

An Uncertainty Quantification and Aggregation Framework for System Performance Assessment in Industrial Maintenance

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Abstract

The exponential increase in technological complexity of modern engineering systems necessitates rigorous and accurate maintenance planning to determine optimum equipment availability and turnaround time whilst allowing for overruns and unforeseen costs. Quality and availability of quantitative data, as well as qualitative expert opinion and experience expose uncertainties that can result in under or over estimation of the above factors. Uncertainty quantification in complex engineering systems should consider inter-connected components and associated processes from a combination of quantitative and qualitative (compound) perspectives. This paper presents a framework to quantify and aggregate compound uncertainties and to be assessed against a predetermined acceptable level of uncertainty. This will provide maintenance planners with a confident, comprehensive view of parameters surrounding the above factors to improve decision making capabilities. The framework was validated by assessing individual and compound uncertainties in a bespoke heat exchanger test rig comprised of subsystem modules interact in a non-linear manner, as well as subjective opinions and actions of operators. The results demonstrate the framework's ability to effectively quantify these factors with an indication of their impact on the system. Future work will include further validation with more complex case studies and development of methods to forecast the quantified uncertainty through the in-service phase of an asset's life cycle.

Keywords: Coefficient of variation; Complex engineering system; Heat exchanger; Pedigree matrix; Uncertainty quantification

1. Introduction

The maintenance of complex systems in an industrial engineering setting sees multiple sub-elements interacting simultaneously and nonlinearly with each other and the environment on multiple levels [1,2]. This includes equipment and workers operating with common material and information flow [3]. Complex engineering systems (CES) inherit a range of uncertainties from factors such as quality and availability of quantitative equipment data and the qualitative influence of workers, expert opinion, experience and environmental conditions. These uncertainties can lead to under or over estimation of maintenance costs, reliability measurement, equipment availability and delays in maintenance scheduling. Research into uncertainty quantification (UQ) around CES generally only considers quantitative, measured data [2,4,5].

Methods to quantify compound effects of different types of uncertainty (combining quantitative and qualitative) are necessary to capture their full system impact. This impact influences system reliability and, therefore, decisions made around system maintenance. The challenge grows in the context of CES with interrelationships between sub-elements. Here, different data recording methods used and assumptions made prompt the

different types of uncertainty represented by different probability distribution functions (PDFs).

This paper presents a 5-step framework to quantify and aggregate compound uncertainties to enhance system performance assessment. This will provide maintenance planners with a confident, comprehensive view of parameters surrounding the above factors to improve decision making capabilities.

A literature review into uncertainty classification in the context of this paper and techniques to combine quantitative and qualitative uncertainties is depicted in Section 2. Details of framework development are described in Section 3 along with key mathematical formulae, functions and assumptions made. Section 4 applies the framework to a case study utilising a bespoke heat exchanger test rig developed at Cranfield University [6]. Individual uncertainties from quantitative and qualitative sources are assessed and aggregated to give a confident indication of system performance. Section 5 discusses case study results, strengths and limitations of the framework along with conclusions and future work in this area.

2. Literature review

2.1. Uncertainty classification

Uncertainty is the degree of information, or lack of information, known about a given entity; be it measured data, equipment state, environmental conditions or accuracy of expert opinion. Error is the difference between the recorded and true value of a measured entity. The resulting risk is the probability of loss or gain of the value of that entity [7,8]. It is important to look beyond the probabilistic world and embrace subjective and expert opinions [7].

A confident uncertainty estimate can be positively utilised to aid decision making. Two key types of uncertainty are described in the Guide to the Expression of Uncertainty in Measurement (GUM): Type A, sourced from quantitative measured data, expressed by the standard deviation of the dataset; and Type B, which considers qualitative technical and expert knowledge or experience as well as environmental conditions [1,9–11]. Implementations of the GUM are explored in Section 2.2. It is necessary to distinguish types of uncertainty to reduce risk and avoid under or over-estimation or of the probability of failure in a system [12–14]. This paper will hence refer to Type A as ‘quantitative’ and Type B as ‘qualitative’ uncertainty.

