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Voltage Fluctuations Caused by Groups of Wind Turbines

by

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Doctoral Thesis
Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy of Loughborough University
21st February 2000

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Abstract

Wind turbines connected to the distribution network can be the cause of voltage fluctuations and resulting fluctuations in the light intensity emitted by light bulbs. These fluctuations may cause people disturbance.

A model has been developed to obtain a flicker prediction which is useful in the design process of a wind farm. The model is based exclusively in the frequency domain (FD). This new approach allows very fast and efficient evaluation. The impact of individual parameters is often easier to recognise and evaluate in a FD-representation.

The following factors leading to flicker disturbances from a wind farm have been considered in detail:

The wind spectrum: Effects of terrain and wind farm wakes on the wind turbulence spectrum have been considered and existing models have been expanded.

The wind coherence: A new coherence model for large separation distances has been derived for use within a wind farm. Effects of the terrain on the coherence of power produced by turbines within a wind farm have been considered.

The wind turbine: A simplified dynamic wind turbine model allows the prediction of turbine specific contributions to flicker for a variety of wind turbines using a minimal set of parameters.

The flickermeter: Flicker measurements are found to sometimes neglect the impact of low frequency voltage variations. These are found to be very important for the correct flicker prediction. A new FD-flickermeter has been developed.

The model has been validated against experimental data and a sensitivity analysis shows which parameters are most likely to influence the voltage flicker and which are best altered to minimise the flicker.

Keywords: power quality, flicker, turbulence spectrum, coherence, complex terrain, wind farm, wind turbine, flickermeter
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Chapter 1

Introduction

It was recognised from an early stage of the wind energy industry that an increased wind power generation requires us to reconsider the concepts for the existing electricity production and distribution [Davitian78, Larsson78]. Many of the best wind turbine sites in Europe are now located in remote areas, often in complex terrain, such as hill tops or mountain ridges. In these areas, which are traditionally supplied by weak grids, the power quality resulting from the interconnection of wind farms becomes an important factor for the design of wind farms.

1.1 Power Quality

Three areas of potential problems have been identified: power fluctuations, harmonics and voltage fluctuations [Haubricht92, Gardner94, Craig95a].

Power fluctuations The fluctuations of wind speed cause variations in wind power output which have to be compensated for by other generation units. These fluctuations range from 10 minutes to some hours and have been studied by [Cliff78, Sørensen78] and more recently by [Beyer89, McNerney92, Steinberger-Willms93].

Harmonics Harmonics are caused by transient events like the (dis)connection of a turbine to the grid or the switching processes of a power electronic device. Adequate filtering technique can reduce the harmonic components, which
would otherwise cause additional heating to electrical components (see for example [Ekanayake97]). The publication [Bossanyi92] discusses in more detail harmonics caused by wind turbines.

Voltage fluctuations Wind turbines are frequently connected at the far end of the existing grid. Here the grid is quite often 'weak' in the sense that the voltage level is sensitive to load variations. Disturbances of the voltage level at a frequency of 0.001 – 25Hz result in changes of the light intensity of light bulbs. A review of studies on the disturbance this 'flicker' causes to people can be found in [Walker79].

This thesis concentrates on flicker caused by voltage fluctuations from grid connected wind turbines during normal operation.

1.2 Voltage fluctuations

Voltage fluctuations cause fluctuations in the light emitted from incandescent light bulbs. Such fluctuations of the voltage level, which would otherwise be considered small, cause disturbance to people. To assess this flicker, incandescent light bulbs of 230V/60W are used as reference light source, less disturbance is experienced for 120V/60W (USA) light bulbs or fluorescent lights (see [Bhargava93] for fluorescent flicker), lamp dimmers on the other hand can increase the flicker according to [Halpin99].

Flicker caused by wind turbines has been studied in field experiments. The background flicker level seems to be not significantly increased by wind turbines in some cases [Davidson96, Craig95c] but there are indications that the flicker level is increased near to and above the allowed limits in other cases [Sørensen96a].

It is important for the planning process of a wind farm development to know the flicker caused by the development before installing the turbines. For an assessment of the flicker in the development stage, only data which are in the public domain, or made available to the wind farm developer by the turbine manufacturer, can be used. The available data generally include wind speed and wind direction distribution, turbulence level, topological data, wind farm layout, type of turbines,
power curve and voltage level and impedance of the network. Often very little reliable data is available on the turbine itself.

If a flicker prediction carried out during the planning process predicts flicker in excess of the allowed limits, counter-measures can be included in the design of a wind farm, see for example [ElSharkawi95] or [Carstens97].

1.3 Flicker prediction models

Several methods have been proposed to predict the flicker in wind farms. A very simple model of the power output fluctuations in a wind farm caused by wind speed fluctuations has been published in [Welfonder97]. No turbine data is considered but the flicker is directly linked to the wind turbulence spectrum.

A more robust method to predict the flicker has been proposed in [VDEW94] and [Klosse97b, Klosse97a]. This method is based on two turbine-specific constants which are derived from power quality measurements on the turbines. However, the proposed flicker coefficient is not a constant but a function of wind speed and turbulence conditions during the measurement period [Sørensen95a, Amaris99].

A number of different publications present models of wind turbines in time domain models of varying sophistication which can also be used to calculate the flicker effect, for example [Spruce93], [Polhaus93], [Infield94], [Barnowski96], [Bossanyi98], [Veganzones99] or [Nicolás99]. Many of these models have evolved from a relatively sophisticated model for one component (grid, generator, turbine), might ignore some important factors totally and lack detail in other areas.

1.4 Advanced prediction model

In the following it is assumed that the flicker of lightbulbs is connected by a causal chain of elements with the fluctuations of the wind resource:

\[ P_{st}^2 = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) \, dn \]  

(1.1)

with the following functions and operators in the frequency domain:
Some of these operations are not linear and approximations have been introduced in some places. However, all elements of the chain can be described in the frequency domain producing a powerful tool for the flicker assessment. In comparison to the above mentioned models the approach used in this thesis has the following advantages:

**Speed of simulation** The model is the first wind turbine flicker calculation model set in the frequency domain (FD). This enables significantly faster processing than an equivalent time domain model.

**Intuitive approach** The FD domain approach allows a more intuitive appreciation of the physical concepts involved.

**Degree of sophistication** The degree of sophistication is adjusted to the problem. All input parameters needed are readily available to a wind farm developer.

**Width of approach** The developed model spans the whole causal chain leading to flicker from the influence of meteorological parameters to the network and the flickermeter calculation.

**Validation** The suggested model is tested and validated against experimental data.

**Originality** Some of the components like the flickermeter algorithm and the coherence model for flat and complex terrain are new and have been developed specifically for this application.

This thesis opens the possibility of a fast flicker prediction in the design stage of a wind farm using a minimum set of data which are in the public domain or can easily be obtained by a wind farm developer.
1.5 Structure of this thesis

The chain of factors contributing to flicker covers such contrasting areas as meteorology, wind turbine characteristics, wind farm losses and electrical integration. The process can be understood as fluctuations passed down a chain of different components, each amplifying certain frequencies, damping others and possibly adding new characteristic frequencies. No part of this chain can be seen as solely responsible for voltage flicker but changes in some parameters are more important than in others. The analysis presented here identifies the sensitivity of the voltage flicker to changes in the various input parameters.

To accommodate and integrate the wide range of different topics involved the thesis is split into three parts and a number of chapters each discussing individual elements of the chain. Each of these chapters can be read and understood separately but should at the same time be seen as integral part of the thesis as a whole.

Part I Wind field modelling: A brief introduction to wind energy meteorology (chap. 2) defines some of the basic terms and models used. The turbulence spectrum (chap. 3) and the spatial coherence (chap. 4 and chap. 5) describe the stochastic structure of the wind field which is the prime cause for any fluctuations.

Part II Wind - turbine - grid interaction: The wind turbine model (chap. 6) is presented and the measurement of flicker is discussed (chap. 7). A new flicker prediction methodology (chap. 8) is presented. The chapter (chap. 9) describes the impact of a wind farm on the predicted flicker.

Part III Model results and validation: The model predictions are compared with measurements (chap. 10). A summary of results and models is given in the last chapter (chap. 11). Conclusions are drawn and recommendations for further research and development given.

The thesis is completed by this introduction, appendices, bibliography and the index register at the end of the thesis.
Part I

Wind field modelling
Chapter 2

Wind energy meteorology

To understand the nature of wind power generation it will be necessary to briefly introduce some definitions related to the wind resource. Readers who have already an understanding of the basic concepts may omit this chapter.

The work [Hoven57] showed the existence of a 'spectral gap' in the wind fluctuations, an interval having less fluctuations, spanning from minutes to hours. With the existence of such a spectral gap a Reynolds separation makes sense, dividing the description of the wind field in a slowly varying 'mean wind speed' and fluctuations 'turbulence' around this mean value.

