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DEVELOPING TEACHING OF MATHEMATICS TO ENGINEERING STUDENTS: TEACHER RESEARCH,  
STUDENT EPISTEMOLOGY  
AND MATHEMATICAL COMPETENCE

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**Résumé**

Un projet de recherche “Engineering Students Understanding Mathematics” (ESUM) a montré des rapports tendus entre les perspectives théoriques des professeurs, l’organisation de l’enseignement et les stratégies d’apprentissage des étudiants. Dans mon équipe de recherche, nous cherchons à améliorer notre enseignement des étudiants futurs ingénieurs et de développer leur compréhension des concepts de mathématiques. Étant donné nos résultats du côté « étudiant », et les possibilités et limites du côté institutionnel, nous explorons une gamme de ressources et d’approches de l’enseignement utilisant les perspectives théoriques autour de l’« inquiry community », la genèse documentaire, et la théorie de l’activité.

A research project “Engineering Students Understanding Mathematics” (ESUM) has shown tensions between teachers’ theoretical perspectives and design of teaching and students strategic approach to learning. In my research team we seek to improve our teaching of engineering students and to develop their conceptual understandings of mathematics. Given our findings on student perspectives, and the institutional affordances and constraints, we are exploring a range of resources and teaching approaches using theoretical perspectives of inquiry community, documental genesis and activity theory.

**ESUM -- Engineering Students Understanding Mathematics**

The ESUM project at Loughborough University was designed to achieve more conceptual learning of mathematics for materials engineering students in their first year at university, supported by funding from the national HESTEM programme<sup>1</sup>. It was created as a developmental research project involving both insider and outsider research and it involved an innovation in learning and teaching. This included: inquiry-based questions and tasks; use of a GeoGebra environment for the learning of functions; small group activity in tutorials and an assessed small group project.

Our research questions were as follows:

- How can we enable engineering students’ more conceptual understanding of mathematics
- What teaching approach is appropriate (and why)? -- Design
- What means do we have of perceiving students’ outcomes? -- Methodology.
- What outcomes do we achieve? -- Evaluation

The project was constructed within a sociocultural frame. Thus, in seeking mathematical understanding, we are interested not only in cognitive processes, but in the whole context and culture in which we are active. This includes: the mathematical meanings we make; the

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<sup>1</sup> HESTEM is the national Higher Education programme relating to Science, Technology, Engineering and Mathematics with the purpose of promoting greater participation and success in these areas of education. We received funding from this programme via the Royal Academy of Engineering in the UK.

perspectives of students and teachers; the institutional dynamics and constraints that we experience and our various worlds in and beyond the institutional setting.

The setting is a University with a 3-year BSc degree in Materials Engineering and first year students taking part in a one year module in mathematics – we focused our research on the first semester only. The module has two lectures and one tutorial per week (each 50 minutes) which have been in the past rather traditional in style. The university uses a virtual learning environment (VLE) known as LEARN which is used to provide a variety of means of communication with students, particularly holding notes and resources. Assessment of the module in the past has been by exam (60%) and computer based tests (40%). We are aware that teaching, in the past, has involved rather instrumental approaches to mathematics (Skemp 1977; Hiebert, 1986, Artigue, Batanero & Kent, 2007).

The ESUM project involved a teaching team of three people, all experienced teachers, with responsibility for: interpretation of curriculum, design of questions/tasks/group project (with the help also of PhD students), design of the innovation and an associated teaching approach, teaching the module (one member of the team has been the lecturer). The research team has comprised four people, the teaching team plus a research officer (employed using the HESTEM funding) with responsibility for the design of the research, for engaging in research *in practice* (insider research) and for conducting research *on practice* (outsider research). Insiders have been mainly those in the teaching team; outsider research has involved mainly the research officer in collecting data, with analysis undertaken by several of the team.

The innovation has included inquiry-based questions and tasks; use of a GeoGebra environment (particularly for the learning of functions); small group activity in tutorials and an assessed small group project. The latter resulted in a revised assessment consisting of the group project (20%), computer-based tests (20%), and exam (60%). I present three examples below (a, b and c) to show what I mean by inquiry-based tasks.

**a) an open question/task in a lecture**

Think about what we mean by a function and write down two examples. Try to make them different examples.