2.2. Combining quantitative and qualitative uncertainty

The GUM has been implemented in a range of UQ-related applications [1,2,9,10,15,16]. These typically follow 5 core stages [1,9,15]: (1) Identify the measurand; (2) Identify uncertainty sources and associated PDFs; (3) Quantify uncertainties (simulation); (4) Aggregate uncertainties; (5) Report analysis results. Coverage factors are applied to accommodate for qualitative estimates, which can lead to underestimation of total uncertainty and cannot be confidently applied in dynamic CES [17,18].

The pedigree approach is a widely renowned and verified method to equate qualitative estimates in line with quantitative data. First proposed by Funtowicz and Ravetz [19], the approach comprises of a matrix to score expert knowledge and opinion according to predefined criteria to permit quantitative performance and reliability assessment. The approach has been applied in environmental fields such as meteorology, oil & gas and genealogy [11,16,18,20–22]. It can be applied on its own as well as part of an approach to standardise combined uncertainty dimensions via 5 qualifiers: Numeral, Unit, Spread, Assessment and Pedigree (NUSAP) [11,16,20,23].

Ciroth et al. [16] presented a process to improve uncertainty estimation by gauging qualitative uncertainty factors through the pedigree approach for flow data in a multidimensional database. Estimates were attributed by their geometric standard deviation (GSD), where inputs fit to the multiplicative lognormal distribution. It is stated that the arithmetic standard deviation used to attribute uncertainty has the disadvantage of relying on the scale (unit) of data in a linear manner [16,24]. Therefore, for the analysis of data from varying sources and measured in different units, uncertainty factors need to be independent of scaling effects. Using GSD as the uncertainty measure overcomes this scale dependency.

To enable uncertainties to be aggregated where data sources do not follow a lognormal distribution, a ratio between the standard deviation and the mean is obtained via the coefficient of variation (CV) [24,25]. These can be given as a percentage to represent uncertainty. Muller et al. [24] defined a set of equations to convert symmetric and asymmetric PDFs to their respective CVs, shown in Table 1.

Given as a dimensionless measure of variability, the CV can be used as a measure of uncertainty for each input and aggregated to give a representative total. The application of the pedigree approach and CVs in the context of this paper is detailed in Section 3. It is highlighted that while this method can provide confident approximations when data characteristics are unknown, it should not be used in place of raw data and statistics when sufficient data is available [24].

2.3. Research gaps

The majority UQ approaches follow variations of the 5-stage process defined in the GUM [15,26]. Considerations of qualitative uncertainty are best made through the pedigree approach. Identification of the most appropriate PDF to represent each input is key to assess its uncertainty [21,27]. The approach described by Ciroth et al. [16] and Muller et al. [24] can be used to quantify and aggregate compound uncertainties through their respective CVs, applied to a range of symmetric and asymmetric PDFs. However, the consideration of qualitative factors through their GSD in the pedigree approach assumes that such factors can only be lognormally distributed [24]. While formulae to denote inputs of varying PDFs by their respective CVs are defined, a method to aggregate CVs from a mix of symmetric and asymmetric PDFs in a compound manner is unclear. This is necessary to establish compound uncertainty estimates represented by different PDFs with a high level of confidence.

Table 1. Probability distribution function (PDF) and relative coefficient of variation (CV) calculations [16,24]

