

Introduction

The mechanisms responsible for ignition of explosive materials in response to low energy stimuli, known as “insults” in the literature, are still not well understood. It is in general believed that explosive ignition is of thermal origin, with mechanical energy being converted into heat energy in localised regions, forming so-called “hot spots”. Modelling such ignition mechanisms using numerical hydrocodes proves to offer some considerable challenges [1]. To supplement the numerical approach, we seek to develop analytical solutions. Here we develop a model to investigate how an instability caused by thermal softening may give rise to localised plastic deformation which may lead to subsequent ignition.

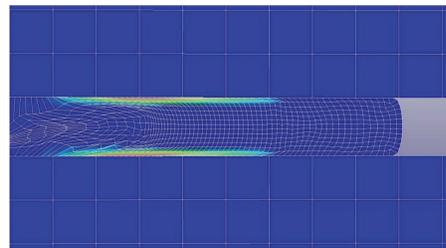


Figure 1: Shear bands arising from friction adjacent to the plates by which an explosive cylinder is being pinched. Contours are of an empirical ignition parameter, with red corresponding to high values of the parameter showing ignition. Avoiding issues such as mesh distortion is a prime motivation for looking at shear analytically. Figure taken from [2], with permission.

We exploit an idealised one-dimensional geometry (see Figure 2), using asymptotic methods to obtain a reduced system of equations which govern the growth rate of perturbations to a uniform shearing motion. This allows us to study the growth and initiation of shear bands in a reactive material. It is found that the growth of perturbations to the temperature and stress are controlled by three dimensionless parameters. This is in contrast to the inert case, in which the growth is controlled by a single parameter [3].

Constitutive Model

The strain-rate model proposed by DiLellio and Olmstead [3] accounts for thermal softening and strain rate dependence, and takes the following exponential form

$$\dot{\gamma}(s, T) = \dot{\Gamma}^* \exp\{-[B_1^{-1}(T_p - T) + B_2^{-1}(s_p - s)]\},$$

where $\dot{\Gamma}^*$, T_p and s_p are the reference strain-rate, temperature and stress, respectively. The parameters B_1 and B_2 are related to strain-rate sensitivity and thermal sensitivity.

Governing Equations

We investigate the standard one-dimensional model for the formation of a shear band, with addition of a chemical reaction modelled using a nonlinear Arrhenius source term (shown in red).

In dimensional form the governing equations read:

$$\begin{aligned} \rho v_t &= s_y, & \text{momentum balance;} \\ \rho c T_t &= \kappa T_{yy} + \beta s \dot{\gamma} + \rho \Omega A e^{-\frac{E}{RT}}, & \text{energy balance;} \\ s_t &= G(v_y - \dot{\gamma}), & \text{elastic relationship;} \\ \dot{\gamma} &= \dot{\gamma}(s, T, \gamma), & \text{plastic flow law;} \\ \gamma &= \int_0^t \dot{\gamma}(t') dt', & \text{plastic strain.} \end{aligned}$$

Here the dependent variables are velocity v , stress s , temperature T , and plastic strain γ . The plastic strain γ is determined by integration of strain rate. The material constants ρ , G , c , κ , β , Ω , A , E and R , are the density, elastic shear modulus, specific heat, thermal conductivity, Taylor-Quinney coefficient, heat of reaction, rate constant, activation energy and molar gas constant, respectively.

Model Schematic

Schematic of uniform shearing vs. a shear band centred about $y = 0$.

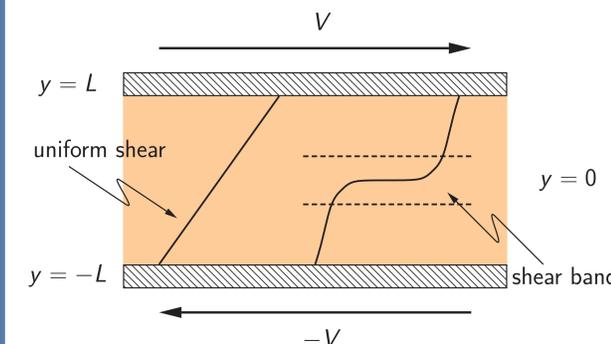


Figure 2: Solid lines show typical velocity profiles. Localised plastic deformation and/or reaction is induced by an inhomogeneity in the thermal flux along the centreline. The thin zone of localisation is identified as a boundary layer, and the problem is treated as a perturbation to the elastic solution. It is assumed that the perturbations will be uniform in the shearing direction so that a one-dimensional model is appropriate.

Results

Here we give results obtained from the boundary layer analysis. Representative parameter values have been chosen using data from [3], [4] and [5]. The parameter Λ_t has been held fixed at a typical value of 0.01.

