_{m2} + \frac{1}{C_{m2}}} \quad (5.21)
\]

The same result could be attained by the introduction of a parallel inductor to resonate out \( C_{\text{load}} \) at the frequency of interest. This solution is fundamentally limited by the use of commercial or MEMS inductors, which exhibit quality factors \( (Q) \) between 50 and 200. Inductors with such low \( Q \)s introduce a small resistance in parallel to \( C_{\text{load}} \), shunting the large current built up by the series resonator to ground. Instead, if the inductor in parallel with \( C_{\text{load}} \) is implemented with a very high FoM resonator, the aforementioned approach can function efficiently in a small form factor.

In this section, an analytical model is derived for the L-network topology, and differences with the series approach are highlighted. A numerical optimization is later performed to identify the
optimal design point for the series and parallel resonators. In conclusion, an example based on measured data is provided to validate the numerical model developed.

5.3.1. Analytical Modeling

An analytical model for the L-network configuration can be easily derived assuming that the static capacitances of the series and parallel resonators are equal \( C_{01} = C_{02} \). The admittances of the the series resonator \( Y_A \), Fig. 5.20a) and of the loaded parallel resonators \( Y_B \), Fig. 5.20a) can be expressed according to Eq. 5.22 and 5.23. The total admittance can be readily calculated as the series of the two admittances (Eq. 5.24), and is represented in Fig. 5.20b.

\[
Y_A = \frac{1}{Z_A} = j\omega C_0 + \frac{1}{R_m + j\omega L_m + \frac{1}{j\omega C_m}} \quad (5.22)
\]

\[
Y_B = \frac{1}{Z_B} = j\omega C_0 + j\omega C_{load} + \frac{1}{R_m + j\omega L_m + \frac{1}{j\omega C_m}} \quad (5.23)
\]

\[
\frac{1}{Y} = \frac{1}{Y_A} + \frac{1}{Y_B} \quad (5.24)
\]
As reported in Fig. 5.20b, the admittance presents two operating resonances and two anti-resonances. The first operating resonance lies at the natural frequency of the system ($\omega_s$), while the second one ($\omega_{RF}$) can be expressed by solving Eq. 5.25. Such frequency is expressed by Eq. 5.26, and closely resemble the operating frequency reported in Section 2.1.3 for a matching network based on a series resonator (Eq. 2.12). $\alpha$ represents the ratio between $C_{load}$ and $C_0$ as previously discussed.

$$2j\omega C_0 \left( 1 - \frac{\omega^2}{\omega_s^2} \right) + j\omega C_{load} \left( 1 - \frac{\omega^2}{\omega_s^2} \right) + 2j\omega C_m = 0 \quad (5.25)$$

$$\omega_{RF} = \omega_s \sqrt{\frac{2C_0 + C_{load} + 2C_m}{2C_0 + C_{load}}} = \omega_s \sqrt{\frac{2 + \alpha + 2K^2}{2 + \alpha}} \quad (5.26)$$

Evaluating the real component of the inverse of $Y_A (Z_A)$ leads to Eq. 5.27, which represents the insertion losses of the series resonator when operating at $\omega_{RF}$. By manipulating Eq. 5.28, it is possible to define a pulling factor for $Z_A (k_{pa})$. Similarly to the $k_p$ described for the series resonator matching network, $k_{pa}$ multiplies the motional resistance ($R_m$) when the device is configured in series with a capacitive load.

$$\text{Re}\{Z_A\} = \frac{K^2}{\omega_s C_0 Q_s} \frac{1}{\left( 1 + K^2 - \frac{2 + \alpha + 2K^2}{2 + \alpha} \right)^2} = k_{pa} R_m \quad (5.27)$$

$$k_{pa} = \frac{2 + \alpha^2}{\alpha^2} \quad (5.28)$$

The real component of $Z_A$ is represented by a slightly different equation (Eq. 5.29). Curiously, the derived pulling factor results identical to $k_{pa}$ (Eq. 5.30), meaning that both the resonators introduce the same losses when operating at $\omega_{RF}$.

$$\text{Re}\{Z_B\} = \frac{K^2}{\omega_s C_0 Q_s} \frac{1}{\left( 1 + \alpha \left( 1 - \frac{2 + \alpha + 2K^2}{2 + \alpha} \right) + K^2 \right)^2} = k_{pb} R_m \quad (5.29)$$
Both $k_{pA}$ and $k_{pB}$ are higher than the pulling factor for the series topology (Eq. 2.14). Therefore, the gain attained by a L-network topology is a trade-off between the larger losses introduced by the series-parallel resonators and the effective load synthetized by the combination of the parallel resonator and $C_{\text{load}}$ (which is smaller and allows for higher gains). Even if an analytical derivation of the optimal $C_0$ can be found, the complexity of the mathematical problem fostered the development of a numerical method to identify the impact of the key network parameters on the maximum attainable gain.