Figure 2.1: Spectral Gap. The spectrum of wind fluctuations at Brookhaven, USA shows a spectral gap, from [Hoven57].
2.1 Equations of motion

The Navier Stokes equations describe the turbulent motion of an ideal gas, under the assumption that the local changes of pressure, temperature and density are small compared to the respective mean values [Boussinesq77]. When mean and fluctuating components are separated, the Reynolds equations are obtained. These two sets of equations build the foundation for all theoretical models of the wind flow.

2.1.1 Navier Stokes equations

The Navier Stokes equations follow from momentum, continuity and energy balance and the assumptions made above. For co-ordinates $q_i$ and velocities $W_i$ in the directions of these co-ordinates the equations are (Einstein’s convention on summation applies) [Jensen82]:

$$\frac{\partial W_i}{\partial t} + W_j \frac{\partial W_i}{\partial q_j} = -\frac{1}{\rho_0} \frac{\partial P'}{\partial q_i} + X_i + \mu \frac{\partial^2 W_i}{\partial q_j \partial q_j}$$  \hspace{1cm} (2.1)

$$\frac{\partial W_i}{\partial q_i} = 0$$  \hspace{1cm} (2.2)

$$\frac{\partial T'}{\partial t} + W_i \frac{\partial T'}{\partial q_i} = \nu \frac{\partial^2 T'}{\partial q_j \partial q_j}$$  \hspace{1cm} (2.3)

with:

- $T'$: fluctuations in temperature
- $P'$: fluctuations in pressure
- $\mu$: viscosity
- $\nu$: thermal diffusivity
- $X_i$: gravitation and Coriolis forces
2.1.2 Reynolds Equations

Often the Gravitation and Coriolis terms and the thermal energy transport equation are neglected. The Reynolds separation between fluctuating $W_i'$ and mean components $\overline{W_i}$ of the wind velocity $W_i$ is:

$$W_i := \overline{W_i} + W_i'$$

Using this in eq. 2.1 and eq. 2.2 yields:

$$\frac{\overline{W_i} \partial \overline{W_i}}{\partial q_i} + \overline{W_j} \frac{\partial \overline{W_i}}{\partial q_j} = \epsilon \frac{\partial^2 \overline{W_j}}{\partial q_j \partial q_i}$$  \hspace{1cm} (2.4)

$$\frac{\partial \overline{W_i}}{\partial q_i} = 0$$  \hspace{1cm} (2.5)

with:

$\epsilon$ : eddy viscosity (turbulence closure)

In the remainder of the text, instead of the generalized description above, a cartesian coordinate system is used with $u,v,w$ being the mean wind speeds in $x,y,z$ direction. $x$ being along the wind direction, $z$ the vertical component and $y$ perpendicular to $x$ and $z$.

2.2 Mean wind speed and profile

The mean wind speed at a location varies with time. These variations of the mean wind speed can be classified according to the typical time scale involved. The wind speed increases generally with height and can be described by a statistical distribution function.

2.2.1 Variations in mean wind speed

Long term variations in the wind climate are very difficult to predict and depend on the global climatic change scenario. The difficulties involved with the evalu-
ation and extrapolation of long records of mean wind speed have been discussed for example by [Palutikof87, Palutikof92] and [Tetzlaff92].

Seasonal variations of the wind resource are relatively well documented and part of standard models. Statistical data about the seasonal variation and the characteristic statistical fingerprints for different stations in Europe is available eg. from the European Wind Atlas [Troen89].

Utilities generally operate on basis of day to day and hourly demand forecasts. In addition to the general demand forecasts a wind power production forecast is used by some utilities. Diurnal patterns are mostly caused by local thermal activity resulting in sea-land or mountain valley winds. Operational problems and strategies have been discussed eg. in [Davitian78, Schlueter84, Knudsen90]. The prediction of hourly variations is currently being investigated by a number of research groups (referenced in [Landberg97]).

2.2.2 Probability distribution

Based on the existance of a spectral gap (see page 7), mean wind speed and fluctuations around this mean wind speed can be separated. The averaging period used for this separation varies between 10 minutes and 1 hour. Throughout this thesis averages over periods of 10 minutes are used and are referred to as mean wind speed or just wind speed.

The probability of occurance of a wind speed over a year can be approximated by a Weibull distribution. The European Wind Atlas uses a sectorwise wind direction frequency $P_i$ and a sectorwise Weibull wind speed distributions $W_i(u)$ to describe the wind regime at a given site:

$$W_i(u) = \frac{k_{wi}}{A_i} \left( \frac{u}{A_i} \right)^{k_{wi}-1} e^{-\frac{u}{A_i}}$$

with:

$A_i$: scale factor
$u$: mean wind speed
$k_{wi}$: shape factor

The sum $\sum_i W_i(u)P_i$ of sectorwise frequency distributions is not necessarily a Weibull distribution. Although the Weibull function is an adequate statistical
description for a large number of sites in northern Europe it is often appropriate to use a measured wind speed distribution instead.

### 2.2.3 Logarithmic Wind Profile

The wind profile describes the variation of wind speed with height. Historically the wind profile has often been described by a power law (for example [Harris68]). The coefficients used in the power law apply only to the specific local conditions and are thus general of limited use. This type of empirical relationship has been replaced in wind engineering by the more general applicable logarithmic profile.

\[
    u(z) = \frac{u_*}{\kappa} \left[ \ln \left( \frac{z}{z_0} \right) - \Psi_s \right]
\]

(2.7)

with:

- \(u(z)\) : wind speed at height \(z\)
- \(z_0\) : roughness length
- \(u_*\) : friction velocity
- \(\kappa\) : von Karman constant (0.4)
- \(\Psi_s\) : stability correction
- \(z\) : height above ground

The wind speed profile (eq. 2.7) is given as a function of the atmospheric stability, see [Stull88] for details. Changes in atmospheric stability are especially important to describe the wind profile in low wind speeds and high thermal activity. For high wind speeds, that are of importance in wind energy engineering, neutral stability or near neutral stability (\(\Psi_s = 0\)) can be assumed. Expressions for the stability correction (\(\Psi_s\)) of the profile can be found in [Fordham85, Frost94, Aasen95].

To extrapolate the wind speed \(u(z)\) from a reference height \(z_r\) and reference wind speed \(u_r\) the eq. 2.7 can be used as follows:

\[
    \frac{u(z)}{u_r} = \frac{\log \frac{z}{z_0}}{\log \frac{z_r}{z_0}}
\]

(2.8)

The selection of a roughness length \(z_0\) for a given area is usually done with the help published reference tables. Differences in the definitions have been discussed by [Wieringa92] and are reflected for example in the different descriptions published eg. by [Troen89] and [ESDU93c].
The wind profile at a given point is a function of the upwind terrain roughness length. A sectorwise logarithmic profile in non homogeneous terrain does not normally lead to a logarithmic profile over all wind directions.

2.3 Turbulence and Variance

The fluctuations of instantaneous wind speeds around a mean wind speed is called turbulence. The fluctuations occur at characteristic length scales which are equivalent to the size of circulations (eddies) in the wind field. Taylor's Hypothesis describes eddies which are 'frozen' while they advect past a sensor. In this approximation spatial and temporal extension (or frequency) of the turbulence are two equivalent descriptions. Turbulence intensity is defined as the standard deviation $\sigma$ of a time series of instantaneous wind speed divided by its mean $u$:

$$I := \frac{\sigma}{u}$$ (2.9)

Turbulence at a given wind speed is represented by square of the standard deviation, the variance $\sigma^2$. The distribution of the variance over a range of frequencies is described in a turbulence spectrum. The level of turbulence is influenced by multiple factors such as the roughness of the terrain, the topography, obstacles or the wake of other wind turbines.

2.3.1 Turbulence profile

An empirical relationship between the friction velocity $u_*$ in eq. 2.7 and the standard deviation of the wind speed has been found to be typical for FSU (flat smooth uniform) terrain [Petersen98]:

$$\sigma = u_* \cdot 2.5$$ (2.10)

The standard deviation is, in this approximation, independent of the height above ground. The turbulence intensity is then obtained from eq. 2.7:

$$I(z) = \frac{\sigma}{u(z)} = \frac{1}{\ln \left( \frac{z}{z_0} \right)}$$ (2.11)
This is in reasonable agreement with experimental results e.g. by [Bergström94]. Alternatively to eq. 2.7, a modified wind profile could be used to derive eq. 2.11. An alternative empirical correction to the turbulence profile is proposed by [Frost94]. For a discussion of the dependence of $\sigma$ on the averaging period and stability see [Skupniewicz89]. For the purpose of assessing the flicker from wind turbines it is suggested that the turbulence intensity is either determined by the use empirical data measured on the site in question, or alternatively by eq. 2.11 from the roughness length.

