

GAMMA-RAY BURSTS, SUPERNOVA KICKS, AND GRAVITATIONAL RADIATION

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ABSTRACT

We suggest that the collapsing core of a massive rotating star may fragment to produce two or more compact objects. Their coalescence under gravitational radiation gives the resulting black hole or neutron star a significant kick velocity, which may explain those observed in pulsars. A gamma-ray burst can result only when this kick is small. Thus, only a small fraction of core-collapse supernovae produce gamma-ray bursts. The burst may be delayed significantly (hours to days) after the supernova, as suggested by recent observations. If our picture is correct, core-collapse supernovae should be significant sources of gravitational radiation with a chirp signal similar to a coalescing neutron star binary.

Subject headings: accretion, accretion disks — binaries: close — gamma rays: bursts — gravitational waves — stars: neutron — supernovae: general

1. INTRODUCTION

It is now widely believed that long (≥ 5 s) gamma-ray bursts (GRBs) are produced by a class of supernovae (SNe) known as collapsars or hypernovae (Woosley 1993; MacFadyen & Woosley 1999; Paczyński 1998). The collapse of the rotating core of a massive star is assumed to lead to the formation of a black hole, the remaining core material having enough angular momentum to form a massive accreting torus around it. Powered by either neutrino annihilation or MHD processes, matter is expected to be expelled with high Lorentz factors once an evacuated channel along the rotation axis of the core has formed.

Direct evidence for the association of SNe and GRBs comes from the detection of bumps in the afterglow of several GRBs (e.g., Price et al. 2002 and references therein) and the recent detection of SN ejecta in the X-ray afterglow of GRB 011211 by Reeves et al. (2002). However, the hypernova class is extremely small: even allowing for the probable beaming of GRBs (Frail et al. 2001), the fraction of hypernovae among SNe cannot be greater than about 10^{-3} . Evidently, the production of a GRB by an SN is a very rare event. What causes this rarity is unclear.

The X-ray observation of an SN-GRB association by Reeves et al. (2002) throws up a further puzzle. Light-travel arguments give a size of 10^{14} – 10^{15} cm for the reprocessing region producing the X-ray spectrum, depending on the beaming. This is much larger than the radius of the progenitor star and must be associated with the SN outflow. Indeed, the measured blue-shift of the spectrum with respect to the known GRB redshift implies an outflow velocity of $\sim 0.1c$. But these two measurements together require that the GRB occurred between 10 hr and 4 days after the SN. This is clearly incompatible with the simplest version of the hypernova model (MacFadyen & Woosley 1999).

In this Letter, we offer a solution to both problems. We reconsider the collapse of a rotating core and suggest by analogy with simulations of star formation that this may produce two or more compact objects. This idea was first investigated by Berezhinskii et al. (1988) and Imshennik (1992) and was developed further in later papers (e.g., Zabrodina & Imshennik 2000; Colpi & Wasserman 2002). The subsequent coalescence of these objects can power a GRB (Paczynski 1986), accounting for the SN-GRB delay (although see also Vietri & Stella 1998

and Rees & Mészáros 2000). The merger itself will generally give the black hole resulting from the collapse a significant velocity (“kick”). This may be the explanation for the kicks observed in pulsars (Arzoumanian, Chernoff, & Cordes 2002). Following the suggestion of MacFadyen & Woosley (1999) that GRB production will be adversely affected by such kicks, we show that only a small fraction of core-collapse SNe will produce GRBs. These are likely to be a subset of those producing a massive black hole ($\geq 12 M_{\odot}$).

