Optimal Inheritance Taxation under Uncertainty and Tax Frictions

Joey Pickens

Spring 2018

Abstract

The current literature in optimal inheritance taxation does not address two important issues. First, there is significant uncertainty surrounding the receipt of a bequest by heirs. This uncertainty can arise from complications in inheritance law, prospects of future taxes, and conflict among the recipients. Such factors can cause individuals to change how much they consume during their life and how much they leave for their heirs. Second, the costs associated with assessing taxable property and collecting taxes on that property can be significant.

The paper considers a non-overlapping generations model adapted from a 2013 paper by Thomas Piketty and Emmanuel Saez. In this model, utility maximizing individuals receive utility from consumption, labor, and bequests left to their heirs and government taxes inheritance and labor income in such a way that they can maximize social welfare through a fixed, redistributive lump sum. We focus on how changes in uncertainty, inheritance tax frictions, and labor-income tax frictions effect the optimal inheritance tax.

We find that an increase in uncertainty causes individuals to consume more and leave less for their heirs. This increased uncertainty also lowers the optimal inheritance tax. We also find that increasing the friction on bequest tax decreases the optimal bequest tax, and increasing the friction on labor income tax increases the optimal bequest tax.²

²This paper is a thesis for my MS in Mathematical Sciences at Carnegie Mellon University (CMU). I would like to give thanks to Professor Laurence Ales of the CMU economics
Contents

1 Introduction ............................................. 3
  1.1 Justification of Additions to the Piketty-Saez Model . 3
    1.1.1 Uncertainty of Bequest Reception ............... 3
    1.1.2 Frictions on Taxes .......................... 6
  1.2 Literature Review ................................. 9

2 The Piketty-Saez Model (2013): Optimal Inheritance Tax with Bequests in the Utility 10
  2.1 The Individual .................................... 11
    2.1.1 Lifetime Resources and Utility .............. 11
    2.1.2 Individual Maximization Problem .......... 12
    2.1.3 First Order Condition for Bequests Left . 13
  2.2 Government Policy ................................. 13
  2.3 Defining an Equilibrium ........................... 14
    2.3.1 The Equilibrium ............................... 14

3 Additions to the Piketty-Saez Model 15
  3.1 Uncertainty of Bequest Reception .................. 15
  3.2 Frictions on Taxes ................................. 16
  3.3 Ratio of Bequests Left to Bequests Received .... 17

4 Steady-Steady Welfare Maximization 17
  4.1 Elasticity Parameters .............................. 18
  4.2 Distributional Parameters ........................ 19
  4.3 Derivation of Optimal Linear Tax on Capitalized Bequests 20
  4.4 A Special Case: The Piketty-Saez model .......... 23

department for advising me throughout this project and Professor Giovanni Leoni of the CMU mathematics department for helping me build a mathematical foundation with five incredible semesters of analysis.
1 Introduction

A bequest is money or property that is left by a deceased person for their heirs. It is typical for a government to tax this bequest, and add it to their general revenues. According to the tax foundation, as of 2015, 19 of the 34 OECD countries taxed inheritance [12].

Optimal taxation is a topic in economic research that uses theory to find the tax rate or system that will optimize a normative notion of social welfare given an economy with certain constraints. In their 2013 paper “A Theory of Optimal Inheritance Taxation,” Thomas Piketty and Emmanuel Saez create an optimal tax framework to analyze bequest taxes [23]. Throughout the paper, we will refer to this framework as the ‘Piketty-Saez’ model.

The Piketty-Saez model includes a social welfare maximizing government who can place linear taxes on bequests and labor income and redistribute this money through a uniform lump sum transfer. Individuals are utility maximizers that have consumption, labor, and bequest left (to their heirs) in their utility function.

I spend the rest of this section justifying additions I made to the Piketty-Saez model and reviewing relevant literature. In Section 2, I introduce the details of the Piketty-Saez model. In Section 3, I implement my additions into the Piketty-Saez framework and in Section 4, I show how to derive the optimal bequest tax that maximizes steady-state welfare. In Section 5, I explore some implications of the steady-state optimum.

1.1 Justification of Additions to the Piketty-Saez Model

This paper makes two major additions to the Piketty-Saez model: uncertainty over whether the bequest will be received and frictions on the income and bequest tax. We justify these additions below.

1.1.1 Uncertainty of Bequest Reception

There is potential for much unpredictability surrounding the leaving of bequests. Such unpredictability can come from several sources, among them intestate succession law, family disputes, and future tax rates.
Intestate Succession and Family Disputes  Intestate succession laws deal with the legal proceeding of what happens when a person dies and does not have a will. Even if a person does have a will, if they do not keep it up to date, they could be left with an inheritor or executor (someone who handles the affairs of a deceased person) that they wanted at one time, but not at the time they died. In the US, depending on the state, if an inheritor passes away without a will and there are no other relatives alive, an entire estate can be left to the government [16]. While having an outdated will is necessary to see such effects, intestate succession laws create potential for a bequest to be inappropriately managed, undesirably allocated, or left to the state.

For many families, inheritance can be a source of tension and argument. In many cases, a deceased person leaves the affairs of their estate to multiple family members, that is, appoints multiple executors. These family members may disagree on how to handle an estate, and this can strain relationships across a family. Even if the estate ended up how the deceased person intended, adverse effects on family relationships could cancel any benefits gained from the inheritors.

The Future of Inheritance Tax  A last source of uncertainty surrounding bequests is the future of inheritance tax law. The institution of inheritance is recognized by many to contribute to the persistence of economic inequality. For this reason, inheritance tax can be seen as a social policy to reducing inequality.

Opinions on Fairness  In western countries like the US and the UK, raising the inheritance tax is often supported by politicians on the political left and lowering or abolishing the inheritance tax is often supported by politicians on the political right. For example, during the 2016 US presidential election, leading GOP candidates Donald Trump and Ted Cruz supported a repeal of the estate tax while leading Democrat candidates Hillary Clinton and Bernie Sanders supported an estate tax increase [1, 24].

