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Close encounters in young stellar clusters: implications for planetary systems in the solar neighbourhood

Daniel Malmberg,[1★] Francesca De Angeli,[2] Melvyn B. Davies,[1] Ross P. Church,[1] Dougal Mackey[3] and Mark I. Wilkinson[4]

[1]*Lund Observatory, Box 43, SE–221 00, Lund, Sweden*
[2]*Institute of Astronomy, Madingley Road, Cambridge CB3 OHA*
[3]*Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ*
[4]*Department of Physics and Astronomy, University of Leicester, Leicester LE1 7RH*

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ABSTRACT

The stars that populate the solar neighbourhood were formed in stellar clusters. Through *N*-body simulations of these clusters, we measure the rate of close encounters between stars. By monitoring the interaction histories of each star, we investigate the singleton fraction in the solar neighbourhood. A *singleton* is a star which formed as a single star, has never experienced any close encounters with other stars or binaries, or undergone an exchange encounter with a binary. We find that, of the stars which formed as single stars, a significant fraction is not singletons once the clusters have dispersed. If some of these stars had planetary systems, with properties similar to those of the Solar System, the planets' orbits may have been perturbed by the effects of close encounters with other stars or the effects of a companion star within a binary. Such perturbations can lead to strong planet–planet interactions which eject several planets, leaving the remaining planets on eccentric orbits. Some of the single stars exchange into binaries. Most of these binaries are broken up via subsequent interactions within the cluster, but some remain intact beyond the lifetime of the cluster. The properties of these binaries are similar to those of the observed binary systems containing extrasolar planets. Thus, dynamical processes in young stellar clusters will alter significantly any population of Solar System-like planetary systems. In addition, beginning with a population of planetary systems exactly resembling the Solar System around single stars, dynamical encounters in young stellar clusters may produce at least some of the extrasolar planetary systems observed in the solar neighbourhood.

Key words: stellar dynamics – celestial mechanics – binaries: general – planetary systems – open clusters and associations: general.

1 INTRODUCTION

Stars form in clusters or groups. Thus in their early lives they are in a potentially very crowded environment. In such environments close encounters with other stars and binaries will be frequent. These close encounters can strongly affect the stability of planetary systems around the stars (de La Fuente Marcos & de La Fuente Marcos 1997; Laughlin & Adams 1998; Hurley & Shara 2002; Pfahl & Muterspaugh 2006; Spurzem et al. 2006). It has been suggested, for example, that the hot Jupiter orbiting the triple star HD 188753 is the result of stellar interactions in a young stellar cluster (Pfahl 2005; Portegies Zwart & McMillan 2005). In order to understand how important dynamical effects in young stellar clusters are for the population of extrasolar planets, we must first know how frequent the interactions between stars in such clusters are. Furthermore, we need to understand which kind of interactions are most important, close encounters or exchange encounters with binaries. If many stars undergo encounters which change the dynamics of planetary systems, our Solar System might in fact be fairly rare. To understand how rare, it is necessary to know the fraction of stars that have never undergone close encounters with other stars.

In this paper we consider the fraction of single stars in the field today that are singletons. A *singleton* is a star which formed single, has not undergone any close encounters with other stars or binaries and has never been exchanged into a binary. We define a close encounter as when two objects pass within 1000 au of each other; see Section 2.

We perform *N*-body simulations of open clusters, varying the initial number of stars and the initial half-mass radius. During the simulations we follow the interaction history of each individual star and therefore we are able to determine the interaction histories of

★E-mail: danielm@astro.lu.se

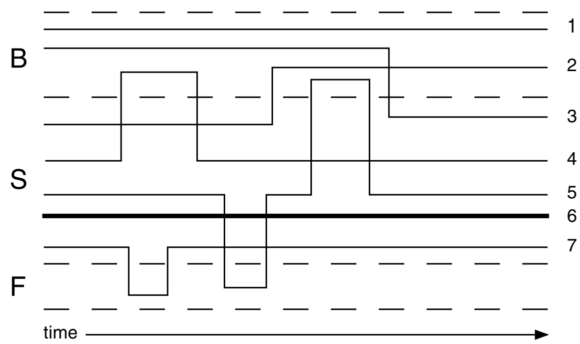

Figure 1. Sketch of different possible interaction histories for a star in a stellar cluster. The dashed lines separate the different 'interaction states' a star can be in. B stands for bound (i.e. binary or triple), S for single and F for fly-by, where we have defined a fly-by as when two stars pass within 1000 au of each other. (1) Star which is in bound systems during the lifetime of the cluster. (2) Star which is initially single, but exchanges into a binary and remains in it for the remaining lifetime of the cluster. (3) Star which is in a primordial binary that is broken up. (4) Initially single star which is first exchanged in and then later out of a binary. (5) Initially single star which first has a close encounter with another star or binary and then is exchanged in and out of a binary. (6) Initially single star which never has a close encounter with another star, a *singleton*. (7) Initially a single star which has a close encounter with another star.

all the stars in the clusters. This allows us not only to determine the singleton fraction in the solar neighbourhood, but also to better understand which kind of interactions are most important in young stellar clusters.

We make the hypothesis that planetary systems only form around single stars, with similar masses and orbits as the planets in our Solar System. If so, we would expect the fraction of Solar System-like planetary systems around other stars in the solar neighbourhood to be proportional to the singleton fraction. All other planetary systems will have been to some degree altered by dynamical effects in the cluster in which their host star formed.

Interactions between stars can be divided up into close encounters and exchange encounters involving binaries. In an exchange encounter an incoming single star replaces one of the stars in a binary. We plot an overview of some of the different possible interaction histories in Fig. 1. Close encounters can, depending on the minimum distance between the stars, either substantially change the orbits of the planets or just slightly perturb the orbit of the outermost planet in the system (Heggie & Rasio 1996; Adams et al. 2006; Spurzem et al. 2006). Such a perturbation might later cause large changes in the planetary system via planet–planet interactions.

If a planet-hosting star, initially single, is exchanged into a binary, the orbits of the planets can be strongly perturbed. The Kozai Mechanism (Kozai 1962) drives cyclical changes in the eccentricities of planets in a system where the binary orbit is inclined with respect to that of the planets. This can via planet–planet interactions lead to the expulsion of several planets, leaving the remaining planets on eccentric orbits (Malmberg, Davies & Chambers 2006).

From the properties of the extrasolar planets detected so far (see e.g. Luhman et al. 2007) we expect many systems to consist of one or two planets on eccentric orbits close to the host star. Dynamical interactions between stars could thus potentially produce the observed properties of some of the extrasolar planets.

In Section 2 we show that the encounter time-scales in clusters of typical sizes are interesting. We then, in Section 3, describe the N-body code used and the simulations performed. In Section 4 we analyse the results of the runs and in Section 5 we discuss their implications. Finally in Section 6 we summarize our results.

