1 Expanded derivations for Feder et al, 2019

Here we show an extended derivation of $F_{ST}(t)$ (equation 3). Since $F_{ST} = \frac{h_b - h_w}{h_b}$, we need to determine the heterozygosities in our simplified model both within and between subpopulations. There are three alleles to consider with reference to a focal subpopulation: the locally derived allele, which is at frequency $f(t)F(t)$, the non-locally derived allele, which is at frequency $f(t)(1 - F(t))$ and the wildtype allele, which is at frequency $1 - f(t)$. Because the compartments are symmetric, it is sufficient to determine the within-subpopulation heterozygosity in the focal compartment:

$$h_w = 1 - [f(t)F(t)]^2 - [f(t)(1 - f(t))]^2 - (1 - f(t))^2.$$

To determine the between subpopulation heterozygosity, we take advantage of symmetries in the allele frequencies. Allele A (local to population A), is at frequency $f_l(t)$ in subpopulation A and at frequency $f_{nl}(t)$ in subpopulation B. Similarly, allele B (local to population B) is at frequency $f_{nl}(t)$ in subpopulation A and at frequency $f_l(t)$ in subpopulation B. The wildtype allele is at the same frequency $1 - f(t)$ across both subpopulations. Therefore,

$$h_b = 1 - 2f(t)F(t) \times f(t)[1 - F(t)] - (1 - f(t))^2].$$

Therefore, the numerator of $F_{ST}(t)$ is given as follows:

$$h_b - h_w = f(t)^2[1 - 2F(t)]^2 \tag{S1}$$

and the denominator of $F_{ST}(t)$ is:

$$h_b = f(t)[2 - 2F(t)(1 - F(t))f(t) - f(t)]. \tag{S2}$$

Combining equations S1 and S2, we see equation 3:

$$F_{ST}(t) = \frac{(1 - 2F(t))^2 f(t)}{2 - 2f(t)F(t)(1 - F(t)) - f(t)}.$$

To determine t_{max}, we take the derivative of $F_{ST}(t)$ with respect to time t, and assume that at $t = 0$, $F(0) = 0$ (the non-local allele is initially not present in the population), and $f(0) = 1/N$ (the frequency of the local allele is at a single copy). Under these conditions, we observe the following simplifications:

$$\begin{aligned} F(t) =& 0.5 \times (1 - e^{-2mt}) \\ f(t) =& \frac{e^{st}}{e^{st} + N - 1}. \end{aligned} \tag{S3}$$

Differentiating $F_{ST}(t)$ with respect to time can be done in the program Mathematica, although the closed form is not instructive:

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> f[t_] := Exp[s*t]/( Exp[s*t] + N - 1 )
> F[t_] := 0.5 + -0.5 * Exp[-2*m*t]
> Fst[t_] := (1 - 2*F[t])^2 * f[t]/
    ( 2 - 2*f[t] * F[t] * (1 - F[t])  - f[t])
> Solve[Fst'[t] == 0, t]
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Solving the derivative for 0 results in the form $t_{max} = \frac{1}{s} \log((\frac{s}{m} - 4)(N - 1))$ (equation 4). Plugging t_{max} back into equation 3 results in $F_{ST}(t_{max})$ (equation 5).