Distribution	Parameters	Deterministic value	PDF	CV calculation
Lognormal	x : Input dataset μ_g : Geometric mean σ_g : Geometric standard deviation (GSD)	Median: μ_g	$f(x, \mu_g, \sigma_g) = \frac{\exp\left(-\frac{(\ln x - \ln \mu_g)^2}{2 \ln^2 \sigma_g}\right)}{\sqrt{2\pi} \ln \sigma_g}$	$CV = \sqrt{\exp(\ln^2 \sigma_g) - 1}$
Normal	x : Input dataset μ : Arithmetic mean σ : Arithmetic standard deviation	Mean: μ	$f(x, \mu, \sigma) = \frac{\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{\sigma\sqrt{2\pi}}$	$CV = \frac{\sigma}{\mu}$
Uniform	x : Input dataset a : Minimum value b : Maximum value	Mean: $\frac{a+b}{2}$	$f(x, a, b) = \frac{1}{b-a}$ for $a < x < b$ otherwise, $f(x, a, b) = 0$	$CV = \frac{b-a}{\sqrt{3}(b+a)}$
Triangular	x : Input dataset a : Minimum value b : Maximum value c : Most likely value	Most likely value: c	$f(x, a, b, c) = \frac{2(x-a)}{(b-a)(c-a)}$ for $a < x < c$ $f(x, a, b, c) = \frac{2(b-x)}{(b-a)(b-c)}$ for $c < x < b$ otherwise, $f(x, a, b, c) = 0$	$CV = \frac{\sqrt{a^2+b^2+c^2-ab-ac-cb}}{\sqrt{2}(a+b+c)}$

3. Framework development: Compound uncertainty aggregation

To provide an assessment of system performance considering individual and aggregated uncertainty within the system, addressing the research gaps above, the 5-step framework was developed in MATLAB, described below and illustrated in Fig. 1. This was applied in a case study considering key parameters identified within the system, detailed in Section 4.

Step 1 identifies and groups the uncertainty sources as inputs according to their type – quantitative or qualitative.

Step 2 calculates relevant statistical parameters for each input via Monte Carlo simulation and the pedigree matrix for respective types, elaborated as follows:

Step 2a: The recorded quantitative data is imported as single column arrays, concatenated in a cell array to allow inputs with a varying number of data points to be considered under a single array. Any non-numeric (NaN) values are removed. The arithmetic and geometric mean and standard deviation are calculated, as well as maximum and minimum values of each input variable. Monte Carlo simulations are run for lognormal, normal and uniform PDFs for a defined number of points (default 10,000). The standard deviation is then calculated using the simulated data for each distribution type. For lognormal parameters, the mean and standard deviation is given as geometric. Normal and uniform distribution parameters are arithmetic.

Step 2b: The qualitative factors are scored by predefined pedigree criteria. The ideal case has minimal uncertainty and a pedigree score of 1.

Scores of 2-5 have progressively higher uncertainties owing to their representative criteria. The scores for each factor correspond to an uncertainty indicator. These were defined according to the subjective impact each factor has on the system.

The GSD of each factor is calculated from the indicator values obtained from one or multiple sources. If the uncertainty indicators are obtained from a single source, the GSD is given as its square root. If they are obtained from multiple sources, the GSD is given by Eq. 1, modelled by the lognormal distribution [16,24,28]. Ideally, and especially in the case of CES, the definition of pedigree criteria and related uncertainty indicators should be made by a diverse selection of suitably qualified individuals.

$$\sigma_g = \exp\left(\sqrt{\frac{1}{n} \times \sum_{i=1}^n \ln\left(\frac{x_i}{\bar{x}_g}\right)^2}\right) \tag{1}$$

Where:

σ_g = GSD, n = number of inputs, x_i = dataset, \bar{x}_g = geometric mean of dataset

The GSD of less ideal indicators is given as a ratio of the calculated GSD and that of the ideal score for each input, meaning that it is always equal to or greater than 1 [16].

Step 3 calculates the CV for each input. In order for uncertainties from different data types represented by different PDFs to be aggregated, they must be considered on an equal scale. This is achieved through the CV, explained in Section 2.2, the formulae for which are illustrated in Table 1 [24].