**b) a lecture or tutorial task**

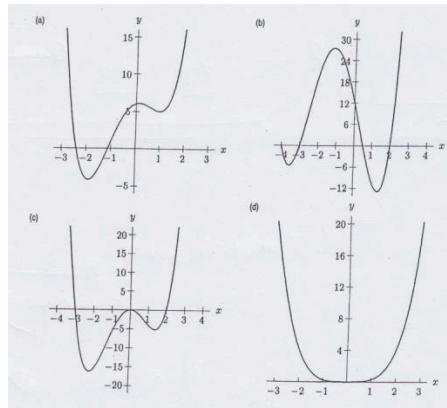
Given the function  $f(x) = x^2 - 3x + 4$ , sketch the function on a pair of axes.

- i) Find the equation of a line that crosses this curve where  $x=1$  and  $x=2$
- ii) Find the equation of a line of gradient 3 that crosses the curve twice
- iii) Find the equation of a line of gradient -3 that does not cross the curve

**c) a tutorial task – to be undertaken in small groups**

Use sliders in GeoGebra to determine which of the graphs below could represent the function  $y=ax^4+bx^3+cx^2+dx+e$

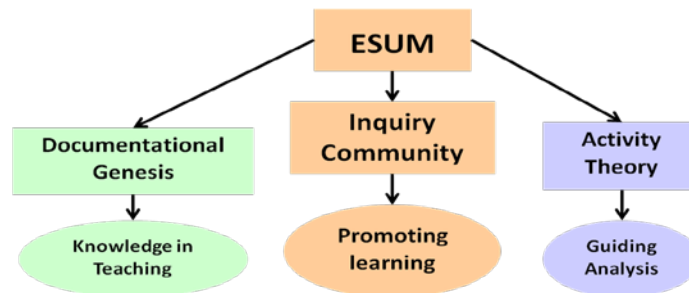
Here a, b, c, d and e are real numbers, and  $a \neq 0$ . Explain your thinking.



It can be seen that task c was designed with the intention that students would use GeoGebra to explore possibilities

### Theoretical perspectives

We have based our work within three areas of theory: inquiry community, documentational genesis and activity theory as represented in the following diagram.



Ideas of inquiry community are central to the entire philosophy of the project and to our practices in learning and teaching; documentational genesis helps to theorise the nature and development of teaching, and activity theory has proved extremely valuable as a tool to aid analyses.

#### *Inquiry community*

Our theory of inquiry community has been based on ideas of Community of Practice (CoP) as expressed by Wenger (1998). We see learning as a process of participation and reification within a community which has shared objectives and working practices (in Wenger's terms Joint Enterprise, Mutual Engagement, and Shared Repertoire). Group and individual identities are formed through engagement, imagination and alignment (literally a process of lining up with the norms and expectations within the practice). All of these concepts carry over into the conceptualisation of a community of inquiry, except the notion of alignment. The problem is that, in aligning with norms and expectations of a practice, it is possible to perpetuate aspects of the practice that are unhelpful in achieving certain goals. For example, in learning and teaching mathematics, we might characterise a mathematics classroom as a community of practice in which we can identify many of the aspects of engagement, repertoire and enterprise of which Wenger speaks. Nevertheless we might also recognise that the community is not achieving the best possible practice in terms of a goals related to students' conceptual understanding of mathematics. It might be that a rather instrumental level of understanding is accepted by all concerned, and we know that this is true in many classrooms.

In an inquiry community, inquiry is a fundamental concept related to ways of working to promote conceptual understandings. Where mathematics is concerned, inquiry in mathematics involves asking questions and seeking answers, recognising problems and seeking solutions, investigating, exploring, conjecturing and seeking justifications for conjectures. When students engage together in such activity they engage deeply with mathematical processes and concepts and address the fundamentals of mathematics rather than just staying at surface (instrumental) level. Similarly in mathematics teaching, when teachers ask questions about their practice and explore ideas in the classroom they get into depth in conceptualising teaching processes and really thinking through what approaches can promote/foster students' conceptual learning. Thus we seek opportunities to promote an inquiry base in both learning mathematics and teaching mathematics. The result of this is that we are not satisfied in just aligning with existing norms and ways of working – we seek an inquiry way of being in which we question the status quo and seek to know more about what is possible in order to make it more effective. We talk about this as a process of critical alignment: questioning what we do as we do it and seeking to know more and to do things in better ways to promote our goals. We can talk about inquiry and critical alignment as developing ways of being in practice. Here theory and practice are related fundamentally both in terms of what we do and how we think about what we do (Jaworski 2006, 2008). Thus, to summarise, in a community of inquiry we