### 5.3.2. Numerical Optimization

The voltage gain provided by a L-network for a fixed $C_{\text{load}}$ of 1 pF is numerically simulated with MATLAB®. To simplify the problem, all the variables are fixed except for the static capacitance of the series ($C_{01}$) and parallel resonators ($C_{02}$). The natural frequency ($f_s$) is fixed to 50 MHz, while $Q_s$ and $k_t^2$ are respectively set to 5,000 and 30%, according to the experimental data showcased in Chapter 4. The static capacitances of the series ($C_{01}$) and of the parallel resonator ($C_{02}$) are then swept from 0 to 1 pF in steps of 10 fF. As clearly shown by the contour plot of the gain as a function of $C_{01}$ and $C_{02}$ (Fig. 5.21), a global optimal point exists when $C_{01}$ and $C_{02}$ approach 400 fF. Two cases of interest can be extracted from Fig. 5.21. In the first case (Fig. 5.22, blue line) the static capacitance of the two resonator is set equal ($C_{01} = C_{02}$), while in the second case (Fig. 5.22a, red line) the static capacitance of the parallel resonator is set to 0 (to simulate a series approach).
This comparison serves to benchmark the benefits introduced by the L-network over the series-only configuration.

As clearly shown in Fig. 5.22, the L-network offers two main advantages over a simple series configuration. If the capacitive load is large enough, the parallel-series topology provides a larger gain (+30%). Furthermore, the optimal point is achieved for smaller resonator static capacitance (400 fF vs. 740 fF). Smaller capacitance resonators have generally shown higher $Q_\text{r}$, both for individual devices and arrays [79]. It is interesting to note that the optimal solution occurs for two identical size resonators, which makes the L-network more amenable to large scale manufacturing.

Ultimately, the L-network also shows robustness in regards to frequency mismatch between series and parallel elements. Frequency mismatch has proven to be a relevant problem for arrays with large number of elements, as discussed in Chapter 4. As highlighted in Fig. 5.22b, the gain provided by the proposed configuration is almost independent of any frequency mismatch between the series and parallel resonators in the range of ± 20,000 parts per million (ppm). Beyond this frequency range, the gain decreases monotonically until the L-network does not offer any advantage over a single series configuration.

![Figure 5.21: Contour plot of the simulated gain as a function of the static capacitance of R1 ($C_{01}$) and R2 ($C_{02}$).](image)
Figure 5.22: (a) Voltage gain as a function of the static capacitance for $C_{01} = C_{02}$ (blue line), representing a L-network implementing two identical resonators, and for $C_{02} = 0$ (red line), which represents a series approach; and (b) maximum gain as a frequency mismatch between $R_1$ and $R_2$; and (c) zoom-in of the central region of Fig. 5.22b.

5.3.3. System Demonstration

To validate the theoretical predictions described in Section 5.3.2, two identical resonators made out of 2 µm of X-cut Lithium Niobate on Silicon and operating around 50 MHz are characterized in air with the same approach described in Section 4.1.3. The two resonators exhibit quite similar performance with <10% variations in $Q_s$, <3% variations in $k_t^2$, identical $C_0$, and only 140 ppm of frequency difference (Fig. 5.23). It is important to note that the static capacitance of these devices does not match the optimal $C_0$ identified by numerical optimization. Nonetheless, as reported in Fig. 5.22a, this is still an operating point in which the L-network shows its benefits.

The gain is subsequently calculated with the same approach reported in Section 5.1. Starting from the measured Y-parameters matrix, an ideal voltage source (with 50 Ω impedance) and a capacitor with $C_{load} = 1$ pF are added to the input and output port respectively, and the gain across the capacitor extracted. As reported in Fig. 5.24, the L-network configuration shows a gain of 46
V/V, resulting in a net improvement of 30% compared to the best single resonator (\(G = 35\) V/V).

These results outperform the gain achieved by any other similar technology for \(C_{\text{load}} = 1\) pF.

![Graph](image)

**Figure 5.23:** Admittance responses of the resonators implemented in the L-network simulations.

![Graph](image)

**Figure 5.24:** Simulated gains based on \(Y_{12}\) data (solid line) and BVD fitting (dashed line) for the L-network configuration and the series configuration implementing the devices reported in Fig. 5.23. Discrepancies in the pulled frequencies between the series and the L-network approach are due to uncertainty in the BVD fitted parameters.
CHAPTER 6: CONCLUSIONS AND FUTURE WORK

This dissertation presented the modeling, fabrication and characterization of high-performance X-cut Lithium Niobate Laterally Vibrating Resonators (LVRs) operating between 50 MHz and 550 MHz, to be implemented in the RFFE matching network of ultra-low power WuRx. Record-breaking Figure of Merits greater than 2,500 ($Q_s > 8,000$ and $k_t^2 > 28\%$) were demonstrated in vacuum on resonators operating around 100 MHz, the highest ever recorded for devices operating at Very High Frequency (VHF). The main sources of damping were characterized to identify an optimal device geometry as a function of design constrains and frequency of operation. Resonators aspect ratio ($L/W$) was proven to be fundamental in minimizing the anchor losses by reducing energy leakage through the substrate. $Q_s$ greater than 26,000 were demonstrated at cryogenic temperature (10 K) on optimized devices. A tradeoff between anchor losses and electrical loading was demonstrated for resonators with a slender (high $L/W$ ratio) geometry at different frequencies. The existence of this tradeoff severely limits the performance of these devices above 500 MHz due to the larger impact of any series resistance ($R_s$) on the motional resistance ($R_m$), which is frequency dependent.