### 2.3.2 Turbulence Spectrum

Transferring a time series by means of a Fourier Transformation ([Bendat71, Press94]) into the frequency domain is a powerful tool for the analysis of its stochastic properties. The representation in the frequency domain is referred to as power spectral density (PSD), autospectrum or simply spectrum of a time series. The integral over the spectrum yields the variance of the time series. The variance obtained for the spectrum of a time series of wind speed measurements is:

$$\sigma^2 = \int_0^\infty S(n)dn = \int_0^\infty n \cdot S(n)d\ln(n) \quad (2.12)$$

with:

- $S$ : wind turbulence spectrum
- $n$ : frequency in Hz
- $\sigma^2$ : variance of time series

Normalising equation eq. 2.12 by the variance gives a non-dimensional spectral representation which is used throughout this thesis (Appendix B). In a graphical representation of a spectrum the area underneath the graph represents the variance of the time series. The shape of the spectrum provides information about the part of this variance associated with a certain frequency. This is if the spectrum is plotted as $nS(n)$ over a logarithmic frequency scale or as $S(n)$ over a linear scale.
2.3.3 Turbulent length scale

The turbulence of a flow is not a property of the fluid but of the flow. Turbulence closures shernes and empirical approaches that allow a mathematical description of the turbulent flow often involve the definition of a length scale. These length scales need to be adjusted to best describe the flow under investigation.

Length scales commonly used in wind engineering to describe the turbulent air flow are the half width of a hill, the height of a point of interest or the width of an object. The standard [IEC97] defines a turbulent length scale parameter $A_1$ as function of the hub height of a wind turbine $z_{hub}$:

\[
A_1 = \begin{cases} 
0.7z_{hub} & \text{for } z_{hub} < 30m \\
21m & \text{for } z_{hub} \geq 30m
\end{cases}
\] (2.13)

2.3.4 Universal shape of the spectrum

Associated with length scales smaller than the measurement height is the so called inertial subrange or Kolmogorov range. Here, in a dynamic equilibrium, eddies decay from large eddies to ever smaller eddies. Only then does a dissipation take place converting the turbulent kinetic energy into heat. Eddies in the 'Kolmogorov range' are isotropic and Taylor's hypothesis is valid. It can be shown that the high frequency end of a wind turbulence spectrum $S$ should then follow a power law of $n^{-\frac{5}{3}}$ (cf. [Jensen82]).

The location of the maximum of the spectrum, in the representation introduced in sec. 2.3.2, is determined either by a fit of the theoretical spectrum to the data in the inertial subrange, e.g. [Morfiadakis96], by fitting the maximum point of the measured spectrum to the maximum of the theoretical model e.g. [Founda93] and [Founda97], or by means of empirical approximation, [Frost94], a review of these methods can be found in [Olesen84].
2.4 Special turbulence spectra

The turbulence spectra for various length scales and velocities collapses to a universal shape. Although the precise form of the spectra is not of decisive nature in the given context, some forms of the turbulence spectrum have been used widely throughout the literature. These are the expressions introduced by [von Kármán30] and [Kaimal72].

2.4.1 Von Kármán spectrum

The spectrum recommended by ESDU [ESDU90] is the von Kármán spectrum [von Kármán30, Kármán48]:

\[
\frac{n \cdot S(n)}{\sigma^2} = \frac{4f}{(1 + 70.8f^2)^{5/2}}
\] (2.14)

The von Kármán spectrum is here represented as function of the normalised frequency \( f = \frac{nL_k}{u} \). The length scale \( L_k \) is a free parameter. It can be used to fit the model to a given set of data [Teunissen80]. Empirical formulae for the length scale have been proposed [ESDU72, Freris90] and [ESDU90]. The IEC standard [IEC97] suggests to use with the von Karman model \( L_k = 3.5A_1 \) where \( A_1 \) is the turbulent length scale defined in eq. 2.13.

In the meteorological literature, spectra are often normalised by the friction velocity \( u_*^2 \) (compare sec. 2.2.3) instead of \( \sigma^2 \). It is however difficult to obtain the friction velocity from experiments. Improved agreement with experimental data has been reported by [Founda93, Founda97] when scaling the 'von Kármán' formulation with the variance. Both forms can easily converted into each other if the relation eq. 2.10 is applicable. Here \( \sigma^2 \) is preferred as normalisation parameter because it can be obtained directly from a single point measurement.
2.4.2 Kaimal spectrum

Due to recommendation by [Frost78] and [Frost94] the form proposed by Kaimal in [Kaimal72] and [Kaimal73] has been used in a wide range of publications. The Kaimal spectrum eq. 2.15 (fig. 2.2) is widely accepted to be representative of sites situated in flat terrain (stable to neutral atmospheric conditions):

\[
\frac{n \cdot S(n)}{\sigma^2} = \frac{0.164 \frac{f}{f_0}}{1 + 0.164 \left(\frac{f}{f_0}\right)^{\frac{3}{2}}}
\]  

(2.15)

This Kaimal spectrum is not identical to the 'Kaimal' spectrum suggested in the standard [IEC97]. Differences of the shape proposed by Kaimal or other spectral representations (eg. eq. 2.14) are not very significant here (cf. [Fordham85, Olesen84, Tieleman92]) so either could be used. Kaimal chooses a normalised frequency as von Kármán, but uses the measurement height \( z \) to normalise the frequency:

![Figure 2.2: Kaimal spectrum for different wind speeds. Spectra with different wind speeds as the ones presented here collapse to a single graph when the frequency axis is normalised by the wind speed.](image)

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The reference frequency $f_0$ determines the location of the maximum in the Kaimal spectrum. [Frost94] suggests an estimation for the parameter $f_0$ of the Kaimal spectrum which frees the user from guessing or fitting length scales. The Kaimal spectra shown in this thesis are calculated using this relation:

$$f_0 = 0.0144 \left( \frac{z}{30} \right)^{0.78}$$

(2.17)

Several publications use a Kaimal spectrum which is normalised with the shear stress velocity $u^*$ ([Kaimal72]). Authors using this representation have found the scaling of the curve not appropriate to their models and several correction factors are reported in the literature, e.g. [Teunissen80] and [Knudsen90]. In [Kaimal72] it is acknowledged that the proposed scaling by the shear stress velocity $u^*$ is not very successful and scaling the longitudinal spectra in stable to neutral conditions with the variance of a time series is suggested [Kaimal73]. The results suggest that the vertical standard deviation might yield even better agreement with the data, this parameter however is not available when standard instrumentation (i.e. a cup anemometer) is used.

2.4.3 Modified Kaimal spectrum

As the spectral representation eq. 2.15 is normalised by the variance its integral should be close to unity (see eq. 2.12). By numerical integration it can be shown that the integral is $c_k = 0.9617$. In the following all Kaimal spectra are normalised by this factor so that the modified form of the Kaimal spectrum is:

$$\frac{n \cdot S(n)}{\sigma^2} = \frac{1}{c_k} \frac{0.164 f}{1 + 0.164 \left( \frac{f}{f_0} \right)^{\frac{5}{3}}}$$

(2.18)

$c_k : 0.9617$ Variance correction factor
2.5 Summary - wind energy meteorology

In this chapter the basic concepts used in wind engineering have been introduced, ie. mean wind speed, logarithmic wind profile, roughness length, turbulence and turbulence spectrum. These definitions build the basis for understanding the following chapters.

Different theoretical and empirical models for the turbulence spectrum have been compared. A modified Kaimal turbulence spectrum (sec. 2.4.3) has been derived for use throughout this thesis. This allows, together with theoretical or measured turbulence levels, prediction of the turbulence spectrum for a given site.

From the chain of elements sec. 1.4:

\[ P_{st}^2 = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) \ dn \]

this chapter looked at the turbulence spectrum \( S \) which is the main source of all (stochastic) fluctuations and of voltage flicker in general.

While this chapter has given an introduction to the stochastic properties of the wind field in FSU terrain, the next chapter will discuss the influence of terrain on the wind field.
Chapter 3

Turbulence in complex terrain

The wind turbulence spectrum is described in the frequency domain by shape, position and amplitude. A suitable spectrum for a flat terrain situation has been proposed in sec. 2.4.2. The mean wind speed, turbulence spectrum and the turbulence intensity are modified by topography.

The wind field over a hill is quite complex, fig. 3.1 gives an impression for neutral stability. For the purpose of this thesis a simple model is needed to approximate the effects especially in neutral stability and near a hilltop, where wind turbines are most likely to be placed.

Figure 3.1: Wind flow over a hill. Sketch of the wind flow over a hill in neutral stability, from [Stull88]
3.1 Mean wind speed over topography

In recent years, experimental programs have been undertaken to improve our understanding of the atmospheric processes in complex terrain. In particular, the ASCOT program [Blumen90] and the measurements in Scotland at Askervein Hill [Taylor87b] contributed to the testing of theoretical models.