Despite this split, inheritance tax is currently seen as unfair by citizens in the US and the UK on both the left and right. A March 2015 YouGov poll in the UK found that 59 percent of voters thought the estate tax was unfair including 51 percent of Labour party members and 48 percent of Liberal Democrat party members [25]. A March 2016 Gallup poll in the US found 54 percent of respondents supported
a full repeal of the estate tax while only 19 percent were against it [1]. This occurred at a time when only 44 percent of voters identified as or leaned Republican (from a September 2016 poll) and inheritance tax was a partisan issue in the 2016 presidential election [2].

Despite the current political climate not being favorable to an inheritance tax increase, an experiment by economists Ilyana Kuziemko, Michael Norton, Emmanuel Saez, and Stefanie Stantcheva suggest that people can easily change their mind about the issue [21, 26]. The experiment surveyed 10,000 individuals on how government policy should react to increasing inequality. One group of participants were shown a presentation on the history of inequality before they took the survey while another group was not. Part of the presentation explained how few people are subject to an estate tax under current law. The researchers found that showing this presentation to participants more than tripled the likelihood that they would support increasing the estate tax. The support rose from 17 percent in the control group to 53 percent in the treatment group. In hypothesizing over causes of this disparity, they claimed that the view of inheritance as being lucky and a birthright as opposed requiring effort facilitated support for a higher estate tax. They point to a 2005 paper by Alberto Alesina and George-Marios Angeletos for a theoretical justification of this hypothesis [9].

**Limiting Principles** While is it clear that opinions on the estate tax for some people can be easily changed, the limiting principles of voters and politicians alike are less clear on this issue. Although the limiting principle for those on the right clearly does not exist (many advocate abolishing the estate tax), it is unclear whether a limiting principal exists on the left with regard to raising the estate tax. In the 2016 US presidential election, Democrat Party nominee Hillary Clinton initially advocated for an increase of the estate tax from 40 to 45 percent. However, she eventually adopted Bernie Sanders position of an increase in the tax to 65 percent.

Although limiting principals for the estate tax are unclear for politicians on the left, some on the left have advocated for the most extreme position on this issue. In his article “Why Isn’t Inheritance Tax 100%?,” Dan Matthews of Forbes argues that an estate tax is the most fair type of tax because “it is the only tax that penalizes the dead and not the living” [22]. In a similarly titled article “Why not fund the welfare state with a 100% inheritance tax?,” Abi Wilkinson of
The Guardian claims that funding social programs is more important than the wishes of the dead [27]. Even though these writers are not politicians and their opinions do not represent mainstream thought, a support base for a 100 percent inheritance tax appears to exist on the left.

**Consequences of Uncertainty**  Given unclear limiting principals for those on the left and easily changed minds of the public, it is plausible to believe that the institution of inheritance may be abolished in the near future. While we would admit that such a policy does not seem likely in the US or UK anytime soon, it is plausible to consider an economy where people think this may happen before they retire and make their working, consumption, and estate plans accordingly.

While the Piketty-Saez model assumes that the government always chooses the bequest tax to maximize social welfare and that individuals know what this tax will be when they pass away, it is plausible to consider an economy where people operate under the uncertainty described above. Such uncertainty would result in individuals leaving less in bequests and consuming more than they would under certainty.

### 1.1.2 Frictions on Taxes

While we formally define a tax friction in a later section, it can be thought of as any inefficiency in the way that taxes are collected or redistributed. Here, we introduce several potential sources of this friction including costs associated with collection, law enforcement, administration, and compliance.

**Implementation and Collection Costs**  The implementation of a tax system cost money and such costs are often measured by governments. In the United States, the Internal Revenue Service (IRS) is the government agency that collects taxes and resolves instances of fraudulent tax filings. In 2017, the IRS had 79,300 employees and its annual budget was $11.5 billion. In the same year, it collected $3.4 trillion in revenue, which corresponds to a cost of 34 cents to collect 100 dollars [7, 6]. This is a small cost, but we could imagine a less efficient tax collecting agency. Moreover, we could imagine an agency that is more efficient at collecting one type of tax than another.
We are interested in the relative efficiency of collecting labor income tax and estate tax. Of the 2017 IRS budget, 21 percent was used for taxpayer services, 41 percent was used for enforcement activities like examinations and collections (34 percent), and 38 percent was used for operational support and business system modernization [6]. Unfortunately, the IRS does not have data on the costs of collecting different types of taxes, but it is plausible to assume that examination and collection costs of an estate tax are significantly different than that of an income tax.

Other Administrative Costs In addition to these direct costs of tax collection agencies, one could consider other expenditures as administrative costs of government. The Piketty-Saez model assumes that all money collected in taxes is redistributed back to the taxpayer, but in modern governmental systems, this is not the case. In addition to the collection of taxes, other non-redistributive functions of government could be seen as a friction of government service. Moreover, because different sources of tax revenue can be earmarked for unique purposes, a non-redistributive function of government could be considered a friction of one type of tax but not another.

To make this more clear, we provide an example. In the United States, property taxes are often used to fund local school systems. Assume that teachers in a particular district are required to be a part of a teachers union and pay a union membership fee every year. This mandatory membership fee could be considered a friction on property tax because property tax ultimately funds teacher salaries, part of which goes to union dues.

Compliance Costs Another source of friction for a tax system is the cost of compliance. In the US, according to estimates from the Office of Information and Regulatory Affairs, over 2.6 billion hours were spent complying with individual income tax law in 2016 [18]. In the same year, the IRS received over 152 million income tax returns [3]. This corresponds to an average compliance time of approximately 17.4 hours per individual taxpayer per year. According to OECD statistics, the average American worked 1783 hours in 2016 [5]. If we assume this compliance time would otherwise be work time, these statistics correspond to an approximate one percent reduction in work time. This loss of working time can be seen as a friction on the income tax.
While compliance costs are significant for labor income tax, some estimates of estate tax compliance dwarf that of income tax. In the US in 2016, there were 12,411 estate tax returns filed [8]. According to estimates from the US Office of Information and Regulatory Affairs, almost 376 million hours were spent complying with estate tax law in 2016, which corresponds to an incredible 30,000 hours per tax return [18]. The compliance costs were estimated to be 19.5 billion dollars in that year, which is more than the 18.3 billion that was generated in estate tax revenue [18, 8]. This suggests that the compliance costs of the estate tax are higher than the revenue collected.