The need for capacitance matching in the matching network for IoT application fostered the investigation of arrays of optimized resonators with larger $C_0$, which showcased lower performance than the individual devices. Frequency mismatch between elements was identified as the main source of $Q_s$, and its impact investigated with a Monte Carlo statistical approach as a function of the resonator size.
Different MEMS-based matching network topologies were investigated to exploit the passive voltage amplification generated at resonance by fabricated devices when implemented in an RFFE matching network. To overcome the limitations of the limited static capacitances (50 – 100 fF) attained by high performance X-cut LVRs, a novel series-parallel matching network configuration (L-network) was investigated. This configuration showcased a 30% passive voltage amplification improvement (46 V/V) compared to a simple series topology (35 V/V) at 50 MHz, for a capacitive load of 1 pF.

Ultimately, the impact of spurious modes on the gain provided by X-cut LVRs when implemented in a matching network was investigated. Guidelines on the spurious modes quality factor, electromechanical coupling and resonant frequency were defined to minimize their impact on the passive voltage amplification.

6.1. Future Research Direction

The limited static capacitance showcased by demonstrated X-cut LN LVRs and the presence of severe in-band spurious modes and multiple higher order harmonics exhibiting high $Q$, pose serious challenges for their implementation in commercial application.

As highlighted in Chapter 3 and 4, the excitation mode exploited by these high performance devices limits the maximum attainable capacitance per unit area. Arrays performance can be improved according to the models described in Section 4.3.5, with a more rigorous control of the fabrication process.

The incredibly high $Q$, and $k^2$ showcased in the present work are nonetheless promising for the investigation of resonators fabricated with different cuts and orientations on different substrates,
thanks to the improvement of thin film machining processes. The high intrinsic $Q_s$ and $k_t^2$ demonstrated could also be exploited by applications in which capacitance matching is not a requirement. If temperature stability is required, the large recorded TCF must be addressed by proper compensation.

A better understanding of spurious modes and how to suppress them is desirable for applications requiring a clean, broad bandwidth. The present work offers some hints on possible approach to minimize their impact.

In conclusion, a deeper investigation on losses mechanism as a function of frequency could provide better insights on the full potential of the technology discussed in this dissertation.
## APPENDIX A: FABRICATION PROCESSES

### A.1. Aluminum Nitride (AlN)

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Notes | 1 µm AlN thick film

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<td>300 s</td>
<td>100 nm</td>
</tr>
</tbody>
</table>

Notes | Sputter etch step (100 s) before deposition
## A.2. Scandium-doped Aluminum Nitride (Sc$_{0.2}$Al$_{0.2}$N)

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Material</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sputtering</td>
<td>Pt</td>
<td>6J</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr pre-sputtering</td>
<td>-</td>
<td>200</td>
<td>300</td>
<td>25 - 5</td>
</tr>
<tr>
<td>Cr sputtering</td>
<td>10</td>
<td>100</td>
<td>35</td>
<td>25 - 5</td>
</tr>
<tr>
<td>Pt pre-sputtering</td>
<td>-</td>
<td>50</td>
<td>30</td>
<td>65 - 21</td>
</tr>
<tr>
<td>Pt sputtering</td>
<td>100</td>
<td>50</td>
<td>230</td>
<td>65 - 21</td>
</tr>
</tbody>
</table>

**Notes**

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Etching (chlorine RIE)</td>
<td>Versaline</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN etch</td>
<td>600</td>
<td>350</td>
<td>5</td>
<td>5 min</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>15</td>
<td>70</td>
<td>0</td>
<td>Argon flow</td>
<td>3 min</td>
</tr>
</tbody>
</table>

**Notes** 1$^{st}$ generation fab

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Ion milling LN etch</td>
<td>Millatron</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current [µA]</th>
<th>Voltage [V]</th>
<th>Angle</th>
<th>Time</th>
<th>Monitoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>500</td>
<td>22.5</td>
<td>2 h</td>
<td>End point</td>
</tr>
</tbody>
</table>

**Notes** 2$^{nd}$ generation fab

<table>
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<tr>
<th>Step no.</th>
<th>Description</th>
<th>Material</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Sputtering</td>
<td>AlSiCu</td>
<td>CVC</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Recipe</th>
<th>Time</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>AKAlSiCu</td>
<td>300 s</td>
<td>100 nm</td>
</tr>
</tbody>
</table>