2 ESTIMATES OF ENCOUNTER RATES

In this section we derive an expression for the encounter time-scale in a stellar cluster and show that this is typically much shorter than the cluster lifetime. Hence most of the stars in clusters will undergo close encounters with other stars.

The time-scale for a given star to undergo an encounter with another star within a distance $r_{\min}$, may be approximated by (Binney & Tremaine 1987)

$$\tau_{\rm enc} \simeq 3.3 \times 10^7 {\rm yr} \left(\frac{100\ {\rm pc}^{-3}}{n}\right)\left(\frac{v_\infty}{1\ {\rm km\ s}^{-1}}\right)\left(\frac{10^3\ {\rm au}}{r_{\min}}\right)\left(\frac{{\rm M}_\odot}{m_{\rm t}}\right). \quad (1)$$

Here n is the stellar number density in the cluster, v_∞ is the mean relative speed at infinity of the objects in the cluster, $r_{\min}$ is the encounter distance and $m_{\rm t}$ is the total mass of the objects involved in the encounter. The cross-section for an interaction is increased greatly by what is known as gravitational focusing, where stars are deflected towards each other because of their mutual gravitational attraction. This effect is included in the above equation.

In addition to encounters involving two single stars, encounters involving at least one binary will occur. When a binary in a cluster encounters another star, it can be broken up if the kinetic energy of the single star is greater than the binding energy of the binary. A binary which is broken up in an encounter with a third star, whose kinetic energy is equal to the average kinetic energy of the stars in the cluster, is termed soft. Binaries that are more tightly bound will not be broken up, but will instead on average be hardened by encounters with a third star; these are known as hard binaries. If the incoming third star is more massive than one of the components of the original binary, an exchange encounter may occur where the incoming star replaces the least massive star in the binary; the probability of this occurring depends on the masses of the stars involved. Encounters involving a massive single star have a higher probability of leading to an exchange encounter.

The hard–soft boundary lies where the binding energy of the binary is equal to the average kinetic energy of the stars in the cluster. For a cluster in virial equilibrium, the square of the velocity dispersion is equal to $Gm_{\rm cl}/2r_{\rm h}$ (Aarseth 2003), where $m_{\rm cl}$ is the total mass of the cluster and $r_{\rm h}$ is the half-mass radius. We can combine this with the energy for a bound two-body orbit, $E = -Gm_1m_2/2a$, where a is the semimajor axis and m_1 and m_2 are the masses of the bodies to find an expression for the semimajor axis of a binary at the hard–soft boundary:

$$a \approx \frac{r_{\rm h}}{N}, \quad (2)$$

where we have taken $m_1 = m_2 = m_{\rm cl}/N$. As an example we take $r_{\rm h} = 2.5$ pc and $N = 700$, which gives $a \approx 1000$ au. Exchange encounters may thus occur if the minimum encounter distance, $r_{\min}$ is similar to or less than 1000 au (Davies, Benz & Hills 1994).

We can now rewrite equation (1) in more appropriate units for our clusters. We assume that $n_h = 3N/8\pi r_{\rm h}^3$ and $v_\infty = (Gm_{\rm cl}/r_{\rm h})^{1/2}$ (Binney & Tremaine 1987) , where $m_{\rm cl}$ is the total mass of the cluster and r_h is the half-mass radius. Equation (1) then becomes

$$\tau_{\rm enc} \simeq 5 \times 10^7\ {\rm yr} \left(\frac{\bar{m}_*}{1\ {\rm M}_\odot}\right)\left(\frac{r_{\rm h}}{1\ {\rm pc}}\right)^{5/2}\left(\frac{100\ {\rm M}_\odot}{m_{\rm cl}}\right)^{1/2} \times \left(\frac{10^3\ {\rm au}}{r_{\min}}\right)\left(\frac{{\rm M}_\odot}{m_{\rm t}}\right), \quad (3)$$

where $\bar{m}_*$ is the mean mass of the stars. We can now estimate the encounter rates in a stellar cluster. As a first example we assume a mass of 500 $M_\odot$ and a half-mass radius of 0.5 pc (see e.g. Lada & Lada 2003; Porras et al. 2003; Kharchenko et al. 2005). The typical total mass of the objects involved in encounters is for simplicity taken to be 1 $M_\odot$ and the average stellar mass is taken to be $\bar{m}_* = 0.6 M_\odot$. Furthermore, we set $r_{\min}$ equal to 1000 au, as discussed above. Equation (3) then gives $\tau_{\rm enc} \approx 2.4$Myr. Hence, on average, the encounter rate should be about 0.4 encounters per star per Myr. The lifetime of a cluster depends on the number of stars, the cluster's radius and its location within the Galaxy. About 10 per cent of stars are formed in large clusters where $N \geqslant 100$ (Adams & Myers 2001), which live for several 10^8 yr (Adams et al. 2006; Lamers & Gieles 2006). A substantial fraction of the stars formed in these clusters will undergo close encounters with other stars. The clusters in which the remaining 90 per cent of stars form have much shorter lifetimes, on the order of a few Myr, due to that they are much smaller and disperse when the leftover gas from star formation is removed (Adams & Myers 2001; Allen et al. 2007). Many of these clusters have a rather small total mass, which means that the encounter time-scale therein is longer. The encounter time-scale per star per Myr is still shorter than the cluster lifetime, however, and hence a significant fraction of stars in such clusters will have undergone at least one close encounter by the time the cluster has dispersed.

In reality the encounter rate for a given object depends on several things, for example, its individual mass. Furthermore, the cluster's half-mass radius is not constant, but changes with time. The number density is not uniform; the central density is generally higher than the mean density. For these reasons, equation (3) only gives a crude estimate of the interaction rates and N-body simulations are needed to give us a better understanding of the dynamical processes involved.

3 *N*-BODY SIMULATIONS PERFORMED

To perform the simulations described in this paper we used the NBODY6 code, which is a full force-summation direct N-body code. A summary of the NBODYx family of codes can be found in Aarseth (1999) and a complete description of the algorithms used and their implementation in Aarseth (2003). The integration scheme employed in NBODY6 is the Hermite predictor–corrector scheme of Makino (1991) with the Ahmad & Cohen (1973) neighbour scheme. Regularization of motion dominated by the close interaction of a pair of particles – for example, a perturbed binary or hyperbolic encounter – is handled by the Kustaanheimo & Stiefel (1965) regularization scheme. This removes the singularity in the equations of motion and makes the integration more numerically stable and more efficient. For our purposes the usual criteria for regularization described in section 9.3 of Aarseth (2003) are modified somewhat. All binaries and hyperbolic encounters where stars come within a little more than 1000 au of one another are regularized (see the discussion in Section 2). This gives rise to the formation of several transient binaries, that is, pairs of stars which are weakly bound and break up very quickly. The effects on planetary systems in these systems are more similar to those caused by multiple fly-by episodes than the effect of a binary companion. Therefore we defined a bound system as a binary only if it survived for at least five orbital periods, while short-lived systems are classified as multiple fly-bys.