The claim that compliance costs exceed revenue collection is controversial. In their 1999 paper, Rutgers University law professors Charles Davenport and Jay Soled estimate that the cost of compliance for estate taxes is 7 percent of estate tax revenue, which is consistent with the compliance costs of other types of taxes [13, 15]. They claim that the primary reason for such a big disparity between their estimate and other estimates is that around half of the costs associated with estate tax administration and planning would still exist even if there were no estate tax. For example, in the absence of an estate tax, executor and trustees would still need to be chosen, bequests would still need to be allocated, documents for the transfer of property would still need to be prepared, etc. Without entering the discussion of what should and should not count as a compliance cost, we can observe that their is a significant friction on the estate tax and such a friction is worth including in a model of optimal inheritance taxation.

**Integrating Compliance Costs into our Model** In our model, we quantify a friction on taxes by how much potential tax revenue is not redistributed because of administration and compliance costs of the tax code. This could be revenue that directly goes to tax collection costs or additional taxable income that would be accrued in the absence of the tax. Viewing collection costs as a friction in this way is straightforward, but this is not so for individual compliance costs.

In general, there are two ways an individual can handle their taxes: do it themselves or pay someone else to do it. In the former case, this individual will presumably be taking time off work and hence, earning less money and paying less taxes. In other words, the tax revenue generated from this person is lower than it would be if they did not have to spend time doing their taxes. In the latter case, an
individual is paying someone else to do their taxes, but not earning any less money as a result. If we assume that tax compliance costs are tax deductible, then less taxes will also be paid as a result of hiring someone else. Current US policy shows that this is not an unrealistic assumption.

According to the TurboTax website, tax preparation fees are an allowable deduction [4]. However, most people cannot take advantage of it because only preparation fees exceeding two percent of adjusted gross income can be reimbursed. It is realistic to consider an economy where this two percent barrier is removed and hence, all preparation costs are tax deductible.

With these potential frictions in mind, we will introduce a labor income tax and a bequest tax friction into our model. The different frictions will reflect varying administrative costs of the respective taxes.

1.2 Literature Review

The debate on estate taxation focuses on the equity-efficiency tradeoff. On the equity side, the institution of inheritance is partly responsible for the persistence of income inequality, which many economists see as undesirable. On the efficiency side, the burden of an estate tax causes distortions that may lead those effected to make economically inefficient decisions. Also on the efficiency side is the Carnegie effect, which researchers have found to be significant in certain situations [10]. The economic discussion on optimal inheritance taxation features a number different models and results.

Two papers on this topic have concluded that the optimal inheritance tax is zero in the long run. In his 1985 paper, Kenneth Judd showed that under any system which maximized a Paretian notion of social welfare through a convergent redistributive tax policy, the optimal capital income tax is zero in the long run [19]. A 1986 paper by Christophe Chamley has a similar result showing that for individuals that live infinite periods and have a utility function of a general form, the optimal tax rate on capital income tends to zero in the long run [11]. Both of these are infinite horizon models and if we take a dynastic interpretation, we get that the optimal inheritance tax is zero in the long run, because instead of leaving capital for themselves in a

---

2 The idea that the work efforts of an inheritor are harmed by receiving a bequest
future period, they are leaving it to the next generation via bequest.

However, this zero tax result can be overridden for each paper if some key hypothesis are relaxed. For example, Wojciech Kopczuk in his 2001 paper shows that Chamley’s and Judd’s work cannot be used to justify a zero capital income tax in a situation where the government has informational constraints [20]. Kopczuk also concludes that an estate tax may be part of an optimal tax system that favors redistribution and corrects externalities. Additionally, in their 2010 paper, Emmanuel Farhi and Ivan Werning show that in a model with altruistic parents and heterogeneous productivity, the estate tax should be negative and progressive [14]. In other words, the estate tax schedule should be decreasing, convex, and fully negative.

The Piketty-Saez paper shows that formulas for optimal inheritance tax can be expressed in terms of macroeconomic variables like behavioral elasticities and distributional parameters [23]. These formulas capture the equity-efficiency trade-off of inheritance taxation through its inclusion of normative distributional parameters in the formula.

While the literature on optimal taxation is quite broad, we could find no model that accounted for compliance, administrative, or collection costs. Much of the optimal tax literature in this area focuses on the problems of law enforcement and tax evasion. Moreover, we could find no literature that addresses the uncertainty surrounding the leaving of bequests. For these reasons, our model adds to the literature by analyzing the relationship between uncertainty, tax frictions, and the optimal inheritance tax rate.

2 The Piketty-Saez Model (2013): Optimal Inheritance Tax with Bequests in the Utility

Consider a dynamic economy with no growth and a discrete set of generations \((0, 1, 2, ..., t, ..., T)\), each with measure 1.\(^3\) Assume each

\(^3\)We make no initial assumption on whether the number of generations is finite or infinite: \(T \in (\mathbb{N} \cup \{\infty\})\).

\(^4\)More formally: for any generation \(t \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\) with \(t \leq T\), \(\mu(D_t) = \int_{D_t} 1d\mu = 1\), where \(D_t\) is defined to be the set of all individuals in generation \(t\). We do not assert \(\mu\) to be a specific measure and we use the same measure for every generation.
generation lives one period and is replaced by the next generation. Note that we will refer to period \( t \) as the period in which generation \( t \) lives. The economy is made up utility-maximizing individuals and a government that wants to maximize social welfare.

2.1 The Individual

In the first generation \( (t = 1) \), there is a fixed set of individuals.\(^5\) Each of these individuals will be the first in their dynasty, which refers to a direct line of descendant from this first individual. We enumerate the dynasties using subscript \( i \) and refer to the first individual in dynasty \( i \) as individual \( 1i \). Because there is a fixed set of individuals in the first generation and one unique dynasty originating from each of these individuals, it follows that there is a fixed set of dynasties. We define \( D \) to be the set of dynasties \( (D = \{i : \text{dynasty } i \text{ exists}\}) \) and make no stipulation on the size of \( D \) other than it is nonempty \( (|D| \in (\mathbb{N} \cup \{\infty\})) \). For every generation \( 1 \leq t \leq T \), we define \( D_t \) to be all the individuals in generation \( t \): \( D_t = \{\text{individual } ti : i \in D\} \). Because each generation has exactly one representative of each dynasty, \( D_t \) is the same size as \( D \) \((|D_t| = |D|)\).