2. CORE COLLAPSE AND FRAGMENTATION

It is well known that the dynamical collapse of a self-gravitating gas cloud increases the importance of rotation. The ratio of kinetic to gravitational binding energy grows as $\sim 1/r$, where r is the length scale of the collapsing object. Many authors (see Bonnell & Pringle 1995 and references therein) have suggested that this probably leads to fragmentation, seen, for example, in the collapse of molecular clouds to form pre-main-sequence stars. Fragmentation requires that the collapsing core becomes bar-unstable and that any bar lives a few dynamical times. In core collapse to nuclear densities, the second requirement is very likely to be met (Bonnell & Pringle 1995), while the first depends on the equation of state and the initial conditions, including of course the angular momentum of the material destined to be a neutron star or black hole. The exact value of the angular momentum is unknown but could be quite large (see, e.g., Heger, Langer, & Woosley 2000). Determining the precise conditions under which fragmentation occurs requires large-scale numerical simulations, which are under way. For the remainder of this Letter, we consider the case in which two compact objects form with masses and radii M_1, M_2 and R_1, R_2 , with $M_1 > M_2$. In Figure 1, we plot R_2 as a function of M_2 for the equation of state of Shen et al. (1998a, 1998b). From Figure 1, we see that we require a mass $\geq 0.2 M_{\odot}$ —at lower masses, nuclei form even in the center of the stars, and the object has a much larger radius. A similar low-mass limit is obtained for other nuclear equations of state.

3. MERGERS AND GRBs

To have the potential of powering a GRB, the merger of the two orbiting lumps must produce a central object surrounded by a torus. This will happen if at least one of the lumps is a

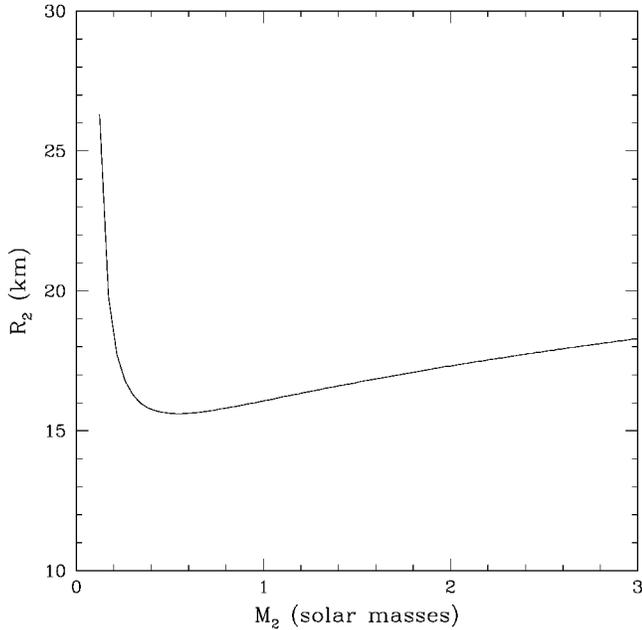


FIG. 1.—Mass-radius relation for cold neutron stars in β -equilibrium using the equation of state of Shen et al. (1998a, 1998b). The endpoint on the low-mass side is reached once nuclei start to form in the center of the star.

neutron star rather than a black hole and if the mass transfer eventually becomes dynamically unstable. For a corotating object less massive than the accretor and filling its Roche lobe, of radius (Paczynski 1971)

$$R_L = 0.462 \left(\frac{M_2}{M_1 + M_2} \right)^{1/3} a \quad (1)$$

(where a is the separation), this requires that R_L moves inward with respect to its radius R_2 . The standard result (e.g., van Teeseling & King 1998) is

$$\frac{\dot{R}_L}{R_L} - \frac{\dot{R}_2}{R_2} = -\frac{2\dot{M}_2}{M_2} \left(\frac{5}{6} + \frac{\zeta}{2} - \frac{M_2}{M_1} \right) + 2\frac{\dot{J}}{J}, \quad (2)$$

where the mass-radius relation is taken as $R_2 \propto M_2^\xi$ and \dot{J} includes all forms of orbital angular momentum loss. This expression shows that dynamical instability must occur if $\zeta < -5/3$, since $\dot{M}_2, \dot{J} < 0$. The mass-radius relation (Fig. 1) now shows that the instability is inevitable for any lobe-filling object since it will occur at the latest once its mass is reduced to $M_2 \approx 0.2 M_\odot$ and the lump begins to expand rapidly on mass loss. The tidal lobe of a noncorotating object is similar in size, so again instability will occur for $M_2 \approx 0.2 M_\odot$. Instability may well happen before this point for other reasons: for example, the orbit may be so close that the accreting matter cannot form a disk around the accretor, adding a dynamical-timescale term to \dot{J} . For sufficiently stiff equations of state, Newtonian tidal effects can also lead to an instability on a dynamical timescale (Lai, Rasio, & Shapiro 1993).