A total of 25 cluster models were computed. They are all listed in Table 1. Models were run with 150, 300, 500, 700 and 1000 stars and at initial half-mass radii of 0.38, 0.77, 1.69, 3.83 and 7.66 pc. For each of the 25 possible parameter combinations 10 realizations were made; that is, 10 cluster models identical except that the initial positions, velocities, stellar masses and binary properties were drawn from different samples of the same distributions. We found that 10 realizations were enough to give us good statistics for the singleton fraction and close encounter rates in the clusters. Adams et al. (2006) found that they needed about 10 times as many realizations to achieve good statistics from their N-body runs. The difference is most likely that our clusters live for about 10 times longer than theirs; we therefore register many more close encounters per run, giving us a large enough sample to get good statistics.

Initial positions of the stars were chosen from the spherically symmetric Plummer (1911) distribution,

$$\rho(\mathbf{r}) = \frac{3m_{\rm cl}}{4\pi r_0^3} \frac{1}{\left[1 + (r/r_0)^2\right]^{5/2}}, \qquad (4)$$

for total cluster mass $m_{\rm cl}$. The scaling factor r_0 is related to the half-mass radius $r_{\rm h}$ via integration by $r_{\rm h} \simeq 1.3 r_0$. This is a standard distribution widely used in stellar cluster models (Heggie & Hut 2003). The assumption of spherical symmetry is in reasonable accord with observations of open clusters, as is the property of central condensation (Lada & Lada 2003). The stellar masses were drawn from the initial mass function (IMF) of Kroupa, Tout & Gilmore (1993) with the masses of binary components being drawn independently from the IMF. The lower mass limit was set to 0.2 $M_\odot$ and the upper mass limit to 5 $M_\odot$. Every third star was part of a

Table 1. Number of stars (column 1), initial and mean half-mass radii (columns 2, 3), number of singletons divided by the number of stars which were single at the end (column 4) and the number of non-singletons divided by the number of stars which were initially single (column 5). The numbers and fractions are the average values from the 10 realizations ran for each cluster. The errors are the s.d. values calculated from the 10 realizations. The number of singletons divided by the number of initially single stars can be found by taking $1 - f_{\rm fb}$.

N	$r_{\rm h,initial}$ (pc)	$r_{\rm h,mean}$ (pc)	$f_{\rm s}$	$f_{\rm fb}$
150	0.38	2.73 ± 0.13	0.56 ± 0.11	0.37 ± 0.11
150	0.77	2.76 ± 0.04	0.66 ± 0.03	0.27 ± 0.04
150	1.69	2.52 ± 0.08	0.82 ± 0.07	0.13 ± 0.07
150	3.83	4.88 ± 0.28	0.97 ± 0.02	0.02 ± 0.02
150	7.66	9.94 ± 0.61	0.97 ± 0.02	0.01 ± 0.02
300	0.38	2.95 ± 0.06	0.38 ± 0.03	0.56 ± 0.03
300	0.77	3.28 ± 0.23	0.61 ± 0.05	0.33 ± 0.05
300	1.69	2.84 ± 0.14	0.76 ± 0.03	0.19 ± 0.03
300	3.83	3.63 ± 0.04	0.93 ± 0.03	0.05 ± 0.03
300	7.66	5.48 ± 0.04	0.99 ± 0.01	0.002 ± 0.002
500	0.38	2.90 ± 0.04	0.41 ± 0.04	0.52 ± 0.04
500	0.77	3.13 ± 0.02	0.55 ± 0.02	0.38 ± 0.02
500	1.69	3.18 ± 0.07	0.71 ± 0.05	0.24 ± 0.05
500	3.83	3.95 ± 0.09	0.90 ± 0.02	0.07 ± 0.02
500	7.66	6.12 ± 0.05	0.99 ± 0.01	0.005 ± 0.002
700	0.38	3.12 ± 0.04	0.17 ± 0.03	0.78 ± 0.03
700	0.77	2.88 ± 0.03	0.42 ± 0.03	0.52 ± 0.03
700	1.69	3.31 ± 0.18	0.66 ± 0.03	0.29 ± 0.03
700	3.83	3.89 ± 0.17	0.87 ± 0.02	0.10 ± 0.03
700	7.66	10.87 ± 0.26	0.99 ± 0.01	0.01 ± 0.01
1000	0.38	2.47 ± 0.17	0.14 ± 0.07	0.83 ± 0.07
1000	0.77	3.08 ± 0.15	0.30 ± 0.04	0.64 ± 0.04
1000	1.69	3.48 ± 0.07	0.59 ± 0.04	0.36 ± 0.04
1000	3.83	4.03 ± 0.21	0.83 ± 0.01	0.13 ± 0.02
1000	7.66	7.42 ± 0.46	0.99 ± 0.01	0.01 ± 0.01

binary so one in five objects (four stars plus a binary) was a binary. Hence the binary fraction, defined as

$$f_b = \frac{N_b}{N_s + N_b} \tag{5}$$

for a population with N_s single stars and N_b binaries, was 0.2. The distribution of initial semimajor axes was flat in log a between 1 and 1000 au. In reality however, both binaries with smaller separations than 1 au and binaries with larger separations than 1000 au exist in clusters. Including these in our simulations would thus give a higher binary fraction, with a value close to the observed one, which is equal to 0.3 (Duquennoy & Mayor 1991), and a lower singleton fraction. Tight binaries will however not affect the dynamical evolution of the cluster significantly, since they will act much like point-like objects and will thus not change the general results of our simulations. We choose not to include them for technical reasons and because as discussed above they would not contribute to the overall results of this paper. The exclusion of wider binaries is justified because their component stars will behave like singletons, in the sense that their binarity will most likely not affect planet formation around them. Such binaries will also be broken up very rapidly in our simulations. The eccentricities were, in accordance with observations, drawn independently from a thermal distribution with d $P(e) = 2e$ (Duquennoy & Mayor 1991), and $0 \leqslant e < 1$, where e is the eccentricity. The stars were taken to be solar metallicity and evolved with the stellar evolution prescription of Hurley, Pols & Tout (2000).

NBODY6 does not contain any treatment of gas hydrodynamics, hence the clusters were considered not to contain any gas. This will greatly increase the lifetimes of the clusters, since more than 50 per cent of the mass in a real cluster may consist of gas and hence the cluster can unbind when it is removed due to stellar winds and supernovae (see e.g. Adams & Myers 2001; Bastian & Goodwin 2006; Goodwin & Bastian 2006). We will explain this more in Section 5. The cluster properties that we have chosen are reasonable given the state of knowledge regarding young Galactic open clusters; see for example Lada & Lada (2003); Adams et al. (2006); Lamers & Gieles (2006).