Each dynasty has exactly one descendant in each generation and we refer to the individual in dynasty \( i \) living in generation \( t \) to be individual \( ti \). For every generation \( 1 \leq t \leq T \) and dynasty \( i \in D \), individual \( ti \) lives out their entire financial life in period \( t \) and dies at the end of period \( t \). Each individual has many functions in the economy including receiving an inheritance, working, and deciding how to spend resources through utility maximization. We analyze these functions next.

2.1.1 Lifetime Resources and Utility

For every \( 1 \leq t \leq T \) and \( i \in D \), individual \( ti \) receives pre-tax inheritance \( b_{ti} \in \mathbb{R}^+ \) from generation \( t - 1 \) at the beginning of period \( t \).\(^6\) The initial distribution of bequests given to the first generation \( \{b_{0i}\} \) is exogenous. However, for \( t > 1 \), all individuals \( ti \) receive bequest \( b_{ti} \) that is given by individual \( (t - 1)i \) who participates in our model and

\(^5\)The only function of generation \( t = 0 \) is to pass on inheritance to the first generation \((t = 1)\).

\(^6\)The set \( \mathbb{R}^+ \) is defined to be \( \{r \in \mathbb{R} : r \geq 0\} \).
chooses $b_{ti}$ to maximize their utility. Inheritances earn an exogenous gross rate of return $R$ each generation.

Individual $ti$ has exogenous pre-tax wage rate $w_{ti} \in \mathbb{R}^+$ drawn from an arbitrary but stationary ergodic distribution. For details about this distribution, see the Appendix. Individual $ti$ chooses to work $l_{ti} \in \mathbb{R}^+$ and earns $y_{L_{ti}} = w_{ti} l_{ti}$ at the end of the period. They split lifetime resources between consumption ($c_{ti} \in \mathbb{R}^+$) and bequests left ($b'_{(t+1)i} \in \mathbb{R}^+$). Lifetime resources are the sum of net-of-tax labor income and capitalized bequests received:

$$Rb_{ti}(1 - \tau_{B_t}) + w_{ti} l_{ti}(1 - \tau_{L_t}) + E_t.$$

Here, $\tau_{B_t} \in [0, 1]$ denotes the linear tax rate on capitalized bequests, $\tau_{L_t} \in [0, 1]$ denotes a linear tax rate on labor income, and $E_t \in \mathbb{R}^+$ denotes a lump sum grant given to all individuals in generation $t$.

Individual $ti$ has utility function $V_{ti} : [0, \infty)^3 \rightarrow [0, \infty)$ that is exogenously drawn from an arbitrary ergodic distribution. This utility function $V_{ti}(c, b, l)$ is increasing in consumption ($c$), increasing in net-of-tax capitalized bequests left ($b$), and decreasing in labor supply ($l$):

$$\frac{\partial V_{ti}}{\partial c} > 0, \quad \frac{\partial V_{ti}}{\partial b} > 0, \quad \frac{\partial V_{ti}}{\partial l} < 0.$$

### 2.1.2 Individual Maximization Problem

Individual $ti$ solves the maximization problem

$$\max_{l_{ti}, c_{ti}, b'_{(t+1)i} \geq 0} p V_{ti}(c_{ti}, Rb'_{(t+1)i}(1 - \tau_{B_{t+1}}), l_{ti}) + (1 - p)V_{ti}(c_{ti}, 0, l_{ti}), \quad (1)$$

and is subject to the budget constraint

$$c_{ti} + b'_{(t+1)i} = Rb_{ti}(1 - \tau_{B_t}) + w_{ti} l_{ti}(1 - \tau_{L_t}) + E_t. \quad (2)$$

The constant $p$ is the probability that a bequest is received by the next generation. We will elaborate on this more in Section 3.1. The individual is maximizing expected utility by considering both the utility experienced if their bequest is received and the utility experienced if their bequest is not received. Moreover, they are aware of the chance that their bequest is not passed on.

---

7See the Appendix for details about the distribution of utility functions
2.1.3 First Order Condition for Bequests Left

To simplify writing this first order condition (FOC), we introduce some new notation. We define

- $V_{ci}$ to be $\frac{\partial V_{ti}}{\partial c}$ applied at $(c_{ti}, Rb'_{(t+1)i}(1 - \tau_{Bt+1}), l_{ti})$,
- $V_{c,0}$ to be $\frac{\partial V_{ti}}{\partial c}$ applied at $(c_{ti}, 0, l_{ti})$,
- $V_{bt}$ to be $\frac{\partial V_{ti}}{\partial b}$ applied at $(c_{ti}, Rb'_{(t+1)i}(1 - \tau_{Bt+1}), l_{ti})$.

The derivation of the FOC for bequests left $b_{(t+1)i}$ requires us to set up a constrained maximization problem with a Lagrangian multiplier. We do this in the Appendix. The resulting FOC is

$$pV_{ci} + (1 - p)V_{c,0} = R(1 - \tau_{Bt+1})pV_{bt}. \quad (3)$$

This implies that when an individual is at a utility maximizing point, the expected utility of consuming one additional unit is the same as the expected utility of leaving one additional unit to their heir.

2.2 Government Policy

The government uses taxation and lump sum redistribution to maximize social welfare, which we define below. For each generation $(t)$, they set a linear labor tax rate $\tau_{Lt}$ and a linear tax on capitalized bequests $\tau_{Bt}$. They distribute all tax proceeds evenly through a lump sum grant $E_t$. While the government could certainly increase social welfare using a non-linear tax scheme or distributing more money to some than others, we restrict government to linear taxes and a uniform redistribution of tax proceeds (a fixed lump sum).\footnote{Because government can only choose linear tax rates and a uniform lump sum for everyone, they could do just as well and almost surely better if they were not restricted in this way. If the government could choose any tax schedule and different lump sums for different people, they have more options to choose from, including linear tax rates and fixed lump sums.}

The government is subject to a period-by-period balanced budget

$$E_t = (1 - \delta_{Bt})\tau_{Bt} Rb'_t + (1 - \delta_{Lt})\tau_{Lt} y_{L_t}. \quad (4)$$
The constants $\delta_{B_t}, \delta_{L_t} \in [0, 1]$ are frictions on the bequest and income tax respectively. We will provide details about these frictions in Section 3.2. Notice that taxes are collected on bequests left by generation $t-1$ ($b'_t$), not the bequests received by generation $t$ ($b_t$). So even if a bequest is not received by the next generation, taxes are paid on it.