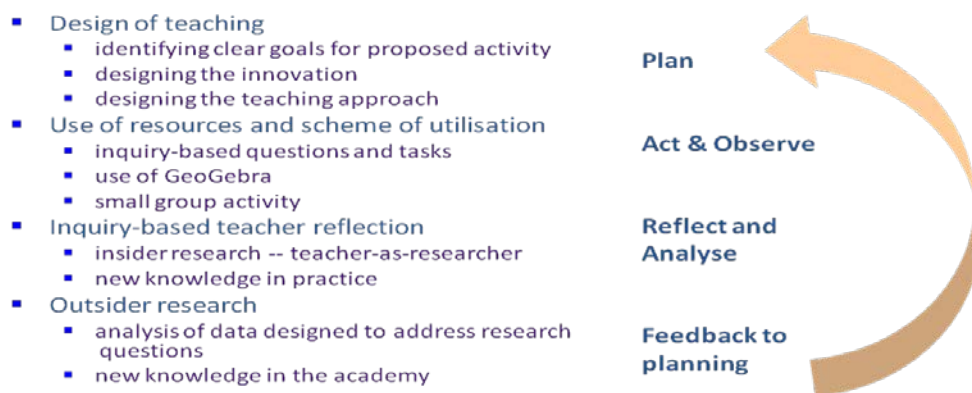
- engage in participation and reification as in CoP;
- address inquiry-based questions to challenge existing ideas and engage in meaning-making more deeply;
- encourage asking of own questions, motivation of wanting to know and looking critically at outcomes;
- develop a critical sense of what we are doing and achieving according to our goals.

### *Documentational Genesis*

Based on the theory of Gueudet & Trouche (2009) documentational genesis is about promoting knowledge in teaching. It involves first of all a clear design of teaching in which we identify clear goals for proposed activity, designing the innovation and designing the teaching approach. This design process takes into account fundamentally the resources that we use and our associated schemes of utilisation – that is, why we use the resources we use and the ways in which we use them. In ESUM our resources include inquiry-based questions and tasks, a GeoGebra environment and small group activity. In each of these we have clear goals for our use of the resource. For example, inquiry-based tasks are designed to promote deep engagement with mathematics as explained above. GeoGebra provides an electronic environment where alternative representations can be seen side by side, to aid visualisation and provide alternative perspectives, and the parameters changed to allow experimentation and inquiry. Small group activity encourages dialogue and negotiation of ideas, addressing meanings and challenging limited perspectives. The ways in which we use these resources are related to our institutionalised setting and the constraints it imposes. So, in a lecture, our approach is modified to fit lecture constraints – e.g., GeoGebra can be used only in demonstration mode; inquiry-based activity only at a level suited to seat work in a lecture hall. However, for tutorials we are able to use a computer laboratory in which each student sits at a computer and movement around the room is possible, so each student can use GeoGebra dynamically to explore in their own ways and discuss these within their group. Decision-making and discussion within a group are promoted within tutorials with the lecturer and a graduate student assistant circulating among groups observing activity and discussing ideas.

The design process relates closely to its implementation in practice and reflection in practice feeds back to inform ongoing design of tasks and teaching approach. The lecturer wrote reflective notes each week which captured issues arising locally. Lecturer and research officer discussed activity after each lecture and tutorial, allowing modifications to be made to practice in the next session. This is a complex inquiry process which forms the basis of what we call developmental research. It is research which leads to new knowledge about teaching and about making sense of the overall complexity in our practice. We see inquiry-based teacher reflection leading to new knowledge in practice (insider research -- teacher-as-researcher), and we see analysis of data gathered from observations on teaching leading to new knowledge in the academy (outsider research).

Thus the process of documentational genesis is a developmental process which provides knowledge in and about the practices in which we engage and simultaneously promotes development in practice so that our practice improves directly in relation to our increased knowledge. The following diagram offers an overview of our discussion above.