**Notes** Sputter etch step (100 s) before deposition
### A.3. Y-cut Lithium Niobate (LN)

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Material</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sputtering</td>
<td>Pt</td>
<td>6J</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Process

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr pre-sputtering</td>
<td>-</td>
<td>200</td>
<td>300</td>
<td>25 - 5</td>
</tr>
<tr>
<td>Cr sputtering</td>
<td>10</td>
<td>100</td>
<td>35</td>
<td>25 - 5</td>
</tr>
<tr>
<td>Pt pre-sputtering</td>
<td>-</td>
<td>50</td>
<td>30</td>
<td>65 - 21</td>
</tr>
<tr>
<td>Pt sputtering</td>
<td>100</td>
<td>50</td>
<td>230</td>
<td>65 - 21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Ion milling LN etch</td>
<td>Millatron</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Notes

- 1 µm LN thick film

<table>
<thead>
<tr>
<th>Current [µA]</th>
<th>Voltage [V]</th>
<th>Angle</th>
<th>Time</th>
<th>Monitoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>500</td>
<td>22.5</td>
<td>2 h</td>
<td>End point</td>
</tr>
</tbody>
</table>

### Notes

1 µm LN thick film

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Material</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Reactive sputtering</td>
<td>AlN</td>
<td>Tegal</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Phase

<table>
<thead>
<tr>
<th>Etching</th>
<th>Sputtering</th>
<th>Conditioning</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>1038</td>
<td>200</td>
</tr>
</tbody>
</table>

#### Power [W]

- 100 (RF)
- 7000 (AC)
- 300 (DC)
- 5000 (AC)

#### Current [A]

- 25 (AC)
- 336 (AC)
- 30 (DC)
- 13.1 (AC)

#### Voltage [V]

- 336 (AC)
- 30 (DC)
- 425 (AC)

#### Ar flow [sccm]

- 32

#### N₂ flow [sccm]

- 15

#### Pressure [mTorr]

- 1.96
- 3.32
- 6.77

### Notes

- 1 µm AlN thick film

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Etching (chlorine RIE)</td>
<td>Versaline</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Recipe

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AlN etch</td>
<td>450</td>
<td>125</td>
<td>5</td>
<td>5 min</td>
</tr>
</tbody>
</table>

#### Cl₂ [sccm]

- 25

#### BCl₃ [sccm]

- 5

#### Ar [sccm]

- 70

#### O₂ [sccm]

- 0

#### Post-etch

<table>
<thead>
<tr>
<th>Post time</th>
<th>Argon flow</th>
<th>Post time</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 min</td>
<td>3 min</td>
<td></td>
</tr>
</tbody>
</table>

### Notes

- Sputter etch step (100 s) before deposition
### A.4. X-cut Lithium Niobate (LN)

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Material</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sputtering</td>
<td>AlSiCu</td>
<td>CVC</td>
<td>-</td>
</tr>
</tbody>
</table>

**Recipe**

<table>
<thead>
<tr>
<th>Material</th>
<th>Time</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>AKAlSiCu</td>
<td>820 s</td>
<td>300 nm</td>
</tr>
</tbody>
</table>

**Notes**  
Sputter etch step (100 s) before deposition

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Etching AlSiCu (chlorine RIE)</td>
<td>Versaline</td>
<td>-</td>
</tr>
</tbody>
</table>

**Recipe**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Al slow</td>
<td>425</td>
<td>25</td>
<td>5</td>
<td>35 s</td>
</tr>
</tbody>
</table>

**Chemicals**

<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>Argon flow</td>
<td>3 min</td>
</tr>
</tbody>
</table>

**Notes**  
Wafer dipped into hot water (80°C) right after vent to prevent Al corrosion

<table>
<thead>
<tr>
<th>Step no.</th>
<th>Description</th>
<th>Tool</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Ion milling LN etch</td>
<td>Millatron</td>
<td>-</td>
</tr>
</tbody>
</table>

**Current [µA]** | **Voltage [V]** | **Angle** | **Time** | **Monitoring** |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>500</td>
<td>22.5</td>
<td>4 h</td>
<td>End point</td>
</tr>
</tbody>
</table>

**Notes**

2 µm LN thick film
APPENDIX B: FREQUENCY PULLING

In this subchapter, the analytical derivation for the pulled anti-resonances ($\omega_p'$ and $\omega_{ps}'$) is reported in detail (Eq. B.1 – B.20). The pulled operating frequencies ($\omega_{RF}'$ and $\omega_{RFs}'$) of the main and spurious mode follow a similar approach. The reference circuits are reported in Fig. B1.