The only way that dynamical instability can be avoided is if (1) both lumps are already black holes or (2) the accretor is a black hole, and the accreting object spirals within its horizon before filling its Roche (or more generally tidal) lobe. This occurs if $R_2 < R_L$ for $a = \eta GM_1/c^2$. With $R_2 = 10^6 R_6$ cm, we

find the following condition:

$$\frac{M_1^3 M_2}{M_1 + M_2} > \left(\frac{7.2 R_6}{\eta} \right)^3. \quad (3)$$

Only the most massive black holes can swallow neutron stars whole. This is true for a wide range of neutron star radii, including $R_6 = 1$. Most mergers result in the dynamical instability of the neutron lump. Thus, fragmentation and subsequent coalescence release enough energy to power a GRB.

4. MERGERS AND KICKS

Simulations of unequal-mass neutron star mergers show that the mass loss from the system is asymmetric. The escaping material originates from the lower mass star and is ejected on a timescale shorter than the orbital period. This provides a thrust to the merged object, which is found to have a velocity $V_{\text{kick}} \sim 800 \text{ km s}^{-1}$ for the case $M_1 = 0.8 M_\odot$ and $M_2 = 0.7 M_\odot$ (S. Rosswog & M. B. Davies 2002, in preparation; but see also Rosswog et al. 2000; Zhuge, Centrella, & McMillan 1994; Rasio & Shapiro 1994; Ruffert & Janka 2001).

In general, we can assume that the ejected material, M_{lost} , is ejected at a speed proportional to $V_{2,\text{orb}}$ (where $V_{2,\text{orb}}$ is the orbital velocity of the secondary, M_2) when the donor finally gets shredded. Combining expressions for $V_{2,\text{orb}}$ and the orbital separation a (assuming the donor fills its Roche lobe) and applying conservation of momentum, we obtain the following expression for the kick given to the merged object:

$$V_{\text{kick}} \propto \left[\frac{GM_1^2}{(M_1 + M_2)R_2} \left(\frac{M_2}{M_1 + M_2} \right)^{1/3} \right]^{1/2} \frac{M_{\text{lost}}}{M_1 + M_2 - M_{\text{lost}}}. \quad (4)$$

For systems in which M_2 has been reduced to the minimum mass of $\sim 0.2 M_\odot$, with $M_1 \gg M_2$, we see that $V_{\text{kick}} \propto M^{-2/3}$, where $M = M_1 + M_2$. We use the result of Rosswog & Davies (as stated above), for $M = 1.5 M_\odot$, to write

$$V_{\text{kick}} = 800 \left(\frac{1.5 M_\odot}{M} \right)^{2/3} \text{ km s}^{-1}. \quad (5)$$

The kick may be slightly lower if the final shredding occurs before M_2 reaches $0.2 M_\odot$. It should also be noted that the speed of the compact object at infinity will be reduced as it is decelerated by the gravitational force of the ejected material. Likely values of the speed of the merged object at infinity lie in the range of $100\text{--}300 \text{ km s}^{-1}$, although the merged object and the ejecta may be bound in some cases.

A remarkably similar kick occurs if both merging objects are black holes, because of the effect of gravitational radiation reaction on the final plunge orbit (Bekenstein 1973; Fitchett 1983; Fitchett & Detweiler 1984). For rapidly spinning holes, as are likely in core collapse, the kick velocity may approach 1500 km s^{-1} (Fitchett 1983). The basic reason for the similarity is that in both cases, the recoil velocity is of order the primary's center-of-mass velocity immediately before the plunge phase.