### ***Developmental Research Methodology***

Developmental research is research which not only charts the developmental process, but also contributes to that development. Our sociocultural frame allows us to address learning and teaching and their development in their full complexity, seeing learning as fundamentally situated in the full social context with its range of communities and cultures. Learning is fundamental to participation in the teaching-learning settings we create and study. We seek to know as much as possible about students' meaning making in mathematics and teachers' learning about teaching within these settings.

As part of the dual nature of developmental research we recognise two research modes, the insider and the outsider. Insider research involves participants in studying and modifying their practice as they engage with it. This includes the design of tasks and approaches, observation of the practices in teaching and learning, reflection and analysis of what has taken place leading to new ways of seeing and doing which are fed back to future planning, dissemination to colleagues locally and further afield through presentation and publication. Outsider research involves at least one researcher who looks at the action as an outsider, and collects and analyses data from the various settings. Data in the ESUM project has included design documentation, student surveys, observation of practice (audio recording) and interviews with students. Analysis has been relevant to the kinds of data. After initial analyses, we have used activity theory as an analytical and organisational tool to make sense of findings, particularly those that seem in tension or contradiction. Findings from analysis at various stages have fed back into the design of tasks and approaches. We have disseminated findings through conferences presentations and publications.

## **Research findings**

### ***From observational and reflective data (comparing with previous cohorts)***

We have made comparisons with previous cohorts of students, either numerically or observationally and reflectively through experience. Monitoring of attendance at lectures and tutorials has shown an increase on previous years with a 75% average attendance as compared with less than 50% in previous years. Scores in tests and the exam have shown higher averages by at least 10% than those with previous cohorts. However, we were not able to access initial qualifications for earlier cohorts, so we have to be aware that this cohort might have been initially better qualified than those earlier.

Observation has shown evidence of student engagement in lectures in response to lecturer's questions and through encouragement to students to ask their own questions. Tutorial structure with students in small groups working on inquiry-based tasks has provided opportunity for deeper mathematical discussions which have been evident in the majority of groups. However, the computer laboratory setting has offered the temptation to engage with social networking sites (e.g. facebook) which has proved a distraction for some students. Increased student involvement in discussion of mathematics questions and tasks has made it possible for the lecturer/tutor to discern students' meaning making in ways which are not possible if there is no such engagement.

### ***From student written (project) work as data***

Project reports which were assessed (20% of overall marks) have shown a serious engagement with the project task. There has been evidence of engagement with inquiry-based questions and with exploration of functions using GeoGebra from which it is possible to discern degrees of understanding. There was evidence of students' appreciation of the contribution GeoGebra offers to promote understanding. The following quotation is typical:

“As a group we looked at many different functions using GeoGebra and found that having a visual representation of graphs in front of us gave a better understanding of the functions and how they worked. In this project the ability to be able to see the graphs that were talked about helped us to spot patterns and trends that would have been impossible to spot without the use of GeoGebra.” [Group F – project report]

### ***From interviews and focus groups data***

In contrast with data from project reports which were written during the semester, interview and focus group data was collected during the progress of the second semester after the developmental study has been completed but before the final exam. Students were more at a distance from their studies in the first semester and were clearly aware of the importance of the exam for their final marks. Also, they were not being assessed on what they said in interviews as they were on what they wrote in project reports. This could have contributed to the honesty/frankness of their responses. Thus student responses, and discrepancies with earlier data have to be seen in these contexts.

In the interviews and focus groups, students showed awareness of the contribution of different kinds of questions and use of GeoGebra to their mathematical understanding. However, they were not convinced that these approaches were most helpful for their learning overall (they showed a more instrumental view of learning than had been discerned during engagement and participation). Students would have liked teaching to be more focused on what they have to know for the exam (a more instrumental view of teaching). We discerned overall that students demonstrated a strategic attitude to their studies. The following quotations are typical:

“I found GeoGebra almost detrimental because it is akin to getting the question and then looking at the answer in the back of the book. I find I can understand the graph better if I take some values for x and some values for y, plot it, work it out then I understand it ... if you just type in some numbers and get a graph then you don’t really see where it came from”. (Focus group 1)

“Understanding maths – that was the point of Geogebra wasn’t it? Just because I understand maths better doesn’t mean I’ll do better in the exam. I have done less past paper practice.” (Focus group 2)

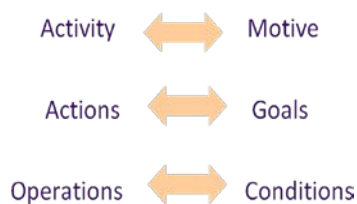
For more detail of research findings see Jaworski and Matthews (2011).