3.1.1 Speed up factor

From the continuity equation eq. 2.5, it follows that when air passes over a hill the wind speed increases, and the logarithmic profile is no longer valid. The acceleration of the air over a hilltop can be described in terms of the 'fractional speed up factor' $\Delta S$, or the 'speed up' $s$:

$$\Delta S = \frac{u(x, z) - u_0(z)}{u_0(z)} = s - 1$$

with:

- $u_0$: undisturbed wind speed

3.1.2 Empirical models

Several empirical models for the speed up over a hill have been proposed [Jackson75], [Lemelin88], [ESDU93b] and [Hassan90]. These models agree that the speed up should be assumed to be proportional to the ratio of the height $h$ of the hill to its dimension (half width) in the flow direction $L_h$ and that the speed up is a function of the relative height $\frac{z}{L_h}$ above ground. The models are then adjusted with a number of parameters $p_i$ to fit either numerical results or wind tunnel experiments, for example with two parameters:

$$\Delta S = p_1 \frac{h}{L_h} \frac{1}{\left(1 + p_2 \frac{z}{L_h}\right)^2}$$

In this work the empirical relations presented by [Lemelin88] are used as an example of an empirical model for the speed up. This 'I,SD' (Lemelin, Surry, Davenport) model gives the following relation for a 3 dimensional axisymmetric hill and a 2 dimensional ridge, with a height to half-width ratio $\frac{h}{L_h} \leq 0.4$. 

20
\[
\Delta S = 2.3 \frac{h}{L_h} \frac{p_1}{(p_3 + 0.4) (1 + 3(p_x/p_1 L_h)^{p_2}) \left(1 + p_3 \frac{x}{L_h}\right)^2}
\]

with:

- \(p_x\): offset from the hilltop
- \(p_s\): aspect ratio of hill
- \(p_1, p_2, p_3\): parameters derived from the hill geometry, here \(p_1 = p_2 = p_3 = 2\)

The LSD model contains a correction for the exact position in relation to an escarpment, taking into account the gradual adaptation of the turbulence and wind profile to local conditions. The LSD model is calibrated against the numerical model MS3DJH/3R [Taylor83]. A model, similar to the LSD model is suggested in [ESDU93b, ESDU92].

Empirical models like the LSD approach allow very fast computation to obtain an accurate estimate of speed up effects. Instead of using approximations which are calibrated against numerical models, the increasing availability of computational power allows today the direct use of the simpler versions of these numerical models.

### 3.1.3 Numerical models

All numerical models are more or less sophisticated approximations to the Navier Stokes equations. The basic model equations are then adjusted to the wind flow of interest using empirical relationships for example to account for the terrain roughness or thermal activities.

**Mass consistent models**

Models for the air flow over a topology generally assume the validity of the continuity equation eq. 2.5. One example for a basic model which is used in wind applications is NOABL [Traci80, Tombrou93]. This type of model does need empirical corrections to account for turbulent or thermal effects and is therefore generally limited in its application to simple terrain.
Models with turbulence closure

Some models go a step further and include the momentum equations eq. 2.4 with a turbulence closure (for closure schemes see [Launder79]). Models of this type are WAsP [Troen89], FLOWSTAR [Inglis92, Carruthers90], MSFD [Beljaars87]. These models do need empirical corrections to account for thermal effects, flow separations, wakes or roughness changes. For a discussion see [Jackson75] and [Jensen88] an overview is given by [Carruthers90] and [Barnard90].

Following its widespread use in wind engineering, the WAsP model is chosen in this thesis. WAsP is a model which uses a potential flow solution for an outer layer and a logarithmic profile in an inner layer. This structure originates from [Jackson75] and the model is in some aspects (i.e. the spectral terrain representation) similar to the MSF3DJH family of models [Beljaars87]. A major feature in WAsP is its computational efficiency based on the use of a polar coordinate system centred around the point of interest and a spectral representation of the terrain.

Predictions of the mean flow in complex terrain have been found to be of uncertain accuracy. The expected model error is in the range of up to 30% ([ESDU93b]), which is quite problematic for commercial wind power applications. No predictions of the turbulence level or structure can be obtained from this type of model.

Models calculating the energy budget

Detailed models use a more complete set of equations, including for example the energy budget eq. 2.3. Applications of this group of models have been published by [Crespo87, Alm93] (PHOENIX). Some complex mesoscale models (see [Wippermann71] for the definition of scales, [Schluenzen84] and [Pielke84] for model equations) have also been applied in complex terrain e.g. [Mengelkamp93]. These models can in theory analyse the turbulent wind field including the dissipation of turbulent kinetic energy and level of turbulence but due to a lack on computational power the possible spatial resolution is very limited. Furthermore the calibration of these models to the precision needed for full scale wind engineering applications is a problem yet to be solved.
3.2 Level of turbulence in complex terrain

The level of turbulence can be increased or decreased by the influence of complex terrain features. Three cases are discussed in the following: a single hill in FSU terrain, a complex terrain area upwind of the point of interest in FSU terrain, a point above 'homogeneous' complex terrain.

3.2.1 Single hill in FSU terrain

For a single hill located in otherwise FSU-terrain [Jackson75] proposed a two layer model. An inner layer with a dynamic equilibrium between generation and dissipation of turbulent eddies and an inviscid outer layer.

An approximation for the upper limit of the inner layer \( l \) is given by [Hassan90] as 1/20 of the half width of a (Gaussian) hill (similar in [Zeman87]). An empirical formula for the inner layer height has also been proposed in [Jensen83a, Jensen84] and used in [Taylor87b, Taylor87a, Mann99]:

\[
\frac{l}{L_h} \ln^2 \left( \frac{l}{z_0} \right) = 2 \kappa^2
\]

(3.3)

For wind energy applications typical heights are above 50 m. An inner layer height at the lower end of the rotor plane corresponds to a hill half-width \( L_h \) of 1000 m. Hence it can be assumed that modern wind turbines located on hills are in most circumstances operating in the outer layer.

The effects in the outer layer can be described by 'rapid distortion' theory [Batchelor54, Townsend54]. Rapid distortion assumes that the turbulent eddies are distorted sufficiently fast that the turbulent energy remains constant and the eddies are stretched in direction of the acceleration. The longitudinal variance of the wind remains nearly constant over the hill while the mean wind speed is increased, therefore the turbulence intensity over the hill decreases. Some turbulent energy is shifted from the longitudinal fluctuations to the lateral and vertical components. This component is generally small and is neglected in the following.
3.2.2 Upwind complex terrain approximation

Complex terrain upwind of a reference site in FSU terrain influences the wind profile at this site. Experimental evidence for this effect is reported by [Panofsky82]. It has been suggested to derive a regional roughness length \( z_{tu} \) equivalent to the roughness length in eq. 2.11 through measurements of the turbulence intensity at sufficient height above ground [Tieleman92, Barthelmie92]:

\[
 z_{tu} = e^{\ln(r) - \frac{s}{c}} \tag{3.4}
\]

[Tieleman92] refers to a 'regional' roughness length for complex terrain in Prices Fork, Virginia of 0.38 instead of the expected 0.025 (for equivalent FSU terrain). [ESDU93b] proposes for hilly area a modified roughness length classification of 3-5 m. If a regional roughness length could be derived directly from topographical data, it would be possible to obtain the turbulence profile from eq. 3.4.

3.2.3 Approximation for homogenous complex terrain

If we assume a complex area which is statistically homogenous in its topography we could derive a regional roughness length for this area. This regional roughness length then affects the wind profile at a height sufficiently high above any local influence of the terrain [Mason85, Hignett94]. This model gives reasonable results for heights above 150 m. It is not of immediate relevance for wind energy applications.

3.3 Turbulence spectrum in complex terrain

One way to modify the spectral shape is to introduce additional parameters into the equation in order to fit the Kaimal shape form or alternatively the von Karman spectrum to the data [Teunissen80, Antoniou91]. This empirical approach is not applicable universally or for the purpose of this work as we need to predict spectra rather than extract it from existing data. The data recorded at a site is generally not suitable for extraction of a wind spectrum characteristic. How to obtain a
prediction for the turbulence spectrum without complex local measurements is discussed in the following.

3.3.1 Spectrum in the inner layer

In the inner layer above complex terrain topology and for frequencies of a length-scale corresponding with the inner layer height it is found that the spectrum adjusts immediately to local features and follows the usual models for FSU terrain. As noted above, the inner layer is not of crucial importance to wind energy applications.

3.3.2 Spectrum in the outer layer

The Atmospheric Boundary Layer over uniform terrain is in a dynamic equilibrium regarding turbulent momentum transfer towards the surface. If a distortion to this equilibrium occurs sufficiently rapidly, the longitudinal fluctuations can be assumed to be constant [ESDU93a]. This 'rapid distortion' in the outer layer as defined by [Batchelor54] (see also [Jackson75]) implies a constant spectral shape for the variance. Thus the spectrum, if represented as a function of the normalised frequency is merely shifted towards lower frequencies. Such a shift towards low frequencies is e.g. observed by [Tieleman92]. The approach of constant fluctuations and varying mean wind speed have successfully been used [Zeman87], [Morfiaadakis96], [Founda93], and [Founda97].