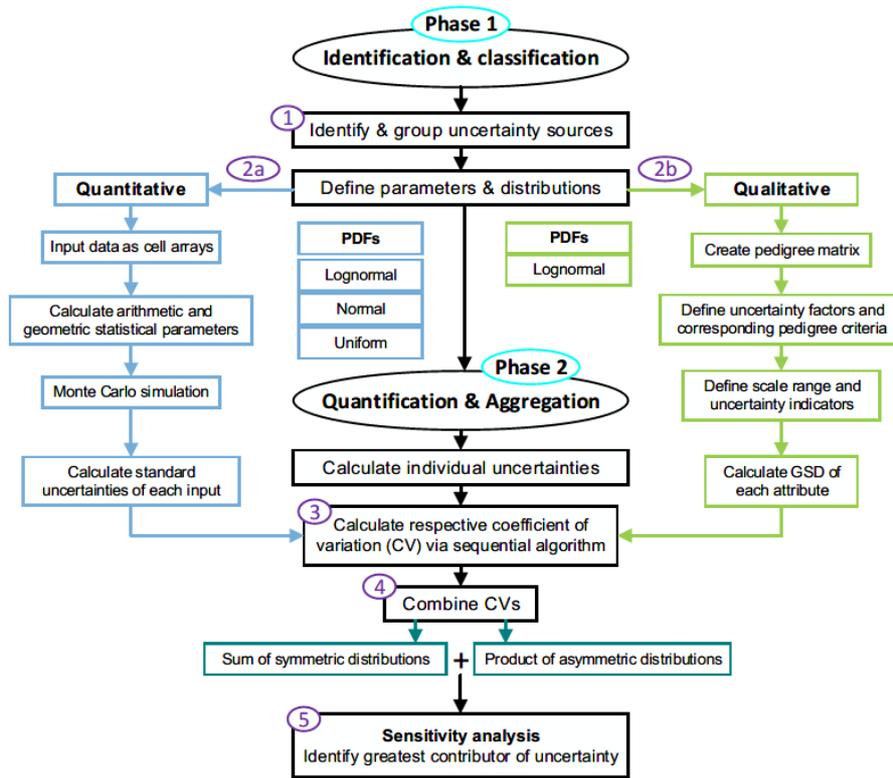


Fig. 1. Framework overview

These are calculated within the framework by a sequential algorithm according to the specified input and distribution type. Summary tables are then generated for the quantitative and qualitative inputs detailing key parameters calculated in Steps 2-3.

Step 4 combines the respective CVs. Normal and uniform distributions are symmetric; the arithmetic mean is equal to the mode and does not change when uncertainty increases [24]. They can therefore be aggregated additively by the root-sum-square (RSS) formula (Eq. 2). Lognormal distributions are asymmetric; the arithmetic mean will change with increasing or decreasing uncertainty. CVs represented by the lognormal distribution, CV_{Ln} , are aggregated multiplicatively by Eq. 3 [24]. To combine these with symmetric distributions, a new arithmetic mean needs to be calculated to account for the shifting uncertainty, given by Eq. 4 [24].

$$CV_{Sym} = \sqrt{\sum_{i=1}^n (CV_i^2)} \quad (2)$$

$$CV_{Ln} = \sqrt{\prod_{i=1}^n (CV_i^2 + 1) - 1} \quad (3)$$

$$\mu_T CV_T = \mu \sqrt{CV_{Sym}^2 + CV_{Logn}^2} \quad (4)$$

To account for this, the framework splits the calculated CVs of quantitative inputs according to distribution type. The sum of symmetric attributed

are added to the product of lognormal attributes by Eq. 5.

$$CV_T = \sqrt{\sum_{i=1}^n (CV_{Sym}^2) + (\prod_{i=1}^n (CV_{Ln}^2 + 1) - 1)} \quad (5)$$

The formulae allow the aggregated CV of both quantitative and qualitative data to be determined as a measure of total uncertainty. Since CV is defined as the ratio between the standard deviation and the mean, the output follows a normal distribution. The uncertainty can be expressed by the standard deviation via Eq. 6.

$$\sigma_T = \mu_T CV_T \quad (6)$$

Step 5 visualises the calculated uncertainties in a scalable manner to assess their impact on the contextual application. The visualisation consists of the total estimated uncertainty along with individual contributions via local sensitivity analysis. This is a widely used method to determine the relative impact of input variables to the compound uncertainty, calculated using partial derivatives [1]. The resulting factors are plotted over the relative total to visualise their impact.