Observations of eccentric binary pulsars show that there is evidence of misalignment angles between the pulsar spin and the orbital angular momentum in some systems (e.g., Hughes & Bailes 1999). This may be a result of the warping of the disk that will occur on a viscous timescale (Pringle 1996). It is possible also that more than one kick mechanism operates.

5. SN KICKS AND GRBs

The torus surrounding the compact object releases its energy into the region along its rotation axis. As pointed out by MacFadyen & Woosley (1999), the production of a GRB will be inhibited if the volume into which the $\nu - \bar{\nu}$ or MHD energy is deposited is increased significantly by the motion of the central object and torus with respect to the surrounding gas. This motion may be the recoil described in the previous section or it may be a kick derived from some other physical mechanism (see, e.g., Lai 2001).

A potential GRB will be extinguished if $V_{\text{kick}} \geq d/\tau_{\text{er}}$, where d is the length scale for energy deposition into a potential fireball and τ_{er} is the timescale for the energy to be released from the torus that is set by the viscous timescale of the torus. Assuming $\tau_{\text{er}} \sim 1$ s and taking $d = 15(M_1/M_\odot)$ km (see figures in Fishbone & Moncrief 1976), a GRB will fail if

$$V_{\text{kick}} \geq 15 \frac{M_1}{M_\odot} \text{ km s}^{-1}. \quad (6)$$

The expression above is plotted in Figure 2 along with the likely kick received by the central object and torus (as given in eq. [5]); note here that for the interesting range of values of M , $M_2 \ll M_1$; hence, $M_1 \approx M$). This figure suggests that the recoil velocity is likely to extinguish any potential GRB when the total mass $M \lesssim 12 M_\odot$. In other words, GRBs will be extinguished when V_{kick} exceeds some particular value, ~ 200 km s $^{-1}$. The exact limiting mass for a GRB is uncertain, but the important point here is that GRBs will only occur if the heaviest fragment is above some limiting mass.

6. THE SN-GRB DELAY AND GRAVITATIONAL-WAVE EMISSION

In the picture presented here, core collapse only produces a GRB in a few cases. Even in these cases, the burst may not follow the collapse immediately but may be delayed while gravitational radiation brings the orbiting fragments into contact. For an initial circular orbit of separation a_0 , this requires a time

$$\tau_{\text{gr}} = 0.18 \frac{(a_0/1000 \text{ km})^4}{m_1 m_2 (m_1 + m_2)} \text{ hr}, \quad (7)$$

where $m_1 = M_1/M_\odot$, etc. We see that a delay of hours is quite possible. Since $\tau_{\text{gr}} \propto a_0^4$, only a small increase in the value of a_0 (say, to 3000 km) will produce a large (factor of ~ 100) increase in the delay between the formation of the two lumps and their coming into contact. To produce a disk of such a radius requires the material to have a specific angular momentum $\sim 2 \times 10^{17}$ cm 2 s $^{-1}$. This compares with values obtained by Heger et al. (2000) of 10^{16} – 10^{17} cm 2 s $^{-1}$ for the cores of massive, rotating stars. Thus, SN-GRB delays of the order inferred by Reeves et al. (2002) in GRB 011211 are at the upper end of what may be regarded as reasonable in this picture.

An obvious corollary of this is that our picture predicts that core-collapse SNe should be strong sources of gravitational radiation and that the signal should resemble a binary in-spiral pattern (see Bonnell & Pringle 1995; van Putten 2001; Fryer, Holz, & Hughes 2002). A neutron star merger should be detectable by the Laser Interferometer Gravitational-Wave Observatory (LIGO) out to 20 Mpc, and by LIGO II out to 300 Mpc. The gravitational-wave signal strength for two point masses in