	Λ_p		
Λ_R	1	10	100
0	—	•••	•••
1	—	—	—
10	—	—	—
15	—	—	—

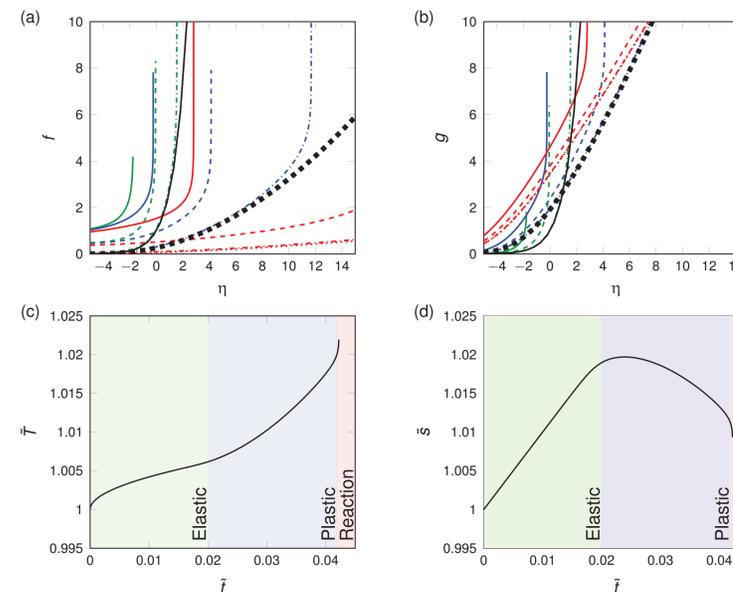


Figure 3: Numerical results showing the evolution of the non-dimensional perturbations to temperature (a) and stress (b) for various Λ_p and Λ_R . The solid black line shows the exact solution $f(\eta) = g(\eta) = e^\eta$ for the special case of $\Lambda_p = 1$ and vanishing reaction ($\Lambda_R = 0$). DiLellio and Olmstead [3] suggest that occurrence of a shear band in the inert case would typically be associated with $\Lambda_p < 10$. This criterion is shown as a thick black dashed line (•••).

Panels (c) and (d) show the time evolution of the non-dimensional centreline temperature $\tilde{T}(\tilde{t})$ and stress $\tilde{s}(\tilde{t})$ calculated from the asymptotic analysis. The process can be split into three stages: elastic, plastic and reaction. The transition times between the stages may be predicted by the asymptotic theory, and show excellent agreement when compared with numerical solution of the full governing equations. It is clear to observe the stress decrease associated with shear banding, and the rapid increase in temperature associated with the commencement of thermal runaway.

Asymptotic Analysis

The analysis results in two coupled equations which control the magnitude of the temperature and stress perturbations, respectively. These read

$$\begin{aligned} f(\eta) &= \int_{-\infty}^{\eta} [\pi(\eta - \eta')]^{-1/2} \left\{ \exp[\eta' + f(\eta') - g(\eta')] \right. \\ &\quad \left. + \Lambda_R \exp[\Lambda_t(\eta' - \eta'_R) + f(\eta') - f(\eta'_R)] \right\} d\eta', \\ g(\eta) &= \Lambda_p \exp[\eta + f(\eta) - g(\eta)]. \end{aligned}$$

Here η is a time-like variable, and η_R a critical reaction timescale. The non-dimensional parameters Λ_p , Λ_R and Λ_t are known in terms of properties of the material and applied deformation, and relate to the plastic behaviour, reaction and shear-rate, respectively.

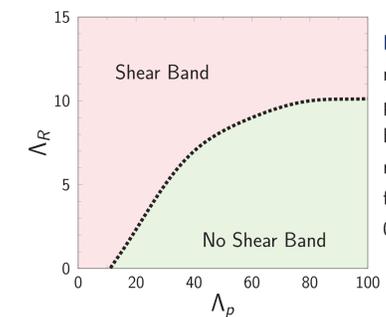


Figure 4: A sketch of the region in the $\Lambda_p - \Lambda_R$ plane in which a shear band is formed. The parameter Λ_t has been held fixed at a typical value of 0.01.

Conclusions

- The reactive shear band has been identified as a boundary layer.
- Asymptotic theory leads to a reduced system of two coupled equations, governed by three dimensionless parameters.
- Critical timescales for onset of plastic work and reaction are well predicted by the asymptotic results.
- Introduction of a reaction leads to shear banding for values of Λ_p outside of the criterion identified in the inert case [3].

References

- [1] J.P. Curtis. A model of explosive ignition due to pinch. In *Thirty-Eighth International Pyrotechnics Seminar*, Denver, Colorado, USA, pages 207–216, 2012.
- [2] J.P. Curtis and J.E. Reaugh. Modelling the ignition of explosives by pinch with the HERMES model. In *42nd International Pyrotechnics Seminar*, 2016.
- [3] J.A. DiLellio and W.E. Olmstead. Shear band formation due to a thermal flux inhomogeneity. *SIAM Journal on applied mathematics*, 57(4):959–971, 1997.
- [4] W.E. Olmstead. Ignition of a combustible half space. *SIAM Journal on Applied Mathematics*, 43(1):1–15, 1983.
- [5] J.M. Powers. Thermal explosion theory for shear localizing energetic solids. *Combust. Theory Modelling*, (3):103–122, 1999.

Acknowledgements

This work was supported by an EPSRC industrial CASE partnership with AWE [grant number EP/L505729/1]. Thanks to colleagues at AWE for useful scientific discussions.