3.3.3 Modification to the spectrum at low frequencies

In addition to the frequency shift in the outer layer it has been suggested that for very low frequencies the velocity fluctuations increase with increased local wind speed [Frank96]. No clear evidence for such a modification has been found for the investigated data or in the literature.
3.4 Turbulence spectrum at Susetter Hill

The data used here to validate the theoretical approach are from a large longitudinal hill, Susetter Hill, in Scotland. Spectra predicted using WAsP and the empirical LSD method are compared with experimental results from Susetter. Susetter Hill is situated in mountainous terrain, the mountains mainly aligned from north to south. Susetter Hill itself is a longitudinal hill with a north-south orientation (fig. 3.2).

3.4.1 Experimental set-up

Three measurement masts were placed in a line from north to south as indicated in the figure as part of a study undertaken by Rutherford Appleton Laboratory. The masts were labelled as mast A (441610/1165560), M (441604/1164800) and B (441610/1164710) with separation distances of 850 m, 760 m, and 90 m. The numbers in brackets refer to the British National Grid (UTM-co-ordinates). Masts A and B were of 15 m height and M had a height of 45 m. Wind speed and wind direction records were recorded simultaneously at the three masts during

![Figure 3.2: Sketch of measurement set-up at Susetter Hill, Shetlands, UK. Simultaneous measurement of wind speed and wind direction have been made at masts located at points B, M and A. The contours are at 10 metres vertical interval, the highest contour being 170 m above sea level.](image)
four periods in January 1991, August 1991 (two periods), and October 1991. The length of the time series were 63901, 62264, 27026 and 24519 samples respectively with a sampling frequency of 0.2 Hz (cup anemometers). The first time series consists of wind speed measurements at 15 m (A), 15 m (B) and 10 m, 25 m, 40 m (M) height and wind direction measurements at 15 m (A), 15 m (B), 15 m (M). The other three time series consist of measurements at 15 m (A), 15 m (B) and 10 m, 45 m (M) height and wind direction measurements at 45 m (M) height. Prior to the evaluation, all data were subjected to the screening and pre-processing procedures described in Appendix B.

To determine if the data have been collected in the outer or inner layer over the hill eq. 3.3 is used. The roughness length for Susetter hill has been estimated from site photographs as \( z_0 = 0.03 \text{m} \) (short grass), it follows that the inner layer exceeds a height \( l \) of 10 m (15 m) if the hill half width \( L \geq 1050 \text{m} \) (1810 m). It is concluded that half width of Susetter hill is below the above derived limits for all wind directions and measurement points. Thus the data under consideration is representative for wind energy applications and can be described using the 'rapid distortion' model.

### 3.4.2 Spectra for winds along the ridge

All time series have been classified in 12 wind direction sectors, each 30 deg wide. At masts B, M, and A, spectra have been calculated for winds of southerly (180±15 deg) direction. The spectra at these masts are presented in fig. 3.3 together with the Kaimal model for flat terrain (thin line) and a fit to the data (thick line).

For these data sets the wind is blowing along the longitudinal hill, striking B (15 m) first, followed by masts M (10 m) and A (15 m). The turbulence spectrum at B and M show a shift to lower frequencies as is expected. With constant variance and increased wind speed the turbulence intensity is reduced and shifted to lower frequencies. The speed up factors corresponding with the shift of the spectrum are 1.4 for mast B and 1.2 for mast M. Mast A does not show the expected shift towards lower frequencies, the corresponding speed up factor is 1.0. For a wind direction from the 180 deg sector the wind is accelerated by the topography at measurement mast B Then, while being moved via mast M towards mast
Figure 3.3: Spectra for wind direction sector 180 deg. Spectra from mast B (upper), mast M (centre), and mast A (lower) with wind coming from the 180 deg sector. The spectral shapes agree with the theoretical model. The position of the spectrum at masts B and M is shifted due to terrain influence towards lower frequencies.
Figure 3.4: Spectra for wind direction sector 150 deg. Spectra from mast B (upper), mast M (centre) mast A (lower) with a wind direction of 150 ± 15 deg are compared with the Kaimal spectrum for flat terrain and a fit of this function to the data. The spectra are shifted, at mast B and M the shift is stronger than at mast A.
A the speed up is reduced and the mean wind speed 'recovers' to its equilibrium value. This results at mast A in turbulence spectra equivalent to those found in flat terrain. The Kaimal '73 model for FSU-terrain fits not only the shape but also the position of the experimental spectrum at mast A very well.

The spectrum from mast A at 15 m is compared in fig. 3.4 with spectra from M at 10 m and B at 15 m for a wind direction of 150 deg. All spectra are shifted towards lower frequencies as predicted. However, mast A shows a smaller shift than for masts B and M. This is also due to the mean wind speed relaxation along the ridge.

### 3.4.3 Spectra for winds perpendicular to the ridge

For westerly wind directions (270 deg) the wind blows perpendicular to the longitudinal axis of Susetter hill (fig. 3.2). For each of the three masts, a spectrum resembling that of a simple two dimensional hill in complex terrain is expected. The spectra at mast B (15 m), mast M (10 m), and mast A (15 m) are presented in fig. 3.5 together with the Kaimal model for flat terrain and a fit of this model to the data.

The spectrum at masts A, B, and M show the expected shift to lower frequencies, associated with terrain induced speed up effects at these points. The corresponding speed up factors calculated for 270 deg by fitting the Kaimal model to the data are very similar for each of the masts 2.0 for B, 2.1 for M, and 2.2 for A.

Lower speed up factors are obtained from the evaluation of the spectra for 240 deg (fig. 3.6). The corresponding values are 1.2, 1.1, and 1.3 for the masts B, M, and A respectively. The wind at 240 deg is no longer perpendicular to the longitudinal Susetter Hill. The third dimension of the hill may become important here, as the air could have been redirected to flow around, rather than over the hill which would result in lower speed up factors.

Due to faulty data from the anemometer at 10 m height, data from mast M for a wind direction of 120 deg was available only at a height of 45 m. This spectrum is compared with spectra measured at 15 m height at A and B in fig. 3.7. The Kaimal model for flat terrain has been fitted to the data, the corresponding speed up factors are mast B: 1.9, mast M: 1.7 and 1.5 for mast A. These values being
Figure 3.5: Spectra for wind direction sector 270 deg. Spectra from mast B (upper), mast M (centre) mast A (lower) with a mean wind direction of 270±15 deg are compared with the Kaimal spectrum for flat terrain and a fit of this function to the data.
Figure 3.6: Spectra for wind direction sector 240 deg. Spectra from mast B (upper), mast M (centre) mast A (lower) with a mean wind direction of 240 ± 15 deg are compared with the Kaimal spectrum for flat terrain and a fit of this function to the data. The spectra are shifted (in normalised frequency co-ordinates) due to the speed up over the hilltop.
Figure 3.7: Spectra for wind direction sector 120 deg. Spectra from mast B (upper), mast M (45m level) (centre) mast A (lower) with a wind direction of 120 ± 15 Deg are compared with the Kaimal spectrum for flat terrain (thin line) and a fit of this function to the data (thick line).
somewhat lower than for a wind direction of 270 deg and somewhat higher than for wind from the 270 deg sector.

### 3.4.4 Comparison with numerical predictions

To validate the model assumptions the speed up factors derived from the measured spectral shift have been compared with models currently used to describe the speed up in complex terrain. The shift of the spectrum and the corresponding speed up factors derived in the last sections from experimental data are compared in tab. 3.1 and fig. 3.8 with the WAsP and LSD model predictions (sec. 3.1.3 and sec. 3.1.2):

<table>
<thead>
<tr>
<th>Sector</th>
<th>Mast A</th>
<th>Mast M</th>
<th>Mast B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WAsP</td>
<td>LSD</td>
<td>WAsP</td>
</tr>
<tr>
<td>000</td>
<td>1.1</td>
<td>1.1</td>
<td>-</td>
</tr>
<tr>
<td>030</td>
<td>1.1</td>
<td>1.3</td>
<td>-</td>
</tr>
<tr>
<td>060</td>
<td>1.3</td>
<td>1.4</td>
<td>-</td>
</tr>
<tr>
<td>090</td>
<td>1.5</td>
<td>1.5</td>
<td>-</td>
</tr>
<tr>
<td>120</td>
<td>1.4</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>150</td>
<td>1.1</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>180</td>
<td>1.1</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>210</td>
<td>1.1</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>240</td>
<td>1.3</td>
<td>1.6</td>
<td>1.3</td>
</tr>
<tr>
<td>270</td>
<td>1.5</td>
<td>1.7</td>
<td>2.2</td>
</tr>
<tr>
<td>300</td>
<td>1.4</td>
<td>1.6</td>
<td>-</td>
</tr>
<tr>
<td>330</td>
<td>1.1</td>
<td>1.3</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 3.1: Speed up factors at Susetter Hill. The speed up calculated using WAsP, LSD are compared with the observed spectral shift.