\[
Y = j\omega C_0 + \frac{1}{R_m + j\omega L_m + \frac{1}{j\omega C_m}} + \frac{1}{R_{ms} + j\omega L_{ms} + \frac{1}{j\omega C_{ms}}}
\]

\[
Y = j\omega C_0 \left(1 - \frac{\omega^2}{\omega_s^2}\right) \left(1 - \frac{\omega^2}{\omega_{ss}^2}\right) + j\omega C_m \left(1 - \frac{\omega^2}{\omega_s^2}\right) + j\omega C_{ms} \left(1 - \frac{\omega^2}{\omega_{ss}^2}\right)
\]

\[
\omega_s = \sqrt{\frac{1}{L_m C_m}}
\]

\[
\omega_{ss} = \sqrt{\frac{1}{L_{ms} C_{ms}}}
\]

\[
j\omega C_0 \left(1 - \frac{\omega^2}{\omega_s^2}\right) \left(1 - \frac{\omega^2}{\omega_{ss}^2}\right) + j\omega C_m \left(1 - \frac{\omega^2}{\omega_s^2}\right) + j\omega C_{ms} \left(1 - \frac{\omega^2}{\omega_{ss}^2}\right) = 0
\]

\[
-\omega^2 \frac{1}{\omega_s^2} \left(1 + \frac{C_{ms}}{C_0}\right) + \frac{1}{\omega_{ss}^2} \left(1 + \frac{C_m}{C_0}\right) \omega_s^2 \left(\frac{C_0 + C_m}{C_0}\right) + \omega_{ss}^2 \left(\frac{C_0 + C_{ms}}{C_0}\right)
\]

\[
\omega_s^2 \left(\frac{C_0 + C_m}{C_0}\right) = \omega_p^2
\]
\[
\omega_{ss} \left( \frac{C_0 + C_m}{C_0} \right) = \omega_{ps}^2 \quad \text{(B.8)}
\]

\[
\omega^4 - \omega^2 (\omega_p^2 + \omega_{ps}^2) + \omega_{ss}^2 \omega_p^2 + \omega_x^2 \omega_{ps}^2 - \omega_x^2 \omega_{ss}^2 = 0 \quad \text{(B.9)}
\]

\[
A\omega^4 - B\omega^2 + C = 0 \quad \text{(B.10)}
\]

\[
x = \omega^2 \quad \text{(B.11)}
\]

\[
Ax^2 + Bx + C = 0 \quad \text{(B.12)}
\]

\[
A = 1 \quad \text{(B.13)}
\]

\[
B = -(\omega_p^2 + \omega_{ps}^2) \quad \text{(B.14)}
\]

\[
C = \omega_{ss}^2 \omega_p^2 + \omega_x^2 \omega_{ps}^2 - \omega_x^2 \omega_{ss}^2 \quad \text{(B.15)}
\]

\[
x_{1,2} = -\frac{B}{2} \pm \sqrt{B^2 - 4AC} \quad \text{(B.16)}
\]

\[
\omega_p' = \sqrt{\frac{\omega_{pc}^2}{2} + \frac{1}{2} \sqrt{(\omega_p^2 + \omega_{ps}^2) - 4(\omega_{ss}^2 \omega_p^2 + \omega_x^2 \omega_{ps}^2 - \omega_x^2 \omega_{ss}^2)}} \quad \text{(B.17)}
\]

\[
\omega_{ps}' = \sqrt{\frac{\omega_{pc}^2}{2} - \frac{1}{2} \sqrt{(\omega_p^2 + \omega_{ps}^2) - 4(\omega_{ss}^2 \omega_p^2 + \omega_x^2 \omega_{ps}^2 - \omega_x^2 \omega_{ss}^2)}} \quad \text{(B.18)}
\]

\[
\omega_{pc} = \frac{\omega_p^2 + \omega_{ps}^2}{2} \quad \text{(B.19)}
\]

\[
\delta_p^2 = \frac{1}{2} \sqrt{(\omega_p^2 + \omega_{ps}^2) - 4(\omega_{ss}^2 \omega_p^2 + \omega_x^2 \omega_{ps}^2 - \omega_x^2 \omega_{ss}^2)} \quad \text{(B.20)}
\]
APPENDIX C: STRESS GRADIENT EXTRACTION

As first step, the deflected beam profile must be extracted over the entire length with a measurement instrument, such as an optical 3D scanner. The second step is removing the part of the measured profile not belonging to the beam (Fig. C.1) via software. In this case, the data were imported from a .csv file.

The remaining curve must be fit to a 2nd order equation (MATLAB® function: `polyfit`) in the form reported by Eq. C.1. The deflection of a cantilever beam due to stress gradient can be calculated according to Eq. C.2, and the stress gradient easily calculated (Eq. C.3 – C.5), where \( \sigma_1 \) is the stress, \( H \) the beam thickness, \( E \) the Young’s modulus, and \( \nu \) the Poisson’s ratio. Fig. C.2 reports the comparison between the measurement profile and the quadratic fitting used to extract the stress gradient (Eq. C.5). The MATLAB® code used for the stress gradient calculation is reported in Table C.1.

\[
y = ax^2 + bx + c
\]  

(C.1)
\[ y = \frac{\sigma_1}{H} \left( 1 - \nu \right) \left( \frac{x^2}{2} \right) \]  
\[ a = \frac{\sigma_1(1-\nu)}{HE} \]  
\[ \sigma_1 = \frac{aHE}{(1-\nu)} \]  
\[ \frac{d\sigma_1}{dy} = \frac{2a_1}{H} \]

Figure C.2: Measured beam deflection and quadratic fitting.