4. Framework implementation

4.1. Case study: Heat exchanger test rig

The framework was applied to a bespoke heat exchanger (HEX) test rig consisting of a hot closed loop system and a cold open loop system, as designed by Addepalli et al. [6], illustrated in Fig. 2. The design comprised of subsystem modules housing component and fluid interactions that interact in a non-linear manner. This, as well as subjective opinions and actions of operators, influence uncertainties in the assessment of system performance and the HEX itself. Oil temperature at the inlet and outlet of the plate-fin type HEX was measured using infrared (IR) passive thermography in real-time [29]. Thermodynamic analysis involving oil viscosity and temperature decrease through connecting pipes was not considered.

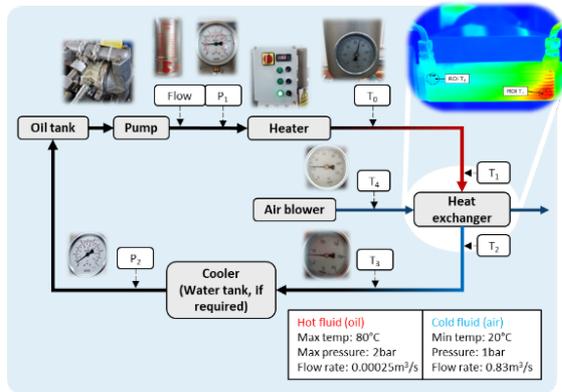


Fig. 2. Oil-air heat exchanger rig design [6]

Recordings were taken once the system had reached operating temperature, 80°C, at steady state. 6 sets of 10 recordings were made for each parameter with 20 minute intervals between each set to allow the temperature to reset.

To enable effective heat transfer, the air blower must be in operation. This created an air blanket around the HEX unit, disrupting the temperature measurements for the IR camera (Fig. 3). Aside from the temperature reading from the IR camera, all parameter measurements were recorded via in-line analogue dials. Many of these dials gave readings on different interval scales varying the measurement accuracy, and therefore resulted in an increased uncertainty. The recorded parameters are illustrated in Fig. 2 and detailed in the following section.

Additional attributes such as parallax error and ambient temperature further increased the uncertainty in measurement. The flow rate of the oil constantly fluctuated from 0-15 L/min owing to the nature of the air-operated pump. This made an accurate reading of mean flowrate unobtainable, so was discounted from the input parameter list.

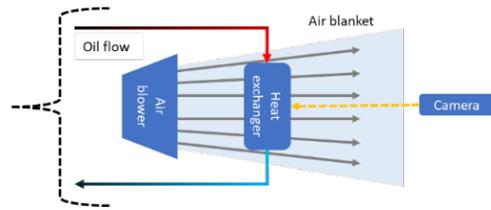


Fig. 3. Representation of air blanket produced by air blower

4.2. Stepped implementation and results

Step 1: 6 quantitative parameters and 5 qualitative factors were identified; described below and summarised in Table 2 and Table 3 respectively.

Temperature readings of the HEX inlet (T_1) and outlet (T_2) given by the IR camera were modelled by the lognormal distribution as the fluid temperature gradually decreased once reaching operating temperature. This gave a skewed result with slightly different mean values for each set of readings. The temperature reading given by the in-line dial after the HEX (T_3) was modelled by the normal distribution as the higher reading interval gave a more constant mean, though with a greater spread in deviation. Oil pressure before passing through the HEX (P_1) was also modelled by the normal distribution. Blower air temperature (T_4) and oil pressure post HEX (P_2) were modelled by the uniform distribution as they were found to be constant throughout.