The speed up factors calculated with WAsP show a symmetric behaviour as would normally be expected from a pure potential flow model. The WAsP model features a constant height internal boundary layer over topographical elements. Only in this internal boundary layer is the potential flow solution modified. Our measurements are located in the outer layer where a potential flow solution is taken.
This separation into two layers was first proposed by [Jackson75]. WAsP includes an empirical formula for the height of the internal layer developed by Jensen [Troen89]. This constant height of the internal Boundary Layer does not allow for a gradual relaxation of the wind profile e.g. over an escarpment, as does the LSD approach.

There is, for speed up factors lower than 1.7, some agreement between the speed-up factors predicted by WAsP and LSD and speed up factors derived from the spectral shift (fig. 3.8). The experimental results provide support for the proposed model to account for the spectral shift. This is not true if the speed up factors derived from the spectral shift are larger than 1.7. The reasons for this need to be investigated further.
3.5 Summary - turbulence in complex terrain

This chapter has analysed the wind field in complex terrain. The speed up due to the terrain and its effect on turbulence and turbulence spectrum have been discussed. In this chapter (based on chap. 2) models have been discussed which can be used for the prediction of spectra in complex terrain. The predictions made have been validated with measured data. From the analysis the following conclusions are drawn:

**Speed up** The effect of a hill on the mean wind speed can be determined by a simple numerical model. Advanced models do take the relaxation of the wind profile over for example escarpments or plateaus into account. Adequate models are available for most situations.

**Turbulence** The ambient turbulence level on site should be obtained from direct measurements. The turbulence level can then be adjusted with the speed up relative to the reference measurement point for the exact position of the wind turbine(s).

**Shape of spectrum** The wind spectrum in complex terrain can be represented by the spectral shapes proposed for flat terrain by von Karman or Kaimal.

**Location of spectrum** The maximum of the spectrum is shifted towards lower frequencies. This is due to the increased mean wind speed over the given terrain.

The modification of the spectrum in complex terrain $H_{KC}$ described in this chapter constitutes the second link in the chain leading to the voltage flicker (eq. 1.1):

$$P_{st}^2 = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) \, dn$$

The influence of the wind farm on the wind spectrum ($H_{CW}$) will be discussed in a later chapter, the next chapters will instead introduce the coherence function. The description of the coherence in flat and complex terrain concludes the description of the wind resource. The coherence function is needed to describe how the wind field interacts with an individual wind turbine ($H_{WP}$) or a group of wind turbines ($H_{FT}$).
Chapter 4

Models for the spatial coherence

Wind time series measured simultaneously at two or more locations are not in general statistically independent. The relation between two time series is described with the help of the coherence function. The coherence function is needed in the context of this thesis to describe two effects:

The wind turbine blades collect the wind energy from the whole rotor area. For two different points in the rotor area it cannot be assumed that the wind speed is statistically independent. To model the wind power output of the rotor the coherence for small separation distances needs to be considered.

The power production from multiple turbines, driven by stochastic, weakly coherent wind signals is advantageous in terms of electrical power quality, compared to a single turbine of the same power. To predict the lumped power output from a group of wind turbines the coherence function for large separation distances needs to be modelled.

4.1 Spatial two point coherence

In this chapter the coherence is introduced and the definition of coherence used throughout this thesis is given. Unfortunately different definitions may be found in some of the literature. The concept of decay parameters will be introduced using for historical reasons a simple exponential model.
Figure 4.1: Symbolic representation of lateral, longitudinal and mixed cases. The two reference points $PY_1$ and $PY_2$ are at equal height above ground and separated by the distance $d$. The arrows indicate the wind direction. The angle between the wind direction and the line separating the two points is denoted $\alpha$.

### 4.1.1 Definitions

The (two point) coherence function $\gamma^2$ at a frequency $n$ is defined as ratio between the cross spectral density $|S_{12}(n)|$ and the product of the power spectral densities $S_1(n)$, $S_2(n)$ of two time series $Y_1(t)$, $Y_2(t)$ measured at the locations $PY_1$ and $PY_2$ [Bendat71]:

$$\gamma^2_{12}(n) = \frac{|S_{12}(n)|^2}{S_1(n)S_2(n)} \tag{4.1}$$

Contrary to the definition used here, some other authors have defined coherence as $\gamma$, rather than $\gamma^2$, e.g. [Frost78, Frost94]. Depending on the point of view $\gamma^2$ and $\gamma$ are referred to as ‘squared coherence’ and ‘root coherence’ respectively [Panofsky84, ESDU91].

Vertical coherence indicates two points separated vertically (e.g. at different levels of a meteorological mast), longitudinal coherence two points in line with the (horizontal) wind direction vector and lateral coherence separations which are horizontal and perpendicular to the wind direction vector (fig. 4.1).
4.1.2 Davenport's coherence model

An exponential model for the coherence function was first proposed by [Davenport61], for vertical coherence and was later also applied to lateral and longitudinal coherence [Davenport67, Pielke70]:

\[ \gamma_i^2(n) = e^{-c_i \frac{dn}{u}} \]  

(4.2)

\( u \) is the wind speed at measurement height \( z \). The 'decay parameters' \( c_i \) have to be determined by experiment. The index \( i = 1 \) is used for longitudinal, \( i = 2 \) for lateral and \( i = 3 \) for vertical separation.

The coherence function is often presented as the logarithm of the coherence over a linear frequency axis. The function should then according to eq. 4.2 be close to a straight line. In this thesis all coherence plots are presented as in fig. 4.2. This allows study of the coherence function over the whole frequency range of interest and the plots are compatible with the presentation of spectra.
4.1.3 Decay parameters

The influence of the decay parameter on the coherence function can be seen in fig. 4.2 where the coherence function is plotted for decay parameters 5, 10 and 20 at 10 m/s wind velocity, 30 m height and 10 m separation distance.

The quoted decay parameters vary due to the above mentioned inconsistency in the literature with respect to the definition of coherence by a factor of 2. All decay parameters in this thesis relate to eq. 4.1 and have been adjusted where necessary.

In the original publication [Davenport61] the wind speed $u_{10}$ at 10 m height is used in eq. 4.2. In subsequent publications e.g. [Pielke70] $u_{10}$ was replaced by the wind speed at measurement height. The older definition has however been used since then in several publications [Shiotani71, Duchêne-Marullaz75, Soucy82, Frost94]. The normalisation with $u_{10}$ leads to an unwanted dependence of the decay factor on the measurement height as has been pointed out by [Kristensen79b].

When using the exponential model, eq. 4.2, for lateral coherence, a wide spread of the coherence decay parameter $C_2$ is observed. The values quoted in literature range from 10 to 60. The experimental results for the longitudinal decay parameter $C_1$ range from 0.4-1.0 for wind tunnel experiments, 2-3 over sea and 4-10 over land. Different models have been proposed to reduce this apparent scatter.

4.2 Coherence model for small separations

To model the wind power output of a rotor a description the coherence over the rotor area is needed. It will be seen that it makes sense to develop two distinct formulations for the large scale inter-turbine coherence and this small scale coherence. Heights from 20 m to 100 m with separation distances from 1 m up to some 40 m are considered to be of small scale.

4.2.1 Idealisations

The theoretical description of turbulence and of spatial coherence is incomplete. Because of the complexity of the problem, theoretical models have to include
far reaching approximations and are often valid only over a very limited region. Besides incompressibility and homogeneity, isotropy of turbulence is assumed initially which is a valid assumption for small separations. Small is to be understood as small relative to the length scale of turbulence, which can be approximated by the height under consideration [Mann91].

4.2.2 Isotropic Turbulence

High spacial coherence is obtained when the separation between the reference points is small compared to the length scale of turbulence. For such small separations isotropy can be assumed and the spatial coherence can be described by a simplified theoretical approach. The spectral tensor $\Phi_{ij}$ is defined as [Kristensen79b]:

$$\Phi_{ij} = \frac{E(k)}{4\pi k^2} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right)$$

where

- $E$: turbulent energy spectrum
- $\delta_{ij}$: Kronecker delta
- $k$: magnitude of $\mathbf{K}$
- $\mathbf{K}$: wave number
- $k_i$: components of $\mathbf{K}$

For a lateral separation $d_2$ the auto- and cross-spectra can be expressed as a function of the spectral tensor:

$$S_{ij} = \int_{-\infty}^{+\infty} d\mathbf{K}_2 \int_{-\infty}^{+\infty} d\mathbf{K}_3 \Phi_{ij} e^{i\mathbf{K}_2 d_2}$$

The evaluation of this integral with a chosen energy spectrum, e.g. Kolmogorov spectrum or von Kármán spectrum, results in an expression for the coherence in terms of Bessel functions of the second kind [Roberts73, Irwin79, Kristensen79b, Mann91]. The form now suggested in [IEC97] is:

$$\gamma^2 = \left( \frac{2^{\frac{1}{3}}}{\Gamma(\frac{5}{6})} \left( \xi^\frac{5}{6} K_{\frac{5}{6}}(\xi) - 0.5\xi^\frac{11}{6} K_{\frac{11}{6}}(\xi) \right) \right)^2$$

with $\xi = 2\pi \left( \left( \frac{nd}{u} \right)^2 + \left( \frac{0.124}{L} \right)^2 \right)^{\frac{1}{2}}$ and:

- $\Gamma$: Gamma function
- $K_{\frac{5}{6}}$: Bessel function
- $L = 3.5 \Lambda_1$: isotropic turbulence integral scale.