The code was run with the standard tidal field prescription (Aarseth 2003; Heggie & Hut 2003) that places the cluster on a circular Galactic orbit in the solar neighbourhood. This leads to an extra acceleration in the Galactic radial direction of $4A(A - B)x$, where A and B are Oort's constants of Galactic rotation, $A = 14.4$ and $B = -12.0$ km s^{-1} kpc^{-1}(Binney & Tremaine 1987). The tidal radius, where the tidal force is equal to the attractive gravitational force of the cluster, is then

$$r_t = \left[\frac{Gm_{cl}}{4A(A - B)}\right]^{1/3}. \tag{6}$$

The presence of the tidal field greatly increases the rate at which stars are lost from the cluster as it reduces the degree to which stars in the outer part of the cluster are bound, although the cluster is not simply truncated at the tidal radius.

4 RESULTS

We simulated clusters over a grid of initial conditions, with 10 realizations each. The lifetimes of the clusters varied, depending both on the number of stars and on the initial half-mass radius (here we define the lifetime of the cluster as the time from the start of the run to when there are only four stars remaining). For example, the clusters with 150 stars and $r_{h,initial} = 7.66$ pc lived for about 200 Myr,

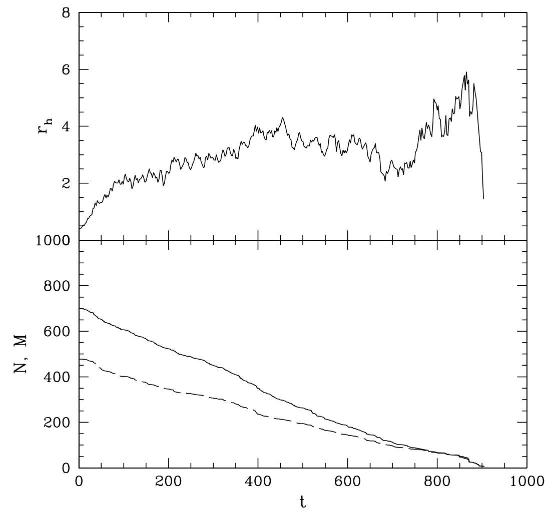

Figure 2. The evolution of the half-mass radius, r_h (in pc), with time, t (in Myr), in our reference cluster ($N = 700$ and $r_{h,initial} = 0.38$ pc) is plotted in the upper panel of this figure. In the lower panel we plot both the number of stars, N (solid line), and the mass, M (in units of $M_\odot$), of the same cluster (dashed line) as a function of time, t (in Myr).

while clusters with 1000 stars and the same half-mass radius lived for about 400 Myr. Hence, the lifetimes slowly increase with the number of stars. The clusters with smaller radii all lived longer, but the lifetimes varied more strongly with the number of stars. For example, the clusters with initially 150 stars and an initial half-mass radius of 0.8 pc lived for about 300 Myr, while the clusters with 1000 stars and the same half-mass radius lived for about 1 Gyr. Note that we have not included the effects of giant molecular clouds (GMCs) or gas remaining from the clusters' formation on the lifetime of the clusters. If included, these would decrease their lifetimes significantly (Goodwin & Bastian 2006; Lamers & Gieles 2006).

In the upper panel of Fig. 2, we plot the evolution of the half-mass radius as a function of time, for a single example of a cluster with $N = 700$ stars and $r_{h,initial} = 0.38$ pc. We will use this specific run as an example several times later in this paper, since it is a typical cluster, and will refer to it as our reference cluster. As can be seen the cluster expands rapidly during the first 200 Myr of the simulation, due to binary heating and mass-loss, whereafter it stabilizes at a half-mass radius close to 3 pc. This initial expansion is seen in all the clusters with an initial half-mass radius less than 2 pc, while the larger clusters' radii vary much more slowly with time. Thus, in the dense clusters, the encounter rates are significantly higher during the first 200 Myr than during the rest of the simulation, and hence most of the non-singletons will be produced early on. Thus, decreasing the lifetime of our clusters by allowing for the effects of GMCs would not significantly change the singleton fraction in them. For all of the runs we calculate the time-averaged half-mass radius, $r_{h,mean}$.

In the lower panel of Fig. 2 we plot the number of stars and the total mass as a function of time in our reference cluster ($N = 700$ and $r_{h,initial} = 0.38$ pc). As can be seen in the figure the decrease in the number of stars, and hence also the mass, is roughly constant with time. The main reason for the mass-loss is the tidal field of the Galaxy. Another thing to note from Fig. 2 is that the mean stellar mass increases with time. This is to be expected, since in the centre of the cluster, where energy equipartition has been achieved,

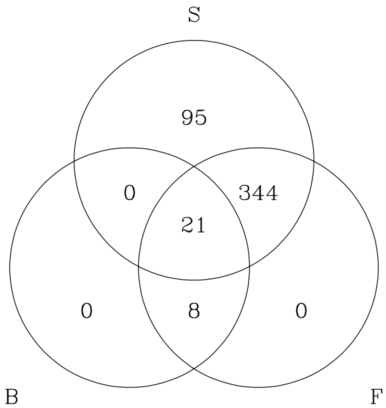

Figure 3. Venn diagram of the stars which were initially single in our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc). The upper circle contains all stars which were single at the end of the run (S), the lower left-hand circle all stars which were in a bound system (i.e. triple or binary) during the run (B) and the lower right-hand circle all the stars which had a close encounter during the run (F) (here defined as when two stars pass within 1000 au of each other). The stars which had no interactions with other stars during the run are therefore those in the upper part of the top circle, hence the number of singletons in this particular run was 95. The binary fraction was 0.2; 232 stars were initially in binary systems.

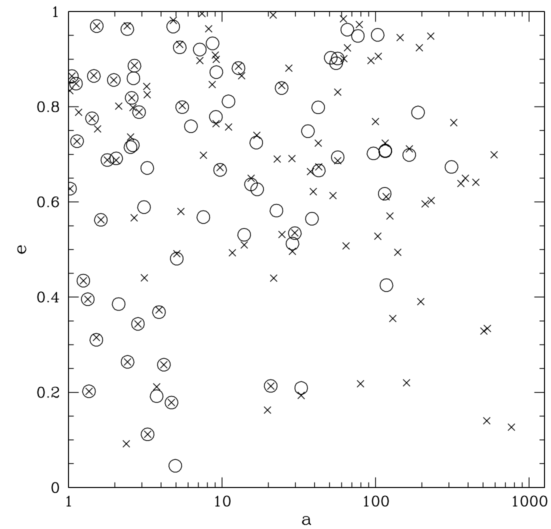

Figure 4. Eccentricity, e, as a function of semimajor axis, a (in au), for the primordial binaries in our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc) at two different times, the start (crosses) and the end (circles). Note that a binary remains in this plot even if it has escaped from the cluster; it is only removed if broken up. Thus the circles represent the population which would be seen in the field.

low-mass stars will have higher velocity dispersions than the high-mass stars and hence will evaporate preferentially.