Social welfare is defined as a weighted sum of individual expected utilities. Each individual $i$ is assigned Pareto weight $\omega_{ti} \geq 0$ by the government and through this mechanism, the government can choose to care more about the utility of some individuals and less about that of others. The social welfare function ($SWF$) is

$$SWF = \max_{\tau_L, \tau_B} \left[ \left( \int_{D_t} \omega_{ti} V^{ti}(Rb_{ti}(1 - \tau_{B_t}) + w_{ti}l_{ti}(1 - \tau_{L_t}) ight. ight.
\left. + E_t - b'_{(t+1)i}, Rb'_{(t+1)i}(1 - \tau_{B_t}), l_{ti} \right) 
\left. + \left( \int_{D_t} (1-p)\omega_{ti} V^{ti}(Rb_{ti}(1 - \tau_{B_t}) + w_{ti}l_{ti}(1 - \tau_{L_t}) 
+ E_t - b'_{(t+1)i}, 0, l_{ti} \right) \right] \right].$$

### 2.3 Defining an Equilibrium

First, we define the following aggregate variables for generation $t$:

- $b_t = \int_{D_t} b_{ti}$ (aggregate bequests received from generation $t-1$),
- $b'_{t+1} = \int_{D_t} b'_{(t+1)i}$ (aggregate bequests left to generation $t+1$),
- $c_t = \int_{D_t} c_{ti}$ (aggregate consumption),
- $y_{L_t} = \int_{D_t} y_{L_{ti}}$ (aggregate labor income).

Because each generation has measure 1, these variables also represent averages.

#### 2.3.1 The Equilibrium

We assume the stochastic process for utility functions ($V^{ti}$) and wage rates ($w_{ti}$) fulfill an ergodicity condition and are such that the economy converges to a unique ergodic steady-state equilibrium. Among other things, this results in a steady-state ergodic equilibrium distribution of bequests and earnings ($b_{ti}, y_{L_{ti}}$). To achieve this equilibrium, tax rates and lump-sum grants must be constant across generations. This equilibrium is independent of the initial distribution of bequests ($b_{0i}$).
and in the long run, the position of each dynasty \(i\) is independent of the initial position \((b_{0i}, y_{L0i})\).\(^9\)

There are a few important consequences of this equilibrium. Aggregate variables for bequests received, bequests left, consumption, and labor income will be consistent across generations. That is, for every pair of generations \(0 \leq t_1, t_2 \leq T\), we have

\[
\begin{align*}
  b_{t_1} &= b_{t_2}, & b'_{t_1} &= b'_{t_2}, & c_{t_1} &= c_{t_2}, & \text{and} & & y_{L_{t_1}} &= y_{L_{t_2}}.
\end{align*}
\]

Also as a result of this equilibrium, government can think about maximizing welfare in a steady state as opposed to maximizing welfare every generation.\(^10\) Therefore, they can choose one \(\tau_B\), one \(\tau_L\), and one \(E\) instead of one for each generation.

3 Additions to the Piketty-Saez Model

We introduce two major nuances into the Piketty-Saez model. First, an individual cannot be sure that their bequest gets passed on to the next generation. Second, there are administrative costs to taxation, both on bequests and labor income. We describe the implementation of these ideas into the model below.

Additionally, the potential difference between the amount of bequests left by one generation and bequests received by the next generation creates the need for additional analysis. This issue is also addressed in this section.

3.1 Uncertainty of Bequest Reception

Assume there is a fixed chance \((1 - p)\) for all individuals that their bequest will not be passed on. In particular, for a fixed \(p \in [0, 1]\), the bequest will be passed on with probability \(p\) and will not be passed on with probability \(1 - p\).

When we refer to a bequest being successfully “passed on to” or “received” by individual \((t + 1)i\), we mean that the bequest was received in the way the bequest leaver intended. That is, individual \((t + 1)i\) received the entire bequest (after tax and interest). If this bequest is “not passed on”, none of it is received by individual \((t + 1)i\).

---

\(^9\)See Appendix for formulation of the equilibrium and its implications.

\(^{10}\)See Appendix for further details.
Our model differentiates in notation between $b'(t+1)_i$, the amount left by individual $ti$, and $b(t+1)_i$, the pre-tax and pre-interest amount actually received by individual $(t+1)i$. With probability $p$, the bequest is received by individual $(t+1)i$ and hence $b(t+1)_i = b'(t+1)_i$. Similarly, with probability $1-p$, the bequest is not received by individual $(t+1)i$ and hence $b(t+1)_i = 0$. While we mentioned in the introduction that individuals could have different sources of uncertainty, for simplicity, we will assume that all individuals across generations have the same perception of the amount of uncertainty and that perception is reflective of reality.

Moreover, we need to make a few important assumptions about bequests not received. First, we make no independence assumptions over the outcomes of different individuals. That is, for all $i, j \in D$ and $1 \leq t_1, t_2 \leq T$, the event that $b(t_1+1)_i = 0$ is independent of the event that $b(t_2+1)_j = 0$. Second, when a bequest leaver considers the event of their bequest not being received, their utility function reflects that zero-bequests are being left. This event would result in no contribution from the bequest being left to utility, that is, the input in the utility function for $b$ would be 0 (we see this in the second term of (1)). Lastly, tax is always collected on a left bequest, even if it is not passed on. Of all the sources of uncertainty discussed in the introduction, none of them would result in taxes not being paid on the estate. As the famous saying goes, “in this world nothing can be said to be certain, except death and taxes.”

### 3.2 Frictions on Taxes

For every unit of taxes paid by individuals, there is a certain amount lost to tax collection costs and other administrative expenditures. We will refer to such losses as frictions. In our model, for a friction $\delta \in [0,1]$, $\delta$ of every unit raised in taxes will not be available for the government to use for lump sum redistribution. We consider two frictions in our model, one on bequest taxes ($\delta_B \in [0,1]$) and another on labor income tax ($\delta_L \in [0,1]$).