Table C.1: MATLAB® code for stress gradient extraction.

```
%Load file
[filename1, pathname] = uigetfile('.csv','Select the CSV file');
filename = strcat(pathname, filename1);
data = xlsread(filename);
x = data(:,1); %x dimension, in meters
y = data(:,2); %y dimension, in meters
dx = x(2) - x(1);
dy = y(2) - y(1);

figure(2)
subplot(2,1,1)
plot(x, y,'LineWidth',3);
title('Original plot');
xlabel('Beam profile [m]')
ylabel('Beam length [m]')
```
set(gcf,'color','white')
grid on
set(gca,'FontSize',13)

uiwait(msgbox('Select start x-value for cropping','modal'));
[x_user, ~] = ginput(1); % Let the user select an x-value from which to crop.
x_tmp = x(x>x_user);
xStart = find(x==x_tmp(1));

uiwait(msgbox('Select end x-value for cropping','modal'));
[x_user, ~] = ginput(1); % Let the user select an x-value from which to crop.
x_tmp = x(x<x_user);
xEnd = find(x==x_tmp(end));

xBeam = x(xStart:xEnd);
yBeam = y(xStart:xEnd);

subplot(2,1,2);
plot(xBeam,yBeam,'LineWidth',3)
title('New plot with cropped values');
xlabel('Beam profile [m]')
ylabel('Beam length [m]')
grid on
set(gca,'FontSize',13)

%Fitting the beam profile
fitCoeff = polyfit (xBeam, yBeam, 2);
yCalc = @(x) fitCoeff(1)*x.^2 + fitCoeff(2)*x + fitCoeff(3);

figure(3)
plot(xBeam, yBeam, 'LineWidth',3)
hold on
plot(xBeam, yCalc(xBeam), 'LineWidth',3)
xlabel('Beam profile [m]')
ylabel('Beam length [m]')
set(gcf,'color','white')
grid on
set(gca,'FontSize',13)
legend('Measurement','Fitting')

%Stress gradient extraction
E = 173e9; %Young's modulus
nu = 0.25; %Poisson's ratio
t = 2.5e-6; %Film thickness

sigma1 = fitCoeff(1)*t*E/(1-nu);

dsigma1 = 2*sigma1/(t*1e6)*1e-6; %[MPa/um]
APPENDIX D: MATRIX ROTATIONS

A generic 2-D rotation of a vector $V$ is reported in Fig. D.1. Assuming $|V| = 1$ and applying a rotation $\theta$ it is possible to calculate the rotation matrix ($a$) by following the steps reported by Eq. D.6 – D.7.

Figure D.1: Generic rotation of a vector $V$.

$$V_x = \cos \phi = \bar{x}$$  \hspace{1cm} (D.1)

$$V_y = \sin \phi = \bar{y}$$  \hspace{1cm} (D.2)

$$V = \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}$$  \hspace{1cm} (D.3)

$$\bar{x}' = \bar{x} \cos \theta - \bar{y} \sin \theta$$  \hspace{1cm} (D.4)

$$\bar{y}' = \bar{x} \sin \theta + \bar{y} \cos \theta$$  \hspace{1cm} (D.5)

$$V' = \begin{bmatrix} \bar{x}' \\ \bar{y}' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = a_x V$$  \hspace{1cm} (D.6)

$$a = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$  \hspace{1cm} (D.7)

2-D rotation can be easily expanded to a 3-D space by following the convention reported in Fig. D.2. The rotation matrices ($a_x$, $a_y$, and $a_z$) around the three axes ($x$, $y$, and $z$) are captured by
Eq. D.8 – D.10. Rotations of vectors around $x$ (YZ plane) and $z$ (XY plane) are positive if counterclockwise, rotations around $y$ (XZ plane) are positive if clockwise.

![Diagram showing rotations around x, y, and z axes.](image)

**Figure D.2:** Convention for rotation around $x$, $y$ and $z$ axes.

\[
\begin{align*}
\mathbf{a}_x &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad (\text{D.8}) \\
\mathbf{a}_y &= \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad (\text{D.9}) \\
\mathbf{a}_z &= \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{D.10})
\end{align*}
\]

Two different references system can be adopted: vector or frame (Fig. D.3). If the rotation is applied to the frame, the rotation matrices can be expressed according to Eq. D.11 – D.13, since $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$.