In Fig. 3 we present a Venn diagram, showing the encounter histories of all the stars which were initially single in our reference cluster. The circle labelled B contains all stars which have been within a bound system (i.e. binary or triple) at some point during the simulation, the circle labelled F contains all the stars which have experienced close encounters with other stars ($r_{\mathrm{min}} \lesssim 1000$ au), and the circle labelled S contains all the stars which were single at the end of the simulation. The number in each category corresponds to the number of stars it contains. One can see, for example, that 21 initially single stars exchanged into binaries. Eight other initially single stars exchanged into binaries and were still in these when lost from the cluster, hence if observed in the field today they would be found in binaries. The number of stars which did not undergo any encounters at all, the singletons, was 95 in total. This is significantly smaller than the initial number of single-stars (468), and is also a small fraction of the number of stars which are single at the end of the run (544).

In Fig. 4 we plot the semimajor axes and eccentricities of the primordial binaries (the binaries which were present at the start of the simulation) in our reference run at the start (crosses) and at the end (circles) of the run. It is important to note that a binary is only removed from this plot if it is broken up; if it escapes from the cluster as a bound system it is still considered to be a binary. Hence, the circles in Fig. 4 would be binaries seen in the field. The hard–soft boundary in this cluster changes with time due to the expansion of the cluster. From equation (2) we obtain its initial position as $a_{\mathrm{hard},t=0} \approx 200$ au and at 200 Myr $a_{\mathrm{hard},t=200\mathrm{Myr}} \approx 1000$ au. One can see that most of the soft binaries are broken up during the cluster's lifetime. Furthermore, we have analysed the energies of the hard binaries in the cluster during the cluster's lifetime and these become harder with time. This is to be expected, and is the result of three-body interactions between stars (Heggie 1975). The net effect is that energy is transferred from the orbits of the hard binaries to the cluster stars, heating the cluster, which leads to the rapid initial expansion seen in the upper panel of Fig. 2.

In Table 1 we list the singleton fraction and the fly-bin fraction for all the different initial conditions. A *fly-bin* is an initially single star which has undergone a fly-by or an exchange encounter with a binary, but is single at the end of the simulation. The fly-bin fraction is the number of fly-bins divided by the number of stars which were initially single. Hence, while the singleton fraction tells us how many single stars in the field today could have unperturbed planetary systems, the fly-bin fraction tells us how many perturbed planetary systems there would be in total, if planetary systems only form around single stars. Each value is the mean of the values for the 10 realizations which we studied; the errors are their s.d. values.

The singleton fraction varies strongly with initial half-mass radius; its dependence on the number of stars is somewhat weaker, but still significant. In all the dense clusters, that is, those with initial half-mass radius smaller than about 2 pc, the singleton fraction is significantly less than unity. In the very dense clusters (i.e. the clusters with large N and small $r_{\mathrm{h,initial}}$), the singleton fraction is around 15 per cent, hence almost all of the stars have experienced close encounters and/or been exchanged into binaries. Note, however, that the singleton fraction is never equal to zero; there are always some stars which have not had close encounters with other stars or binaries.

We have also studied the escape rate of singletons compared to non-singletons from the cluster. One might expect that the singletons would mostly be ejected early on, but this is not the case. Instead the ratio of singletons to non-singletons in the cluster is roughly constant throughout its lifetime except for in the beginning of the simulation, when almost all stars are singletons. However, it is clear from the simulations that most of the singleton stars reside in the cluster's halo, where the number density of stars is lower than in the core and so interaction rates are significantly smaller.

5 DISCUSSION

5.1 Time dependence of interaction rates

We see a rapid initial expansion of the dense clusters, which is predominantly caused by binary heating from three-body interactions within the cluster. As the cluster expands, it loses stars. As the encounter time-scale is proportional to the half-mass radius as $r_{\mathrm{h}}^{5/2}$ and to the cluster's total mass as $m_{\mathrm{cl}}^{-1/2}$ (see equation 3), this leads to a strong decrease of the interaction rates, which in turn means that most of the fly-bins are created in the beginning of the simulation. In this discussion we mainly use the time-averaged half-mass radius to characterize the clusters, but it is important to note that the singleton fraction is mostly sensitive to the initial half-mass radius. Hence, observed clusters with virtually the same mean half-mass radii and ages can produce significantly different singleton fractions, depending on their initial radii. We note again that our simulations do not include the presence of gas in the clusters, which would change the initial expansion rate. Furthermore, the removal of the gas would unbind clusters with inefficient star formation, because most of the mass in such clusters is in the form of gas.

The lack of gas in our simulations means that we overestimate the number of encounters that stars formed in small clusters experience, since such clusters would disperse much faster than the clusters in our simulations, due to the significant loss of cluster mass when the gas is removed after about 5–10 Myr (Adams & Myers 2001). To better understand the rates in these clusters we have therefore also examined the singleton fraction in them 5 Myr into the simulations. At this point the singleton fraction is a factor of two larger than its final value and the number of fly-bys that a typical star experiences is significantly decreased. One should however also take into account the mass dependence of the interaction rates. A more massive star will experience more encounters, since its encounters will be more gravitationally focused, and also because of mass segregation. Since in this paper we want to examine the effects of close encounters on planetary systems, for which we only have good observational statistics around solar mass stars, we should thus primarily look at the interactions of stars with a mass close to 1 $\mathrm{M}_\odot$. For these stars the interaction rates are higher than for the average star, which decreases the singleton fraction. Furthermore, if we calculate the singleton fraction from only the stars with a mass close to 1 $\mathrm{M}_\odot$, the larger clusters in Table 1, expected to have lifetimes on the order of several 10^8 yr even with the inclusion of gas, will have a very low singleton fraction.

Since the interaction rates are much higher at the beginnings of the simulations, most of the close encounters between stars occur early on. Thus, in order to fully understand the effects that fly-by encounters between stars could have on planetary systems, one needs to investigate the effects of close encounters on planetesimal discs. The rates of encounters with binaries, where exchanges occur, are however quite constant during the lifetime of the cluster. This is to be expected, since mass segregation will cause the massive stars to sink into the centre, leading to many three-body interactions with the potential of exchanging the stars in the binaries. Thus, it is reasonable to assume that when an exchange encounter with a binary occurs, the process of planet formation around the single star will have ceased.