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Possible alternatives to the integral over the energy spectrum $E(k)$ in eq. 4.4 have been published by [Roberts73], and [Mann94]. It is argued that of the eddies forming the energy spectrum $E(k)$ only the smaller eddies contribute to the destruction of a larger eddy and thus the integral in eq. 4.4 should only be performed over the high frequency part of the spectrum. These models can also be approximated by exponential forms, for example [Mann91]. An empirical approximation to the formula of [Roberts73] was proposed in [ESDU91]:

$$\gamma \approx e^{-1.15 \left( \frac{2\pi n d}{u} \right)^{1.5}}$$

(4.6)

This approximation is compared in fig. 4.3 with the IEC model (eq. 4.5) and Davenport's coherence model (eq. 4.2). The ESDU approximation is similar to the Davenport model, the IEC model shows lower coherence values at low frequencies. A different approximation to the IEC model was suggested in [IEC97] and similar

![Graph of Coherence models](image)

Figure 4.3: Coherence models, small separation. Davenport's coherence model (eq. 4.2) ($c_i = 10$) is compared with the IEC model (eq. 4.5) and the ESDU approximation (eq. 4.6). The IEC model shows lower coherence at low frequencies.
by Holley and Thresher [McNerney92]:

\[ \gamma = e^{-8.8 \left( \frac{0.12d}{u} \right)^2 + \left( \frac{z}{n} \right)^2} \]  

(4.7)

This IEC approximation is compared with the Davenport model and the IEC Kármán model (eq. 4.5) in fig. 4.4. The agreement with the IEC model is not very good, especially for low frequencies. A better agreement to the IEC model is achieved if it is assumed that the right hand side of eq. 4.7 represents \( \gamma^2 \) instead of \( \gamma \) so that:

\[ \gamma^2 = e^{-8.8 \left( \frac{0.12d}{u} \right)^2 + \left( \frac{z}{n} \right)^2} \]  

(4.8)

The Davenport model (eq. 4.2) predicts the highest coherence at low frequencies followed by the IEC model (eq. 4.5), the improved IEC approximation eq. 4.8 and the IEC approximation (eq. 4.7).

Although the models seem to differ significantly at low frequencies they do, apart

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**Figure 4.4: Davenport and IEC coherence models.** The Davenport model shows highest coherence at low frequencies, it follows the IEC model the modified IEC approximation (see text) and the IEC approximation.
from eq. 4.7, agree reasonably well at frequencies higher than 0.01 Hz. This means that for problems associated with small distances that is below 1 rotor diameter of a wind turbine the choice of the coherence model will not have significant influence on the result of the calculation performed. The evaluation of eq. 4.5 is demanding, the revised IEC approximation (eq. 4.8) has been used instead in this thesis to improve computational efficiency.

4.2.3 Non-isotropic expansions

For separation distances comparable to the turbulent length scale, the isotropy of turbulence is disturbed by the shear stress in the surface layer. A model which assumes uniform shear stress in the vertical direction is discussed by [Mann94] and a formulation was proposed which takes account of blockage by the surface. An alternative approach is to use the two dimensional horizontal energy spectrum in eq. 4.3. This spectrum has been proposed by [Peltier96] and recently compared with experimental data (small separations) by [Tong96].

Despite these recent advances in the description of turbulent spectra, the findings of [Kristensen81] still apply, that models based on eq. 4.3 are not valid for separation distances larger than the appropriate turbulent length scale. This is due to the lack of isotropy in the neutral boundary-layer [Kristensen81, Kristensen83, Kristensen89, Henjes97].

Some idea of the difficulties encountered when analysing such models for large separations is found in the results of [Kristensen81] and [Beyer93], they report high experimental coherences for separation distances up to and in excess of 10 times the measurement height. From any of the above mentioned models a coherence very close to would be expected for all frequencies. Similarly, a high cross-correlation as has been reported for large separations by [Hanna92] would contradict \[ \gamma^2(n) = 0 \].

In the light of the current state of theoretical models, rather simple semi-empirical approaches are discussed in the following paragraphs which do in practice, describe the coherence function for large separations with sufficient accuracy. Flat terrain and neutral stratification are assumed, parameterisations which include stability can be found in [Pielke70], [Ropelewski73], [Berman77] and [Soucy82].
4.3 Coherence model for large separations

For a weak network, the power quality depends on the spacial coherence [Beyer90b, Paulsen89]. Hence the spatial coherence of the wind speed in a wind farm has to be taken into account. Separation distances of wind turbines in a wind farm will always be greater than the height of the turbines above ground. Of interest here is the coherence in near neutral atmospheric conditions i.e. high wind speeds. Heights from 20 m to 100 m with separation distances from 20 m up to some 10 km for the large scale coherence.

Very few authors have published data for horizontal large scale two point coherence. This is believed to be due primarily to the non standard requirement to erect several meteorological towers and record wind speed data from them simultaneously. Based on published data and data recorded at the Rutherford Appleton Laboratory in Oxfordshire, United Kingdom, models proposed by different authors are compared in order to find a model which fits the data over a wide range of parameters.

4.3.1 Model equations

Past experimental results confirm that an exponential model is well suited to approximate the coherence function of the streamwise velocity component. It is known that the exponential decay of the coherence depends on the separation distance $R$, standard deviation $\sigma$ and the measured frequency $n$.

Coherence for large longitudinal separation

In a coordinate system which is moving with the mean wind speed, an appropriate form of the exponential model with arbitrary decay constants $b_i$ is:

$$\gamma_i^2 = e^{-b_i \sigma n d}$$  \hspace{1cm} (4.9)

Now a relation is considered which describes this dependency in a coordinate system where the wind field moves relative to the reference points with wind speed. For non isotropic turbulence different decay constants are expected for each direction. For longitudinal separation it can be assumed that:
\[ \gamma_1^2 = e^{-a_1 \frac{\mathcal{R}}{u}} \]  \hspace{1cm} (4.10)

where \( a_1 \) is a longitudinal decay constant. The formulation of eq. 4.10 is guided by the following argument: The measured frequency is increased as the wind field is shifted across the reference points. The distance any eddy travels before it decays is increased with mean wind speed. Both effects have been compensated for in the model by dividing by the mean wind speed \( u \).

**Coherence for large lateral separation**

For a lateral separation the picture is different as the shift of the wind field in longitudinal direction will affect the measured frequency but not lead to a shift of eddies from one reference point towards the next. It is postulated for the lateral case:

\[ \gamma_2^2 = e^{-a_2 \frac{\mathcal{R}}{u}} \]  \hspace{1cm} (4.11)

The empirical decay constant \( a_2 \) has now unfortunately a dimension \( \frac{L}{m} \). The equations for longitudinal and lateral coherence can be combined to provide an expression including \( \alpha \) the angle between the wind direction and the line connecting the two reference points. Such a formulation is supported by empirical evidence of correlation coefficients measured in the horizontal plane [Shiotani71, Hayashi91]:

\[ \gamma^2(\alpha) = e^{-\sigma \frac{\mathcal{R}}{u} \left( \frac{\sigma_1}{u} \cos \alpha \right)^2 + \left( a_2 \sin \alpha \right)^2} \]  \hspace{1cm} (4.12)

**Differences to Davenport model**

The new model (eq. 4.12) differs from Davenport's model (eq. 4.2) in two major ways, that is in the dependence on wind speed and standard deviation. Davenport formulated his model for vertical separations and it was only later applied to longitudinal and lateral separations. It was not recognised at the time that the shift of the wind field in longitudinal direction would require a separate treatment of longitudinal and lateral coherence. In addition to this modification the turbulence is included in the considerations and this will be shown to be empirically justified.
The new model is compared with models proposed by other authors for longitudinal and lateral coherence, experimental data reported in the literature and some new experimental evidence. The discussion will focus on the comparison of models on the special cases for longitudinal (eq. 4.10) and lateral coherence (eq. 4.11).

4.3.2 Discussion for longitudinal coherence

The decay of the longitudinal component of the coherence has been modelled in the past using separation distance, standard deviation and mean wind speed as parameters. These parameterisations are discussed here for longitudinal separations.

Decay parameter and turbulence intensity

The turbulence intensity has previously been used as a scaling factor by [Ropelewski73]. This scaling successfully collapses data from wind tunnel experiments, experiments over water and experiments over land. The formula for longitudinal coherence proposed by [Ropelewski73] is identical to eq. 4.10 and supports the parametrisation using the standard deviation.