The 5 qualitative factors derived were reliability of data (1), basis of estimate (2), reading accuracy (3), environmental conditions (4) and sample size (5). Each of these were modelled by the lognormal distribution.

Step 2a: A summary of the quantitative parameters is given in Table 2. The reading intervals and errors contributing to qualitative factors are discussed in the following section.

Step 2b: The 5 qualitative factors were scored by defined pedigree criteria detailed in Appendix A. These were based on adjusted examples from literature [11,22,23] to apply to the case study. Uncertainty indicators for all factors for each score are illustrated in Appendix B. For this case study, the uncertainty indicators were obtained from a single source and their GSD is therefore given as its square root. A summary of these parameters is given in Table 3.

Step 3: The summary tables with calculated CV for each input are given in Table 2 and Table 3 for quantitative and qualitative factors respectively.

Step 4: The combined CV of each PDF is shown in Table 4, aggregated for symmetric and asymmetric distributions and total CV.

Table 2. Recorded data and calculated parameters

Parameter	Reading interval	Reading error	Distribution	Mean	Standard deviation	Min	Max	CV
T ₁ , HEx In (°C)	0.1°C	-	Lognormal	64.9839	1.0158	62.6000	66.2000	0.0157
T ₂ , HEx Out (°C)	0.1°C	-	Lognormal	27.4887	1.0280	26.0000	28.7000	0.0276
T ₃ , Temp dial out (°C)	5.0°C	±2.0°C	Normal	35.8333	1.8645	32.5000	37.5000	0.0517
T ₄ , Temp blower (°C)	2.0°C	±0.5°C	Uniform	18.0000	0.0000	18.0000	18.0000	0.0000
P ₁ , Pressure pre-HEx (bar)	0.5 bar	±1.0 bar	Normal	1.7511	0.0765	1.6000	1.8000	0.0432
P ₂ , Pressure post-HEx (bar)	0.5 bar	±0.3 bar	Uniform	0.3000	0.0000	0.3000	0.3000	0.0000

Table 3. Pedigree factors with relating GSD and CV

Factor	Dist.	Ped. score	Un. Ind.	GSD	CV
(1)	Ln	3	1.50	1.2247	0.2048
(2)	Ln	3	1.60	1.2649	0.2383
(3)	Ln	2	1.05	1.0247	0.0244
(4)	Ln	2	1.10	1.0488	0.0477
(5)	Ln	3	1.40	1.1832	0.1694

Step 5: The visualisation in Fig. 4 illustrates the relative CV of each quantitative (blue) qualitative (orange) input against the aggregated total (cream). The colour bar represents the acceptability of the relative factors according to predefined scales. Further development will allow customised scales to be visualised with the total CV from each PDF type and compared through global sensitivity analysis using Sobol indices.

Table 4. CV aggregation results

PDF	CV comb.	CV agg.	CV _T
Ln recorded	0.0317	0.3701	0.3762
Ln pedigree	0.3688		
Norm. recorded	0.0674	0.0676	
Uni. recorded	0.0000		

5. Discussion and conclusions

The framework was designed to enhance system performance assessment through the quantification and aggregation of compound uncertainties. These arise as a result of different recording methods and assumptions made about the system and are modelled by different PDFs.

The use of CV to represent uncertainty enabled effective quantification of compound uncertainties. Previous work in this area enabled the aggregation of individual quantitative uncertainty with qualitative uncertainty through the pedigree matrix [16,24]. The

capability to aggregate uncertainties represented by a mix of symmetric and asymmetric PDFs is further developed by an indication of factors that lie outside acceptable levels, as well as a function to allow the user to view and select the best suited PDF for each input. Acceptable levels of uncertainty are user-defined according to the application and visualised by the colour bar. Benefits of this framework include enhancements to performance assessment and corresponding maintenance planning for CES and respective subsystems.