---

11 A quote from Benjamin Franklin.
3.3 Ratio of Bequests Left to Bequests Received

We introduce a parameter to our model that relates bequests received to bequests left. We define $\gamma_t^{(p)}$ to be the ratio of bequests left (to generation $t + 1$, by generation $t$) to bequests received (by generation $t$, from generation $t - 1$),

$$\gamma_t^{(p)} = \frac{b_{t+1}'}{b_t}.$$  \hspace{1cm} (5)

This ratio will depend on $p$ because the amount of bequests actually received depends on the probability a bequest is passed on. Using the independence assumption on whether bequests are received and the fact that in steady state, aggregate bequests left are the same for each generation, we would expect $\gamma_t^{(p)} = \frac{1}{p}$.  \hspace{1cm} (12)

4 Steady-Steady Welfare Maximization

In this section, we solve the government maximization problem for the long-run steady state equilibrium of the economy. In this problem, the government chooses long-run policies for the lump sum ($E$), linear labor tax rate ($\tau_L$), and linear tax on capitalized bequests ($\tau_B$) to maximize steady-state social welfare. We can update our balanced budget equation (4) to include these long run policies:

$$E_t = (1 - \delta_B)\tau_B Rb_t' + (1 - \delta_L)\tau_L y_L t.$$  \hspace{1cm} (6)

We can also update our social welfare function:

$$SWF = \max_{\tau_L,\tau_B} \left[ \left( \int_{D_1} p\omega_t V^{ti}(Rb_{iti}(1 - \tau_B) + w_{iti}l_{iti}(1 - \tau_L) \\
+ E - b_{(t+1)iti}' Rb_{iti}(1 - \tau_B), l_{iti}) \right) \\
+ \left( \int_{D_1} (1 - p)\omega_t V^{ti}(Rb_{iti}(1 - \tau_B) + w_{iti}l_{iti}(1 - \tau_L) \\
+ E - b_{(t+1)iti}', 0, l_{iti}) \right) \right].$$

12See Appendix for more details.
We assume that $E$ is fixed and so $\tau_L$ and $\tau_B$ are linked to meet the balanced budget. Therefore, we can solve for the optimal $\tau_B$ in terms of $\tau_L$, and several other macro variables.\textsuperscript{13} Some of these other variables are elasticities that measure behavioral responses to taxation and distributional parameters that are normatively defined.

### 4.1 Elasticity Parameters

We define long-run elasticities for bequest flow and labor supply based on aggregate variables.

Define $e_B$ to be the long-run elasticity of aggregate bequest flow (aggregate capital accumulation) with respect to the net-of-bequest-tax rate $1 - \tau_B$. Here, we use that $b_t$ is a function of $1 - \tau_B$ (assuming that $\tau_L$ adjusts). Formally,

$$
e_B = \frac{1 - \tau_B}{b_t} \frac{db_t}{d(1 - \tau_B)} \bigg|_{E}, \tag{7}$$

where the notation identifies that the lump sum $E$ is fixed. Define $e_L$ to be the long-run elasticity of aggregate labor supply with respect to the net-of-labor-tax rate $1 - \tau_L$. Here, we use that $y_{Lt}$ is a function of $1 - \tau_L$ (assuming that $\tau_B$ adjusts). Formally,

$$
e_L = \frac{1 - \tau_L}{y_{Lt}} \frac{dy_{Lt}}{d(1 - \tau_L)} \bigg|_{E}. \tag{8}$$

These elasticities are “policy elasticities” [17] that capture budget neutral joint changes in $(\tau_L, \tau_B)$ and can be estimated empirically.

We also introduce an individual elasticity of bequests received for every individual $ti$,

$$
e_{bi} = \frac{1 - \tau_B}{b_{ti}} \frac{db_{ti}}{d(1 - \tau_B)} \bigg|_{E}. \tag{9}$$

Notice that $e_B$ is the bequest-weighted population average of $e_{bi}$,

$$e_B = \frac{\int b_{ti} e_{bi}}{b_t}. $$

\textsuperscript{13}The optimal $\tau_B$ formula does not include $E$ directly.
Here, we use the fact that each generation has measure 1. We also define $\hat{e}_B$ to be the average of $e_{B_{ti}}$ weighted by $g_{ti}b_{ti}$,

$$\hat{e}_B = \frac{\int_{D_t} g_{ti}b_{ti}e_{b_{ti}}}{b_{t}}.$$  \hspace{1cm} (10)

Note that $\hat{e}_B$ is equal to $e_B$ if individual bequest elasticities $e_{b_{ti}}$ are uncorrelated with $g_{ti}$.\textsuperscript{14}

### 4.2 Distributional Parameters

To capture distributional parameters of earnings, bequests received, and bequests left, we define normalized variables that are functions of the individual Pareto weights among other things. We first assign each individual $ti$ a social marginal welfare weight $g_{ti}^{(p)}$ (that depends on $p$),

$$g_{ti}^{(p)} = \frac{\omega_{ti}[pV_{c}^{ti} + (1-p)V_{c,0}^{ti}]}{\int_j \omega_j[pV_{c}^{tj} + (1-p)V_{c,0}^{tj}]}.$$  \hspace{1cm} (11)

The weights $g_{ti}$ are normalized to sum to 1 in each generation. That is, $\int_{D_t} g_{ti}^{(p)} = 1$ (where we use the fact that each generation has measure 1). Intuitively, $g_{ti}$ measures the social value of increasing the consumption of individual $ti$ by 1 unit. The measurement is relative to distributing the 1 unit equally across all individuals).

To measure the distributional parameter of each variable, we use ratios of the weighted (by $g_{ti}$) population average of that variable to the unweighted population average:

$$\bar{y}_{received}^{(p)} = \frac{\int_{D_t} g_{ti}^{(p)}b_{ti}}{b_{t}}; \hspace{1cm} \bar{y}_{left}^{(p)} = \frac{\int_{D_t} g_{ti}^{(p)}b_{(t+1)i}}{b_{t+1}}; \hspace{1cm} \bar{y}_{L}^{(p)} = \frac{\int_{D_t} g_{ti}^{(p)}y_{L_{ti}}}{y_{L_{t}}}.$$  \hspace{1cm} (12)

These ratios are below 1 if the corresponding variable is lower for those with high social marginal welfare weights and above 1 if higher. For example, if our weights favored the low income earners who also leave and receive smaller bequests, the parameters would be below 1.