### Activity theory—helping to make sense of analysis

The variety of data and outcomes of analysis resulted in a complexity of findings both quantitative and qualitative. We needed to look at this complexity in a holistic way while making sense of its various parts and their sources. For this purpose we followed our general analysis with further analysis based in activity theory.

According to Leont’ev, “Activity is the non-additive, molar unit of life ... it is not a reaction, or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development” (1979, p. 46). Activity is the whole with which we work and in which we participate. ‘We’ are the teachers and researchers, the students, as well as other stakeholders, administrators, policy makers and so on. Included also are interlinking and interacting conditions and the issues that are generated through practical interpretation of theoretical goals and their interaction with the cultures involved. Thus, one reason for employing activity theory is to capture complexity in the wholeness described, as well as to examine specific elements and their contribution to the whole.

We used two models for this purpose. One was the expanded meditational triangle of Engeström (e.g., 1999), and details of our use of this can be found in our contribution to the ATATEMLO symposium held in Paris in 2011<sup>2</sup> (Jaworski et al., 2011). I include here details of our use of the second model which is the three levels of activity proposed by Leont’ev.



Leont’ev, 1979

Leont’ev states that all activity is motivated, although the motive might not be explicit. When it is explicit it is referred to as a motive-goal. Within the activity we can recognize certain actions, each of which is associated with an explicit goal. The activity however, is more than just a sum of the actions. The actions include certain operations which depend on certain conditions. In the tabular form below, we have represented some of the findings expressed above. It should be recognized that while we have clear insights into teaching activity and goals, our portrayal of students’ perceptions and goals in interpretative from our data.

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<sup>2</sup> Activity approaches to technology enhanced mathematics learning orchestration. 23-25 November, 2011, University of Paris, Diderot.



L evel	Teaching	Students
1	<p><b>Activity</b> mathematics teaching-learning.</p> <p><b>Motivated</b> by the desire for students to gain a deep conceptual-relational understanding of mathematics.”</p>	<p><b>Activity</b> learning within the teaching environment and with respect to many external factors</p> <p><b>Motivated</b> by the desire to get a degree in the most student-effective way possible.</p>
2	<p><b>Actions</b> design of tasks and inquiry-based questions</p> <p><b>Goals</b> student engagement, exploration and getting beyond a superficial and/or instrumental view of mathematics.</p> <p><b>Actions</b> use of GeoGebra</p> <p><b>Goals</b> providing an alternative environment for representation of functions offering ways of visualizing functions and gaining insights into function properties and relationships.</p>	<p><b>Actions</b> taking part in the module: attending lectures &amp; tutorials; using the LEARN VLE system; using the HELM books<sup>3</sup>; etc. with goals related to student epistemology.</p> <p><b>Goals</b> intention to attend lectures &amp; tutorials because this is where you are offered what you need to pass the module; clear views on what ought to be on offer and what you expect from your participation; wanting to know what to do and how to do it; wanting to do the minimum amount of work to succeed; wanting to understand; wanting to pass the year’s work.</p>
	<p><b>Operations</b> the kinds of interactions used in lectures to get students to engage and respond, the ways in which questions are used, the operation of group work in tutorials and interactions between teachers and students.</p> <p><b>Conditions</b> all the factors of the university environment that condition and constrain what is possible ...</p>	<p><b>Operations</b> degrees of participation – listening in a lecture, talking with other students about mathematics, reading a HELM book to understand some bit of mathematics, using the LEARN page to access a lecture, Powerpoint etc.</p> <p><b>Conditions</b> timetable pressure, fitting in pieces of coursework from different modules around given deadlines, balancing the academic and the social, getting up late and missing a lecture. ...</p>

Advantages of such analysis are that it draws our attention to key aspects of the activity for the groups who take part in it; it emphasizes key differences in perspectives and cultures that lead to tensions and issues; it provides opportunity to reconsider the teaching approach and utilization schemes; and it increases teaching knowledge. The areas of knowledge include practical knowledge in the design and use of inquiry-based tasks, a greater awareness of characteristics, possibilities for modification such as relating to

- use of GeoGebra in institutional settings – what is possible and how it is perceived by students;
- student degrees of engagement and insights into student epistemology;
- the importance of assessment;
- difficulty in discerning degrees of conceptual understanding .