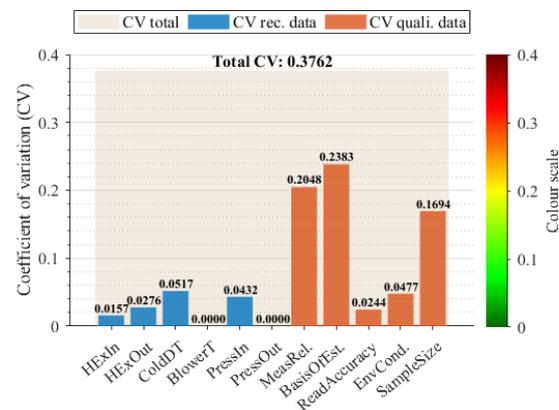


Fig. 4. Aggregated total against individual factors

The presented framework is capable of assessing quantitative uncertainties for differing input dimensions in each dataset. Selection of quantitative inputs and their corresponding PDFs is achieved by calculating all permutations in Step 2a, which are stored in arrays and selected according to user input. Further development is required to determine the GSD of qualitative inputs given my multiple sources such as surveys or interviews. This can be done separately, but an integrated process would be more efficient.

Appendix A. Pedigree criteria

Criteria for heat exchanger case study [11,22,23]

Score Factor	1	2	3	4	5
Measurement reliability (1)	Data is < 2 months old and/or recorded by fully calibrated sensor or fully qualified person	Data is < 6 months old and/or recorded by fully calibrated sensor or fully qualified person	Data is < 12 months old and/or recorded by fully qualified person	Data is > 12 months old and/or recorded by fully qualified person	Age or source of data unknown or > 12 months old
Basis of estimate (2)	Best possible data, use of historical field data, validated tools and independently verified data, given by fully qualified expert	Smaller sample of historic data, parametric estimates, internally verified data, some experience in the area	Limited available data, unverified, inexperienced opinions	Incomplete data, small sample, educated guesses, indirect approximate rule of thumb estimate	No experience in the data
Reading accuracy (3)	Measurements taken using fully calibrated and accurate equipment: $\pm 0.1^{\circ}\text{C}$, ± 0.1 bar	Measurements taken using recently calibrated but less accurate equipment: $\pm 0.5^{\circ}\text{C}$, ± 0.5 bar	Measurements taken using recently calibrated but less accurate equipment: $> \pm 1.0^{\circ}\text{C}$, $> \pm 1.0$ bar	Measurements taken using accurate equipment that may need recalibrating	Measurements taken using un-calibrated and inaccurate equipment
Environmental conditions (4)	Data recorded under specific consistent conditions or a specified range of conditions from area under study	Data recorded in generally consistent conditions with fluctuations specified	Data recorded in generally consistent conditions, changes not specified	Data recorded in a range of unspecified conditions	Data from unknown or distinctly different areas
Sample size (5)	> 20	> 10	> 5	< 5	Unknown

In addition, Step 5 of the framework requires further development to determine acceptable uncertainty parameters to be scaled according to the calculated CV. Validation of the most suitable techniques to determine sensitivity coefficients for each input will further enhance this final step.

The framework was applied to a bespoke heat exchanger test rig which contributed various uncertainties that impact measurement quality and accuracy. To resolve the issues in flow rate and temperature reading through the air blanket produced by the blower, a new centrifugal pump and digital temperature and pressure sensors will be installed improve data quality. In addition, future work will advance the framework further to account for correlations between input parameters, improve derivation of sensitivity factors and develop methods to forecast the quantified uncertainty through the in-service phase of an asset's life cycle.

Appendix B. Pedigree matrix uncertainty indicators for all factors

Factor / score	1	2	3	4	5
(1)	1.00	1.10	1.50	1.70	1.90
(2)	1.00	1.20	1.60	1.80	1.90
(3)	1.00	1.05	1.10	1.40	1.80
(4)	1.00	1.10	1.40	1.70	1.90
(5)	1.00	1.20	1.40	1.60	1.90

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For access to the data underlying this paper, please see the Cranfield University repository, *CORD*, at DOI: 10.17862/cranfield.rd.12906443.

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