![Diagram showing rotation around vector (red) or frame (white).](image)

**Figure D.3:** Convention for rotation around vector (red) or frame (white).
\[
a_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\theta & \sin\theta \\
0 & -\sin\theta & \cos\theta
\end{bmatrix}
\]  \hspace{1cm} (D.11)

\[
a_y = \begin{bmatrix}
\cos\theta & 0 & -\sin\theta \\
0 & 1 & 0 \\
\sin\theta & 0 & \cos\theta
\end{bmatrix}
\]  \hspace{1cm} (D.12)

\[
a_z = \begin{bmatrix}
\cos\theta & \sin\theta & 0 \\
-\sin\theta & \cos\theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (D.13)

For non square matrices, such as the piezoelectric coupling matrices \(d, e, g, \) or \(h\) depending on the equation form), the Bond method can be used to perform rotation and investigate the piezoelectric property of different crystal along specific orientations. The constitutive equations for piezoelectric material include matrices with three different dimensions (Eq. D.14 – Eq. D.15). To rotate matrices such as \(c\) and \(e\) it is necessary to introduce a matrix \(M\), defined according to [89] and expressed as in Eq. D.17.

\[
c = [\text{Stiffness}]_{6 \times 6}
\]  \hspace{1cm} (D.14)

\[
e = [\text{Piezoelectric coupling coeff.}]_{3 \times 6}
\]  \hspace{1cm} (D.15)

\[
\epsilon = [\text{Dielectric constants}]_{3 \times 3}
\]  \hspace{1cm} (D.16)

\[
M = \begin{bmatrix}
2a_{xx} & 2a_{xy} & 2a_{xz} & 2a_{yx} & 2a_{yx} & 2a_{xy} \\
a_{xx} & a_{xy} & a_{xz} & a_{yx} & a_{yx} & a_{xy} \\
2a_{xx} & 2a_{yy} & 2a_{yz} & 2a_{yx} & 2a_{yx} & 2a_{xy} \\
a_{yx} & a_{yy} & a_{yz} & a_{yx} & a_{yx} & a_{xy} \\
a_{xx} & a_{yy} & a_{yz} & a_{yx} & a_{yx} & a_{xy} \\
a_{xx} & a_{yy} & a_{yz} & a_{yx} & a_{yx} & a_{xy}
\end{bmatrix}
\]  \hspace{1cm} (D.17)

Matrices can be rotated according to Eq. D.18 – D.20. In case of concatenated rotations, \(a\) and \(M\) assume the form captured by Eq. D.21 – D.22.

\[
c' = M c M^T
\]  \hspace{1cm} (D.18)

\[
e' = a e M^T
\]  \hspace{1cm} (D.19)

\[
\epsilon' = a \epsilon a^T
\]  \hspace{1cm} (D.20)
\[ a = \prod_{k=n}^{0} a_n = a_n a_{n1} \ldots a_2 a_1 \quad (D.21) \]

\[ M = \prod_{k=n}^{0} M_n = M_n M_{n1} \ldots M_2 M_1 \quad (D.22) \]

An example is the calculation of the \( k_t^2 \) of Y-cut starting from Z-cut Lithium Niobate. As first step, it is necessary to rotate -90° around the \( x \) axis (frame convention) using matrices \( a_x \) and \( M_x \) to obtain Y-cut (Fig. D.4). As second optional step consist in rotating the wafer by -90° around \( y \), using \( a_z \) and \( M_z \) to re-align the coordinate system (Fig. D.5). Ultimately, it is possible to sweep around \( y \) using \( a_z \) and \( M_z \) to calculate the parameters of interest (\( i.e., \ k_t^2 \)), as reported in Section \( X \).

![Figure D.4: Rotation to calculate Y-cut properties from Z-cut matrices.](image1)

![Figure D.5: Reference system reported in [62] for Y-cut LN compared to the one of the present dissertation.](image2)

Another example is the calculation of the \( k_t^2 \) of X-cut starting from Z-cut Lithium Niobate. In this case it is necessary to rotate by -90° (wafer convention) around \( y \) using matrices \( a_y \) and \( M_y \) to obtain X-cut. Then, a rotation of 90° must be performed around \( x \) using \( a_z \) and \( M_z \) to re-align the coordinate system. Ultimately, it is possible to investigate the parameter of interest by performing
a sweep around $x$ (matrices $a_z$ and $M_z$), as reported in Sections 3.2.3 and 3.2.4. Tables D.1 and D.2 reports the MATLAB® code used for the investigation of Y-cut and X-cut material properties.

Figure D.6: Rotation to calculate X-cut properties from Z-cut matrices.

Figure D.7: Reference system reported in [62] for X-cut LN compared to the one of the present dissertation.