5.2 Binary properties

In our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc), 21 stars which were initially single were exchanged into binary systems. This number depends on the initial conditions but is significant for all our initially dense clusters. In Malmberg et al. (2006) it was shown that multiplanetary systems, originally around single stars which were exchanged into binaries, can be strongly affected by companion stars through the Kozai Mechanism (Kozai 1962). For this to occur, however, the inclination between the companion star and the planetary system must be greater than 39°.23, and the semimajor axis of the binary must be sufficiently small. If so the eccentricity of the outer planet oscillates, leading to planet–planet interactions which can eventually cause the expulsion of one or more planets.

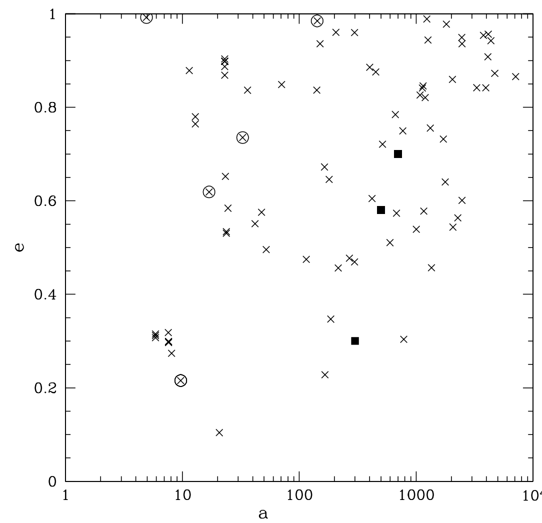

Figure 5. Eccentricity, e, as a function of semimajor axis, a (in au), for the binaries which contain stars that were initially single in our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc). The crosses are all such binaries that existed sometime during the lifetime of the cluster. Those which are also marked with circles survived the end of the run and hence would be seen in the field today. The filled squares are binary systems which we have simulated with the MERCURY integrator. In these binaries, planetary systems like our own Solar System would, if they were present around the single star which exchanged into the binary, be broken up over a time-scale of a few Myr, if the inclination between the planets and the companion star were sufficiently large.

In Fig. 5 we plot the eccentricities and semimajor axes of all the binaries formed in our reference cluster that contain stars which were initially single. Most of these binaries are fairly wide and eccentric. To test the stability of planetary systems orbiting stars in these binaries, we repeated the simulations made in Malmberg et al. (2006), but with different initial conditions for the companion star. The simulations were made using the MERCURY integrator (Chambers 1999; Chambers et al. 2002). The simulated binaries are marked as filled squares in Fig. 5. In all three simulations we started the runs with a solar mass star orbited by four giant planets, in a binary with a companion star of mass equal to 0.5 $\mathrm{M}_\odot$ at an inclination of 70°. The giant planets had the same orbits and masses as the giant planets in the Solar System. The results were very similar to those seen in fig. 2 of Malmberg et al. (2006), with the expulsion of one or more planets. The analysis is somewhat complex however, since the time-scale of the Kozai Mechanism depends on the binary period. If the semimajor axis of the binary is too large, the Kozai Mechanism will be washed out by planet–planet interactions (Innanen et al. 1997). Thus, the wider the binary is, the longer it needs to survive in order for the planetary system to be strongly affected. The lifetimes of the binaries in our clusters vary, from a few 10^5 yr to several

10^8 yr, so some of them are most likely too wide and short lived to cause any significant damage to the planetary system. Furthermore, the inclination between the planets and the companion star must, as mentioned above, be large enough. The orientation of the orbital plane of the binary with respect to the orbital plane of the planets is expected to be uniformly distributed if the system is formed in a three-body encounter. Thus one can show (Malmberg et al. 2006) that 77 per cent of the binaries containing initially single stars will have a high enough inclination between the orbital planes of the planetary system and the companion star to cause damage to the planetary system. Most of the stars which were initially single and have been in binaries were in *several* systems. This is to be expected, since it is the massive stars in the clusters which are preferentially exchanged in and out of binaries. This effect increases the probability that the planetary system around a single star, which has been in a binary during the simulation, has been sufficiently inclined with respect to the companion star. Since the binaries generally get harder during the clusters lifetime, this also increases the probability that such a planetary system will be vulnerable to the Kozai Mechanism since a shorter semimajor axis decreases the Kozai time-scale.

Around one per cent of solar-like stars are known to host so-called hot Jupiters (Marcy et al. 2005). These could be created via tidal interactions with the host star in systems with extremely eccentric orbits (Faber, Rasio & Willems 2005). Such eccentric orbits can be created through the Kozai Mechanism, if the inclination between the orbital planes of the planets and the companion star is close to 90°. The probability that such systems will exist in stellar clusters is proportional to the number of binaries containing initially single stars formed in exchange encounters. Thus the observation that the stars which were initially single and later exchanged into binaries are generally in several binaries increases the chance of creating hot Jupiters in this way. We investigate this in more detail in a forthcoming paper.

From Fig. 5 we can also see that, in several of the binaries containing stars which were initially single, the periastron distance of the companion star was shorter than the semimajor axis of, for example, Neptune (30 au). In these systems the companion star will cause direct damage to any planetary system, through close encounters between it and the planets. This will have severe effects on the stability of the system and could lead to one or more planets being ejected (Holman & Wiegert 1999).

The properties of the binaries known to contain planetary systems in the field today are largely unknown, but for a few systems the semimajor axes and eccentricity have been determined. For a compilation of known systems and their properties see Desidera & Barbieri (2007). These binaries all have similar properties to the binaries created in Fig. 5 and thus the planetary host stars in these systems could potentially have been single stars when the planets formed. The binaries marked with a circle in Fig. 5 are the binaries containing initially single stars which survived to the end of the simulation. There are a total of five such binaries, three of which are made up of two initially single stars each, and two containing one star which was initially single and one star which was in a primordial binary.

Another important feature of the clusters is the binary fraction which they produce. In our clusters the binary fraction is initially 0.2. However, since many binaries break up during the simulation, the resulting binary fraction seen in the field would be lower than this. In our reference cluster, for example, the final binary fraction is 0.13. Current estimates set the binary fraction in the field as high as 0.3 (Duquennoy & Mayor 1991), which indicates that an even higher initial binary fraction should be assumed in our clusters. This is however only partly true. In the field today, binaries with separations both smaller than 1 au and larger than 1000 au exist and these are included in the estimated binary fraction. If these were included in our simulated clusters, the final binary fraction would go up. If we, for example, assume that the distribution of semimajor axes is flat in log a all the way down to 0.1 au, and thus that another 10 per cent of stars in our clusters are in binaries with separation between 0.1 and 1 au, the final binary fraction would be 0.2, assuming that none of those binaries break up during the cluster's lifetime. Including these in our cluster simulations is however not necessary, since they can be treated as single stars from a dynamical viewpoint and thus they will only have a very small effect on the overall evolution of the cluster. Furthermore, including binaries with separations larger than 1000 au in our clusters makes no sense, since these would be broken up almost immediately. In conclusion this means that the binary fraction used in our clusters does in fact produce a final binary fraction very close to the observed one.