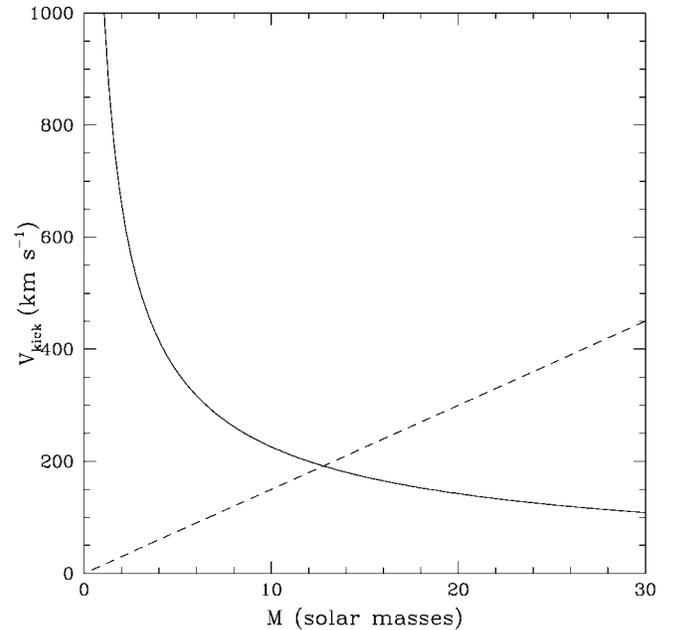


Fig. 2.—Kick received (in units of kilometers per second) by the central object and torus (*solid line*) and the kick required to extinguish a GRB (*dashed line*). Both are plotted as a function of total mass, M (in units of solar mass).

circular orbits with a separation a is given by $h \propto \Omega^2 \mu a^2$, where $\Omega^2 = G(M_1 + M_2)/a^3$ and $\mu = M_1 M_2 / (M_1 + M_2)$. Hence, the detectability of a merger of two compact objects is a sensitive function of their masses. As an example, we consider here that the merger of two half-neutron stars (i.e., $M_1 = M_2 = 0.7 M_\odot$) will be detectable to a distance of 6 Mpc with LIGO and 100 Mpc with LIGO II. We can derive a predicted event rate from an assumed event rate per galaxy (see Phinney 1991). Assuming a formation rate of 10^{-2} yr $^{-1}$ per galaxy, LIGO II should see ~ 400 mergers per year. This is much larger than the number of neutron star merger events per year LIGO II should detect (~ 10).

7. CONCLUSIONS

We have suggested by analogy with large-scale simulations of star formation (M. R. Bate, I. A. Bonnell, & V. Bromm 2002, in preparation)¹ that the core collapse of a massive rotating star may lead to the fragmentation of nuclear-density lumps. The subsequent coalescence of these lumps under gravitational radiation gives the resulting black hole or neutron star a significant kick velocity, compatible with those observed in pulsars (see, e.g., Arzoumanian et al. 2002). A GRB can result only when this kick is small. Thus, only a small fraction of core-collapse SNe produce GRBs. The most likely candidates are those containing massive black holes ($M_1 \gtrsim 12 M_\odot$) that have not formed via the merger of two lower mass black holes. The burst may be delayed significantly (hours to days) after the SN, as suggested by recent observations.

The complexity seen in star formation studies suggests that a large variety of behaviors is likely in core collapse. A GRB appears to require a rather high degree of symmetry and alignment and is therefore a rather unusual outcome. We note that in the case of a kick driven by mass expulsion in a double neutron star merger, the expelled gas may have up to 10 times

¹ See <http://www.ukaff.ac.uk/starcluster>.

the energy of the kinetic energy of the merger product. Given likely initial neutron star kick velocities of $\sim 1000 \text{ km s}^{-1}$, these energies may approach 10^{50} ergs and thus have noticeable effects on the early development of the SN outburst.

A clear test of our picture will be given by gravitational-wave experiments. An observed chirp signal, in which the *total* mass is $\approx 1.4 M_{\odot}$, would be easily explained in our model but

practically impossible to explain via standard neutron star mergers (as predicted from the observed binary pulsars; Phinney 1991).

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