\textsuperscript{14}See Appendix for details.
4.3 Derivation of Optimal Linear Tax on Capitalized Bequests

We now turn to finding a formula for the optimal bequest tax rate. The optimal rate will depend on many of the variables we have introduced, including the tax rate on labor income $\tau_L$, behavioral elasticities $e_B$ and $e_L$, distributional parameters $\bar{b}_{\text{received}}(p)$, $\bar{b}_{\text{left}}(p)$, gross rate of return $R$, measure of uncertainty $p$, tax frictions $\delta_B$ and $\delta_L$, and ratio of bequests left to bequests received $\gamma_t(p)$.

**Proposition 1.** Assume $\tau_L$, $e_B$, $e_L$, $\bar{b}_{\text{received}}(p)$, $\bar{b}_{\text{left}}(p)$, $\bar{y}_L(p)$, $R$, $p$, $\delta_B$, $\delta_L$, and $\gamma_t(p)$ are all given and $R$, $\bar{y}_L(p) > 0$. The optimal linear tax on capitalized bequests $\tau_B$ that maximizes long-run steady-state social welfare with period-by-period budget balance is given by

$$\tau_B = \frac{1 - \frac{1 - \delta_L}{1 - \delta_B} \cdot [1 - \frac{e_L \tau_L}{1 - \tau_L}] \cdot \left[ \frac{\bar{b}_{\text{received}}(p)}{\bar{y}_L(p)} (1 + \hat{e}_B) + \frac{\gamma_t(p)}{R} \cdot \frac{\bar{b}_{\text{left}}(p)}{\bar{y}_L(p)} \right]}{1 + e_B - \frac{1 - \delta_L}{1 - \delta_B} \cdot [1 - \frac{e_L \tau_L}{1 - \tau_L}] \frac{\bar{b}_{\text{received}}(p)}{\bar{y}_L(p)} (1 + \hat{e}_B)}.$$  

(13)

Given the formula, we needed to assume that $R$ and $\bar{y}_L(p)$ are both greater than zero. The assumption that $R > 0$ is easily justified, given that a realistic value of $R$ is close to 1. The assumption that $\bar{y}_L(p) > 0$ is also quite likely, but requires a little analysis. We need there to exist a subset of individuals in generation $t$, $P \subseteq D_t$, with the following three properties: $P$ has positive measure, every individual in $P$ has non-zero Pareto weight ($\omega_{ti} > 0$), and every individual in $P$ has non-zero income $y_{ti}$. In this case, looking at the definitions of $g_{ti}(p)$ (11) and $\bar{y}_L(p)$ (12), we see that $\bar{y}_L(p)$ must be greater than zero.

In this formula derivation, we manipulate several definitions, use the FOC for bequests left, and apply the envelope theorem. We now describe this derivation.

**Proof.** We consider a small reform in the bequest tax $d\tau_B > 0$. For the budget to stay balanced, the labor tax must decrease ($d\tau_L < 0$). Given that the budget is balanced and lump sum is fixed ($dE = 0$) we can derive a useful equality.

Recall the balanced budget equation (6). Differentiating the lump sum $E$ and given that $dE = 0$, we get the following:
0 = dE = (1−δ_B)(Rb'_tdτ_B+τ_BRdb'_t)+(1−δ_L)(y_Ltdτ_L+τ_Ldy_L) \quad (14)

Using the elasticity equations (7) and (8), we get the following two equalities:

\begin{align*}
(1−δ_B)(Rb'_tdτ_B+τ_BRdb'_t) &= (1−δ_B)Rb'_tdτ_B(1−e_B \frac{τ_B}{1−τ_B}), \quad (15) \\
(1−δ_L)(y_Ltdτ_L+τ_Ldy_L) &= (1−δ_L)dτ_Ly_L(1−e_L \frac{τ_L}{1−τ_L}). \quad (16)
\end{align*}

Substituting in \( b'_t \) and \( db'_t \) for \( b_t \) and \( db_t \) in (15), we get

\begin{align*}
(1−δ_B)(Rb'_tdτ_B+τ_BRdb'_t) &= (1−δ_B)Rb'_tdτ_B(1−(\frac{1−τ_B}{b'_t} \frac{db'_t}{d(1−τ_B)} \frac{τ_B}{1−τ_B})) \\
&= (1−δ_B)Rb'_tdτ_B(1−(e_B b'_t \frac{db'_t}{d(1−τ_B)} \frac{τ_B}{1−τ_B})).
\end{align*}

The independence assumption on bequests received gives us that \( b'_t = \frac{db'_t}{d(1−τ_B)} \). Replacing \( \frac{db'_t}{d(1−τ_B)} \) for \( p \) and \( \frac{1}{p} \) respectively,

\begin{align*}
(1−δ_B)(Rb'_tdτ_B+τ_BRdb'_t) &= (1−δ_B)Rb'_tdτ_B(1−e_B \frac{τ_B}{1−τ_B}). \quad (17)
\end{align*}

Substituting (17) and (16) into (14), we get the equality

\begin{align*}
(1−δ_B)Rb'_tdτ_B(1−e_B \frac{τ_B}{1−τ_B}) &= −(1−δ_L)dτ_Ly_L(1−e_L \frac{τ_L}{1−τ_L}). \quad (18)
\end{align*}

Applying the envelope theorem and using the fact that \( b'_{(t+1)i} \) and \( l_{ti} \) are chosen to maximize individual utility, we get that the effect of the reforms (\( dτ_B \) and \( dτ_L \)) on steady-state social welfare is

\begin{align*}
\frac{dSWF}{dτ_B} &= \int_{D_i} \ω_i(pV_{ti}^c+(1−p)V_{ti}^{L_i}) \cdot [Rdb_{ti}(1−τ_B)−Rb_{ti}dτ_B−dτ_Ly_{Li}] \\
&+ \ω_iV_{ti}^{L_i} \cdot (−dτ_BRb'_{(t+1)i}).
\end{align*}