5.3 Fly-bys

In Fig. 6 we plot the rate of close encounters in our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc) at five and 100 Myr. These rates can be compared with what we expect from equation (3). As we showed in Section 2, the interaction rate for a cluster of mass around 500 $\mathrm{M}_{\odot}$ and a half-mass radius of 0.5 pc should be around 0.4 encounters per star per Myr. This is in good agreement with the result seen in Fig. 6 (solid line). Between 5 and 100 Myr, the mass of the cluster decreases from 500 to 400 $\mathrm{M}_{\odot}$ and the half-mass radius increases from about 0.5 to 2 pc. Thus, the rates should go down by a factor of about 40, and from Fig. 6 we see that the decrease is by about a factor of 50, in reasonable agreement with estimates. The rates which we see in our cluster are also in rough agreement with the rates found by Adams et al. (2006) for a similarly sized cluster initially in virial equilibrium.

The effects of fly-bys on planetary systems around the stars are somewhat more complicated to quantify than the effects of binary companions. If one of the stars has a Solar System-like planetary system around it, the fly-by might, if it is close enough, change the eccentricity and semimajor axis of the outer planet enough to cause

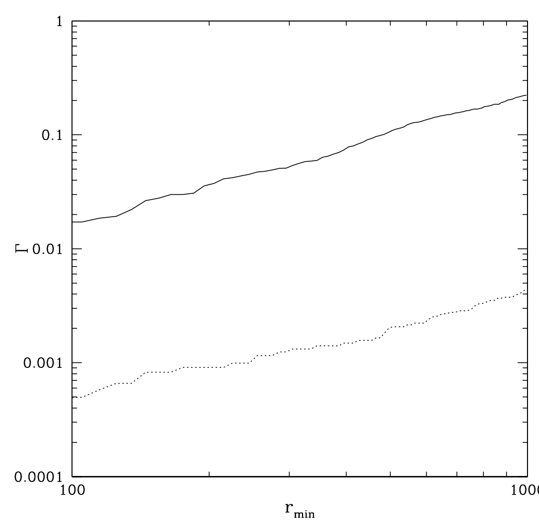

Figure 6. The encounter rate averaged over the whole cluster, Γ (number of encounters per star per Myr), as a function of the minimum encounter distance, r_{min} (in au), in our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc). The solid and dashed lines show the rates at 5 and 100 Myr, respectively.

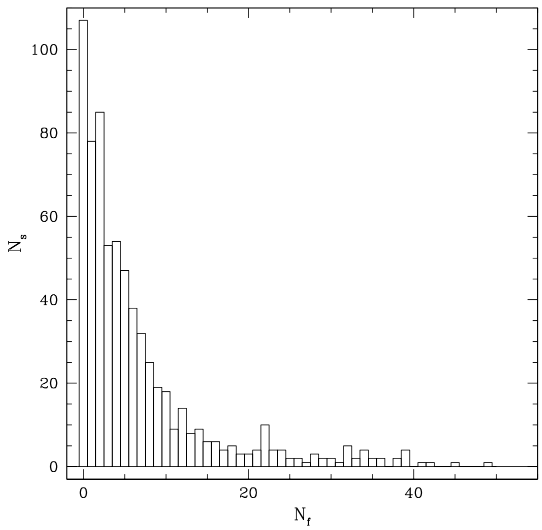

Figure 7. The distribution of the number of fly-bys, N_{f} for all the stars in our reference cluster ($N = 700$ and $r_{\mathrm{h,initial}} = 0.38$ pc).

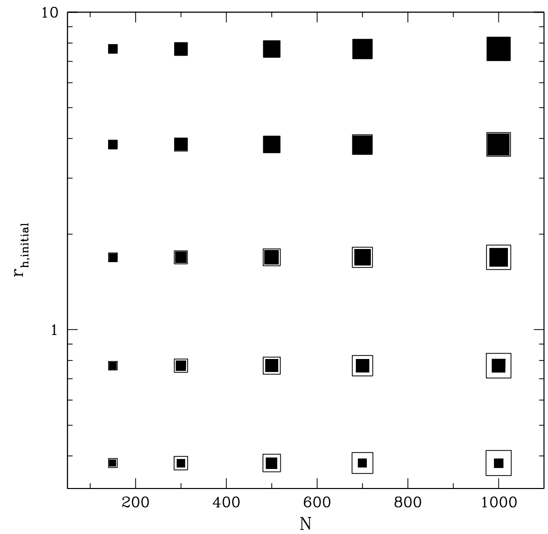

Figure 8. The singleton fraction as a function of the initial number of stars in the cluster, N and initial half-mass radius, $r_{\mathrm{h,initial}}$ in pc. The areas of the open boxes correspond to the number of single stars at the end of the simulations and the areas of the filled boxes within them to the number of singletons, also at the end of the simulations. The values are averaged over the 10 realizations for each initial condition.

chaotic evolution of the system and the expulsion of one or more planets. Adams et al. (2006) calculate the cross-section for the disruption of planetary systems for five different stellar masses, ranging from 0.125 to 2 $\mathrm{M}_{\odot}$. These cross-sections can be used as a starting point to understand the effects on Solar System-like planetary systems from fly-by encounters with other stars/binaries. In order to fully understand these effects it is however also necessary to calculate the cross-sections for the disruption of planetary systems due to secular planet–planet interactions, induced by small perturbations in the orbital elements of the planets, due to encounters with other stars. Analytical formulae for the induced change in eccentricity in a three-body encounter were derived by Heggie & Rasio (1996) and these can be used to analyse this effect. Another method is to include planetary systems in *N*-body simulations of stellar clusters (Hurley & Shara 2002; Spurzem et al. 2006). Such studies have shown that systems like the Solar System, where the planets are on circular orbits, are generally not significantly perturbed by distant encounters, while the effect on already eccentric planets is significantly bigger. Thus the likelihood that a single close encounter between two stars, where one hosts a planetary system like the Solar System, will significantly alter the planetary orbits is rather small. One should however note that most stars which undergo close encounters with other stars in the large clusters in our simulations do so more than once (see Fig. 7). In our reference cluster, for example, a little more than 20 per cent of the stars had experienced 10 or more fly-bys. Thus, even if only one in 10 of the fly-bys causes damage to a planetary system, a significant fraction of the planetary systems in these clusters could be altered from their original state due to the effect of fly-bys. The importance of this effect varies between different clusters due to the differences in lifetimes. Stars within cluster which disperses after 5–10 Myr will not suffer many fly-bys, while those in longer lived clusters will.