Table D.1: MATLAB® code for matrix rotation (Y-cut LN).

```matlab
load("LiNbO3_Z.mat")
density = 4700;
thetaX = -90;
a1 = rotx(thetaX);
M1 = genM(a1);
thetaZ1 = 0;
a2 = rotz(thetaZ1);
M2 = genM(a2);
thetaZ2 = 1:1:180;
for i = 1:length(thetaZ2)
    a3 = rotz(thetaZ2(i));
    M3 = genM(a3);
    a = a3*a2*a1;
    M = M3*M2*M1;
```
\begin{verbatim}
eps0 = LiNbO3_Z.eps0;
c = LiNbO3_Z.c;
e = LiNbO3_Z.e;
eps = LiNbO3_Z.eps;
c = M*c*M';
e = a*e*M';
eps = a*eps*a';
%d = -(c\e)';

kt2_11(i) = (e(1,1)^2/(c(1,1)*eps(1,1)*eps0));
kt2_31(i) = (e(3,1)^2/(c(1,1)*eps(3,3)*eps0));
phv_11(i) = sqrt(c(1,1)/density);
epsR_11(i) = eps(1,1);
epsR_33(i) = eps(3,3);
kt2_16(i) = (e(1,6)^2)/(c(6,6)*eps(1,1)*eps0);
end

function M = genM(a)

    axx = a(1,1);
    ayy = a(1,2);
    axz = a(1,3);
    ayx = a(2,1);
    ayy = a(2,2);
    ayz = a(2,3);
    azx = a(3,1);
    azy = a(3,2);
    azz = a(3,3);

    M1 = [axx^2 axy^2 axz^2
           ayx^2 ayy^2 ayz^2
           axz^2 azy^2 azz^2];

    M2 = 2*[axy*axz axx*axz axx*axy;
           ayy*ayz ayx*ayz ayy*ayx;
           ayz*azz azx*azz azx*azy];

    M3 = [ayx*axz ayy*axy azx*azz
           ayy*axy*axy azx*azx*azz
           axx*axy*axy azx*azx*azy];

    M4 = [axy*azz+axy*azx ayy*azz+axy*azx ayz*azz ayz*azz;
           axx*azz+axy*azz azx*azz+axy*azz ayz*azz ayz*azz;
           axy*azz+axy*azz azx*azx+axy*azz ayz*azx ayz*azx;
           axy*axx+axy*azx axx*axx+axy*axx ayz*axy ayz*axy];

    M = [M1, M2;
         M3, M4];
end
\end{verbatim}
Table D.2: MATLAB® code for matrix rotation (X-cut LN).

```matlab
load("LiNbO3_Z.mat")
density = 4700;

thetaY = -90;
a1 = roty(thetaY);
M1 = genM(a1);

thetaZ1 = 90;
a2 = rotz(thetaZ1);
M2 = genM(a2);

thetaZ2 = 1:1:180;
for i = 1:length(thetaZ2)
    a3 = rotz(thetaZ2(i));
    M3 = genM(a3);
    a = a3*a2*a1;
    M = M3*M2*M1;
    eps0 = LiNbO3_Z.eps0;
c = LiNbO3_Z.c;
e = LiNbO3_Z.e;
eps = LiNbO3_Z.eps;
    c = M*c*M';
e = a*e*M';
egps = a*eps*a';
% d = -(c\e)';
    kt2_11(i) = (e(1,1)^2/(c(1,1)*eps(1,1)*eps0));
    phv_11(i) = sqrt(c(1,1)/density);
    epsR_11(i) = eps(1,1);
    kt2_16(i) = (e(1,6)^2)/(c(6,6)*eps(1,1)*eps0);
end
```
APPENDIX E: OVERETCH MODELING

A simple model to characterize the overetch due to the ion milling is reported in Fig. E.1. The typical angle between the Argon (Ar) ion beam and the rotating wafer surface in the ion milling available at the CMU Nanofab is 22.5°, while the measured etch rate perpendicular to the surface \( a_\perp \) is around 20 nm/min (cfr. Appendix A). Assuming a duty factor \( \delta_c \) of 25%, given by the fact that the stage is rotating at a constant speed, it is possible to estimate the parallel etch rate according to Eq. E.1.

\[
a_\parallel = a_\perp \tan(22.5^\circ) \delta_c = 2 \text{ nm/min} \quad (E.1)
\]

This model does not consider the sidewall sloping of the photoresist \( \phi \), which can be usually estimated between 10° and 20° according to the type of PR, exposure dose, development methods and baking procedure. In this case, the extra angle must be included in the calculation, as captured by Eq. E.2.

\[
a_\parallel' = a_\perp \tan(22.5^\circ + \phi) \delta_c = 4 \text{ nm/min} \quad (E.2)
\]

Figure E.1: Geometrical model for the overetch calculation.
Assuming an etching time of 2 hours, required to remove 2 µm of lithium niobate at 20 nm/min, plus few hundreds nm of Si to allow for an easier XeF₂ release, provides for a total overetch of roughly 0.5 µm. This value is in agreement with the value measured on previously fabricated 50 MHz and 400 MHz devices and must be added to each edge of the resonator plate.
BIBLIOGRAPHY AND REFERENCES


[82] R. Wang, S. A. Bhave, and K. Bhattacharjee, “High $k^2\times Q$, multi-frequency lithium niobate