Using the first order condition for bequests left (3), we can substitute out \( V_{ti}^{L_i} \) and get
\[ dSWF = \int_D \omega_t \left( pV_c^{ti} + (1 - p)V_{c,0}^{ti} \right) \cdot \left[ R\bar{b}_t \tau (1 - \tau_B) - R\bar{b}_t \tau d\tau_B - d\tau_L y_L t_i \right] + \omega_t \left( \frac{pV_c^{ti} + (1 - p)V_{c,0}^{ti}}{pR(1 - \tau_B)} \right) \cdot ( - d\tau_B Rb''_{t+1}(t+1) ). \]

At the optimum \( \tau_B \), we have that \( dSWF = 0 \). Noting this, dividing by \( \int_j \omega_j \left[ pV_c^{tj} + (1 - p)V_{c,0}^{tj} \right] \), and using the definition of \( g_{t_i}^{(p)} \) (11),

\[
0 = \int_D g_{t_i}^{(p)} \cdot \left[ R\bar{b}_t \tau (1 - \tau_B) - R\bar{b}_t \tau d\tau_B - d\tau_L y_L t_i \right] + g_{t_i}^{(p)} \cdot \left( \frac{-d\tau_B Rb''_{t+1}(t+1)}{pR(1 - \tau_B)} \right) - \int_D g_{t_i}^{(p)} \cdot \left[ R\bar{b}_t \tau (1 - \tau_B) - R\bar{b}_t \tau d\tau_B - d\tau_L y_L t_i - \frac{d\tau_B Rb''_{t+1}(t+1)}{p(1 - \tau_B)} \right].
\]

Using the definition of \( e_{bt_i} \) (9), we can simplify to

\[
0 = \int_D g_{t_i}^{(p)} \cdot \left[ -d\tau_B R\bar{b}_t \tau (1 + e_{bt_i}) - d\tau_L y_L t_i = \frac{d\tau_B Rb''_{t+1}(t+1)}{p(1 - \tau_B)} \right].
\]

Rearranging (18) and substituting for \( d\tau_L \), we get

\[
0 = \int_D g_{t_i}^{(p)} \cdot \left[ -d\tau_B R\bar{b}_t \tau (1 + e_{bt_i}) + \frac{1 - \delta_B}{\delta_L} \cdot \frac{1 - e_B \tau_B (1 - \tau_B)}{1 - e_L \tau_L (1 - \tau_L)} \cdot \frac{y_{L,t_i}^{(p)}}{y_L^{(p)}} Rb''_{t+1}(t+1) \right].
\]

Dividing by \( R\bar{b}_t \tau d\tau_B \) and using the definition of \( \hat{e}_B \) (10), \( \gamma_t^{(p)} \) (5), and our three distributional parameters (12), we get

\[
0 = \int_{D,t_i}^{(p)} g_{t_i}^{(p)} \left( 1 + \hat{e}_B + \frac{1 - \delta_B}{\delta_L} \cdot \frac{1 - e_B \tau_B (1 - \tau_B)}{1 - e_L \tau_L (1 - \tau_L)} \cdot \frac{y_{L,t_i}^{(p)}}{y_L^{(p)}} \right) - \frac{\gamma_t^{(p)} g_{t_i}^{(p)}}{R (1 - \tau_B)}. \]

From the independence assumption on bequests left, \( \frac{b_L^{(p)}}{b_t^{(p)}} = \frac{1}{p} \). By rearranging, we obtain the steady-state optimum linear tax rate on capitalized bequests,

\[
\tau_B = \frac{1 - \frac{1 - \delta_B}{1 - \delta_L} \cdot \frac{1 - e_L \tau_L (1 - \tau_L)}{1 - e_L \tau_L (1 - \tau_L)} \cdot \frac{g_{t_i}^{(p)} y_{L,t_i}^{(p)}}{y_L^{(p)}} (1 + \hat{e}_B) + \frac{\gamma_t^{(p)} g_{t_i}^{(p)}}{R (1 - \tau_B)}}{1 + e_B \left( 1 - \frac{1 - \delta_B}{1 - \delta_L} \cdot \frac{1 - e_L \tau_L (1 - \tau_L)}{1 - e_L \tau_L (1 - \tau_L)} \cdot \frac{g_{t_i}^{(p)} y_{L,t_i}^{(p)}}{y_L^{(p)}} (1 + \hat{e}_B) \right)}.
\]

\[ \square \]
4.4 A Special Case: The Piketty-Saez model

The model that we have presented is a more general framework than the Piketty-Saez model. The Piketty-Saez model has three additional assumptions as compared to my model: all bequests left will certainly be passed on to the next generation \( p = 1 \), there are no administrative costs for the linear tax on capitalized bequests \( \delta_B = 0 \), and there are no administrative costs for the linear labor tax \( \delta_L = 0 \).

A result of the steady state optimum is that the amount of bequests left is constant throughout generations. So for every generation \( t \), \( b_{t+1} = b_t \). Further, the assumption that \( p = 1 \) gives me that \( b'_{t+1} = b_{t+1} \) for any \( t \) (because no bequests are taken away in this case). From these two observations, it follows that \( \gamma^{(1)}_{t} = \frac{b'_{t+1}}{b_t} = 1 \).

These assumptions simply our steady-state optimal tax rate (13) to

\[
\tau_B = \frac{1 - [1 - \frac{\epsilon_L \tau_L}{1 - \tau_L}] \left[ \frac{\xi^{(1)}_{received}}{\bar{y}_L^{(1)}} (1 + \hat{e}B) + \frac{1}{\tau_L} \frac{\bar{y}_L^{(1)}}{\xi^{(1)}_{received}} \right]}{1 + \epsilon_B - [1 - \frac{\epsilon_L \tau_L}{1 - \tau_L}] \frac{\bar{y}_L^{(1)}}{\xi^{(1)}_{received}} (1 + \hat{e}B)}.
\]

Note to readers: This is an incomplete version of my paper. It is missing Section 5 (Implications of the Steady State Optimum) which includes analysis of the relationship between variables and graphs of the optimal bequest tax for a special case of the model. It is also missing the Appendix and conclusion.
References