5.4 Singleton fraction of solar mass stars

As was seen earlier in Fig. 2 the half-mass radius of a cluster expands rapidly during the early stages of its evolution due to binary heating and mass-loss. Thereafter the half-mass radius stays roughly constant during the remainder of the cluster's lifetime. Thus the initial expansion is important when we map the simulated clusters on to the observed cluster population. A cluster which is seen to have a half-mass radius of about 3 pc can have had a significantly smaller radius initially. Furthermore, most of the embedded clusters seen in Lada & Lada (2003) will not evolve into open clusters, but instead disperse when their gas is ejected after a few Myr and thus have very short lifetimes.

In Fig. 8 we plot the singleton fraction for each of the clusters in our simulations as a function of the initial number of stars, *N*, and the initial half-mass radius, $r_{\mathrm{h,initial}}$. As can be seen from the figure and from Table 1 the singleton fraction varies slowly with *N* and strongly with $r_{\mathrm{h,initial}}$. Across the range of masses that we simulate, the radii given by Lada & Lada (2003) lie towards the centre of the plot in Fig. 8. In the more massive clusters the singleton fraction is around 0.5. For stars in the mass range $0.8 \leqslant m \leqslant 1.2\,\mathrm{M}_{\odot}$ the singleton fraction is however even lower. For example, in our reference cluster the singleton fraction for stars in this mass range is 0.11, compared to the singleton fraction averaged over all the stars, which is 0.18. Averaging the singleton fraction over *N* and r_{h}, we find that as a lower bound we can say that of the solar mass stars which form in clusters which evolve to become open clusters, more than 50 per cent of the single stars are not singletons. Since about 10 per cent of stars form in such clusters, at least 5 per cent of solar mass stars in the solar neighbourhood are not singletons. Thus, at least 5 per cent of solar mass stars may have had their planetary systems significantly altered through dynamical interactions with other stars and thereby forming at least some of the observed extrasolar planets. Furthermore, stars in the smaller clusters, with $N \leqslant 100$ stars, also undergo close encounters with other objects before the cluster disperses due to the removal of gas. Even though the singleton fractions in these clusters are not as low as our simulations indicate, because real clusters have much shorter lifetimes than in our simulations, it is still not equal to one. We can understand this from analysing the singleton fraction 5 Myr into the simulations of low-mass clusters (see Section 5.1).

In Fig. 9 we plot the singleton fraction as a function of initial number of stars, *N* and the time-averaged half-mass radius, $r_{\mathrm{h,mean}}$.

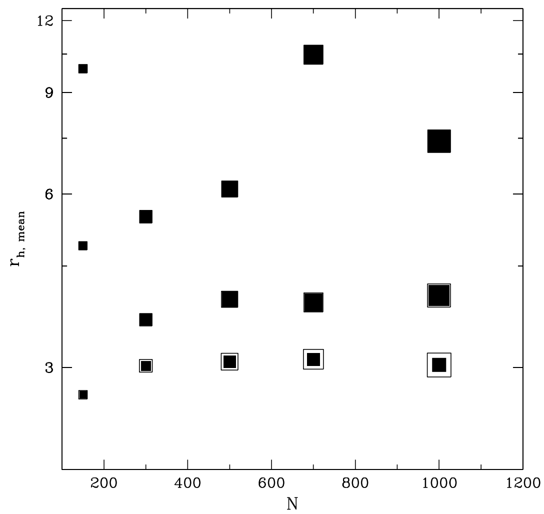

Figure 9. The singleton fraction as a function of the initial number of stars in the cluster, N, and the time-averaged half-mass radius, $r_{\rm h,mean}$, in pc. The areas of the open boxes correspond to the number of single stars at the end of the simulations and the areas of the filled boxes within them to the number of singletons, also at the end of the simulations. For each value of N, the square with smallest $r_{\rm h,mean}$ is the average value of all the realizations with this N and initial half-mass radius of 0.38, 0.77 and 1.69 pc, respectively. For the remaining squares the values are just averaged over the 10 realizations for each set of initial conditions.

As can be seen, all our clusters with initial half-mass radii less than 2 pc expand; their time-averaged half-mass radii are equal to around 3 pc. The properties of these evolved clusters are broadly comparable to those in the open cluster catalogue of Kharchenko et al. (2005).

To understand the effect on the total stellar population in the solar neighbourhood one also needs to take into account the fact that the lifetime of the clusters depends on the star formation efficiency in them. A low star formation efficiency leaves a large portion of the clusters' mass as gas, which means that the cluster will disperse upon gas removal or at least lose a significant fraction of its mass (Goodwin & Bastian 2006). This means that many stars in such clusters will not undergo encounters with other stars. Furthermore, preliminary observations suggests that when including isolated star formation the mean cluster size decrease from the previous estimate of 300 (Adams et al. 2006; Allen et al. 2007). Given these uncertainties it is not possible to give an exact estimate of the singleton fraction of stars in the solar neighbourhood. From the results of our simulations it is however clear that the singleton fraction of solar type stars in the solar neighbourhood is no less than 5 per cent.

This can be compared with the estimated fraction of solar mass stars which are expected to harbour planetary systems of the type so far detected, which is about 10 per cent (Marcy et al. 2005). As discussed above, more than 5 per cent of stars with a mass in the interval between 0.8 and 1.2 $M_\odot$ are not singletons. Thus, the number of stars which could have had their planetary systems perturbed by dynamical interactions in the clusters in which they were formed is comparable to (or slightly smaller than) the number of stars around which, based on observational constraints, we expect to find perturbed planetary systems.

6 SUMMARY

In this paper we have performed N-body simulations of stellar clusters, with properties similar to, but due to the exclusion of gas not identical to, those in which a significant fraction of stars in the solar neighbourhood formed. During the simulations we monitored the interactions of each star with other stars and binaries in the cluster. This allowed us to estimate a lower bound on the singleton fraction in the solar neighbourhood, that is, the fraction of stars which formed single and have never undergone any close encounters or been exchanged into a binary. We find that of the stars with a mass close to 1 $M_\odot$, which were formed as single stars in large clusters, a significant fraction have undergone some type of strong interaction with another star or binary, either a fly-by or an exchange encounter with a binary system. For the total population of solar-like stars in the solar neighbourhood this means that at least five per cent of them have undergone such encounters. If one or more of the stars in such an encounter had a planetary system with properties similar to those of the Solar System, it may have been perturbed by the encounter. Such a perturbation may cause the expulsion of several planets via planet–planet interactions, leaving the remaining planets on eccentric orbits. Furthermore, some of the stars which were initially single exchanged into binaries which remain after the demise of the cluster. These binaries have similar properties to those of observed binary systems containing extrasolar planets. Thus, dynamical processes in young stellar clusters might be responsible for the properties of some of the observed extrasolar planets.

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