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<sup>3</sup> HELM: Helping Engineers Learn Mathematics. A specially prepared set of books which we give to engineering students to support their mathematical study. <http://helm.lboro.ac.uk/>

## **Emerging issues**

### ***Tensions between theory-based design and its practical realisation***

*The institutional dimension:* this includes the fabric and organisation of the university, expectations and norms of practice and ways of thinking about what constitutes good ways of educating students (not necessarily research based). It includes ways of constituting the curriculum and assessment structures and the basis of judgments on good educational outcomes.

*Cultural influences:* this includes the cultures of higher education in the UK, cultures of mathematics and mathematicians, student cultures, and school cultures since these students have just arrived from education in schools. It includes particularly the culture of assessment which we have found to be a most important consideration both in institutional culture and in the perspectives of students.

*Students' strategic approach:* in today's student-aware economic culture, for many students the outcome of university studies, the degree assumes perhaps more importance than the studies themselves. Thus students take a strategic approach to study which is often interpreted as the approach which will most likely result in a high level qualification.

All of these areas could be discussed more fully in terms of our project and its outcomes, the tensions that add to overall complexity and ways in which we can navigate these tensions in our design of teaching. Two are outstanding: the first concerns assessment – students' strategic approach is highly focused on the assessment, particularly the exam. We intend to explore assessment possibilities which are more closely linked to achieving our aims related to conceptual understanding. This may mean challenging existing academic cultures and we aim to do this through innovation and research, and the making explicit of the theories which underpin existing and potential practices. The second concerns the nature of mathematics understanding, particularly conceptual understanding. This has been much harder to recognise than perhaps we expected at the design stage.

### ***Mathematical competence and understanding***

We ask the question, what actually do we mean by 'conceptual understanding' in practice? For a theoretical conceptualisation we draw extensively on the literature, for example, Skemp (1976), Heibert et al (1986) and Artigue, Batanero & Kent (2007). However, while it is relatively easy to say what we mean by conceptual understanding, we have found that it is not so easy to discern it in practice. Institutional constraints militate against a lecturer's deeper knowledge of students: in the time allocation and cohort sizes it is impossible to develop the same knowledge of students that a teacher would have in a school classroom. This is true for all teaching of students in higher education. However, our students are engineering students and the European Society for Engineering Education, SEFI, is also concerned about these issues, asking, What do we mean by mathematical competence? (SEFI, 2011) I quote from their document:

“Mathematical competence is the ability to recognize, use and apply mathematical concepts in relevant contexts and situations which certainly is the predominant goal of the mathematical education for engineers.” (p. 3)

“It is clear that such competencies cannot be obtained by just listening to lectures, so adequate forms of active involvement of students need to be installed.” (p.3)

Competencies include “thinking and reasoning mathematically, posing and solving mathematical problems, modelling, representing, communicating” (p. 9-10)

Such statements might have been written as the basis for the inquiry-based approach we have been taking. Following their introduction, the SEFI document goes further to detail the competencies in the areas stated and we are keen to explore how these competencies relate to

our activity and its outcomes. A question we will ask is, how do competencies relate to understandings or mathematical meanings and how can we recognise and assess them? This is an area for further research.

### **For the future**

As a result of the ESUM project we have new knowledge and further questions. This knowledge is feeding back into new planning and teaching. We are making changes to practical considerations within our theoretical perspectives. We are conscious of institutional and cultural constraints and working on how to address them as a process of critical alignment in teaching and in our documentational genesis. We are planning further research into the nature of assessment and its relation to student activity and epistemology and have already submitted bids for funding for this research. We plan further research into the nature of understanding, perhaps related to “mathematical competence”; this is still at a very early stage.

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