Decay parameter and separation distance

It should be noted that in an experimental validation values of coherence lower than those predicted by eq. 4.12 can occur. This is due to the effect of transverse fluctuations and variations in wind direction which is not included in the analysis. A functional dependence of the decay parameter \( c_1 \) on the separation distance \( R \) has been proposed by several authors [Panofsky75, Perry78, Kristensen79a] to account for this effect. A more recent experimental study for distances from 210 m to 25 km in Lower Saxony, Germany does not support such a parametrisation [Beyer90a, Beyer93].

Decay parameter and mean wind speed

For large longitudinal distances (greater than 300 m) Taylor's hypothesis cannot easily be applied as all original eddies will have been destroyed whilst being
transported over such an extended time period. Consequently it has been argued that the scaling of the distance travelled by an eddy with the wind speed may no longer be appropriate [Handwerker93]. It is believed here that the assumption of 'frozen eddies' can be replaced by a more statistical approach, as suggested by [Fordham85] so that there remains a strong physical argument for the scaling in eq. 4.10 with the mean wind speed.

4.3.3 Discussion for lateral coherence

The decay of the lateral component of the coherence has been modelled in the past using separation distance, standard deviation and mean wind speed as parameters. These parameterisations are discussed here for lateral separations.

Decay parameter and separation distance

Opinions regarding the dependence of the decay parameter on the separation distance are split between the different models. The majority of measurements do not reveal any dependency of the decay parameter on the separation distance. However, [Bowen83] finds $c_2 = c_2(d)$ and [Shiotani80], $c_2 = c_2(d^{0.5})$. A recommendation which attempts to combine the different models was made by [Kristensen79b]. It is considered here that the experimental evidence is generally against such a dependency, so that as in eq. 4.12, it is not included.

Decay parameter and standard deviation

The dependency of the lateral coherence on the standard deviation $\sigma$ has been used in [Roberts73] and [Kristensen89]. As in the case of longitudinal coherence, a reduction of the scatter of experimental decay constants can be observed if the decay constant is assumed to be proportional to the standard deviation $\sigma$. In contrast to the approach chosen for the longitudinal model, [Ropelewski73] recommend the use of the ratio between the longitudinal and lateral length scale as parametrisation of the decay parameter (also in [Berman77]). These length scales are themselves a function of the atmospheric stability and are problematic to identify. A more pragmatic approach is taken here by assuming that both lateral and longitudinal decay depend linearly on the standard deviation.
Decay parameter and wind speed

The coherence function given by eq. 4.11 is, for constant turbulence intensity $\xi_u$, independent of the mean wind speed. This somewhat unusual proposal is supported by an experimental campaign from Butjadingen, Northern Germany. These data show no evidence of a dependence of the coherence on the wind speed as would be expected from eq. 4.2 [Beyer93, Handwerker93]. However, the experiments do not conclusively prove that there is no such dependency. Experimental data published in [Shiotani80] show a slight dependence of the decay parameter $c_2$ on the wind speed, which suggests again that for lateral coherence the scaling with wind speed in eq. 4.2 is inappropriate. The scatter of decay parameters $c_2$ taken from the literature is unaffected by removal of the scaling. Thus, from experimental data alone, it is difficult to decide if such scaling should be applied or not. There is, however, no compelling physical argument to support a dependence of lateral coherence decay on the mean wind velocity. The eddies are not transported any faster over the lateral separation distance with an increase in longitudinal wind speed.
4.4 Coherence in complex terrain

It is assumed in the following that the small scale (up to 1 rotor diameter) vertical and lateral coherence function is not significantly influenced by the terrain. The small scale coherence can then be modelled based on the equation eq. 4.5 which has been presented in the discussion for the flat terrain coherence, sec. 4. The changes which apply to the large scale coherence are discussed here and, based on data from Susetter Hill, an extension to the model equation eq. 4.12 is proposed which allows it to be applied to coherence over a hill.

4.4.1 Earlier investigations

Only three earlier publications are known which deal in some way with horizontal two point coherence in complex terrain. [Perry78] discuss the influence of a nearby large hill on their results. The two publications [Sacre89, Sacré92] study the lateral and vertical coherence over a hill in Southern France (Lastours) for separation distances of 10, 20, and 30 m. The authors compare this data with measurements taken in flat terrain on a bridge. Additionally, one publication studies vertical coherence functions for separations of 1.8 m and 23.4 m from a mountainous and highly complex site (Sky River, California), by [Thomsen96b].

4.4.2 Qualitative discussion

Wind coherence over a hill is subject to a series of changes in the flow pattern which are direct consequences of the accelerated wind speed:

- Turbulent eddies are subject to longitudinal stretching when they pass a hilltop. This is assumed to happen on a timescale which is short in comparison with the eddy lifetime. Eddy stretching taken on its own would lead to a decrease in the measured frequency of fluctuations at a given point.

- The increased mean wind speed transports eddies faster. They will pass the measurement point at an increasing rate. Together the effects of vortex stretching and increased transport velocity on the frequency are expected to eliminate each other.
The models for longitudinal coherence relate the eddy decay time to the distance travelled. With faster transport velocity the distance an eddy is able to travel before it decays is increased. The increase in coherence due to this effect is automatically included in the model.

4.4.3 Model equation

It can be considered that the effect of speed up on the spatial distribution of eddies is to reduce their numbers per unit area. As the spatial variance is decreased fewer eddies contribute to the destruction of large eddies, increasing their average lifetime and thus the coherence over a hilltop. The introduction of a speed up factor eq. 3.1 in eq. 4.12 accounts for the increase in spatial coherence:

$$\gamma^2(\alpha) = e^{-\frac{1}{2} \ln \left( \frac{a_1 \cos \alpha}{a_2 \sin \alpha} \right)^2 + \left( \frac{a_2 \sin \alpha}{a_1 \cos \alpha} \right)^2}$$

(4.13)

This equation is used with the same decay parameters identified for flat terrain eq. 4.12, i.e. $a_1 = 30 \pm 10$ and $a_2 = (35 \pm 10) \left( \frac{\text{sec}}{\text{m}} \right)$. In the following, this semi-empirical model is compared with experimental data from a complex terrain site. The speed up is defined relative to a (virtual) undisturbed wind profile. It should be emphasised here, that the wind speed $U$ and the turbulence intensity $I$ are the local wind speed and turbulence measured at the points under consideration and do not refer to a virtual undisturbed wind speed and its corresponding turbulence. The model is not expected to predict the coherence well if for example the turbulence intensity or the terrain induced speed up changes significantly over the distance considered.
4.5 Summary - models for the spatial coherence

Coherence models for small and large separation, flat and complex terrain have been discussed.

Coherence - small separations Models for small scale, isotropic coherence have been discussed. An approximation proposed in [IEC97] has been modified and selected for use in this thesis (eq. 4.8).

Coherence - large separations Model equations have been developed for coherence of two wind turbines with large separations in flat terrain (eq. 4.12). The model incorporates the influence of the standard deviation and mean wind speed on the coherence. The new model has been discussed in detail in [Schlez98].

Coherence - complex terrain The new model has been modified to take the influence of the terrain expressed by the speedup factor into account (eq. 4.13).

The coherence for small separations was considered reasonable well known and verified. The models for large separation distances on the contrary have been newly proposed here. They are based on simple approximation and partially supported by results reported earlier by other authors. However it was perceived necessary to validate the new model equations against experimental data.
Validation of the coherence model

The validation is split in two parts, data recorded at the Rutherford Appleton Laboratory (RAL), Wind Test Site, Didcot, UK were obtained to test the proposed coherence model. Data from Susetter Hill, Scotland, UK was used to validate the coherence model in complex terrain.

5.1 Experimental set-up at RAL

The data were recorded on three meteorological towers (T2, T3, and T4), see fig. 5.1. T2 is equipped with 2 cup anemometers at heights 7 m and 18.7 m, 2 temperature sensors at 2 m and 18.7 m and a wind vane at 17 m height. T3 has a cup anemometer and a wind vane installed at 18 m height. The tower T4 is equipped with only one anemometer at 18 m height. Data from all three masts and the air pressure were recorded simultaneously in March 1996 at a sampling frequency of 0.2 Hz.

5.1.1 Data selection

Because of the low sampling frequency turbulence intensities were corrected assuming a Kaimal spectrum. The data were split manually into runs exhibiting stationary conditions in terms of wind direction and wind speed. The temperature and wind speed measurements from mast T2 were used to calculate the Richardson number. It should be noted, that due to the lack of homogeneity in the terrain
Figure 5.1: Sketch of the Rutherford Appleton Laboratory test site. Auxiliary buildings (single storey, flat roof) are marked with A, wind turbines (not operating) with WT1-4. Wind speed measurements have been performed at 18 m height at the three towers marked T2, T3 and T4.
upwind of T2, the calculation of the Richardson number is not considered to be very reliable. The records are summarised in tab. 5.1. The wind speed is taken from T4 and the wind direction Θ from T2 because these masts were least affected by obstacles and instrument failures respectively. RUNs numbered 1, 7, 8, 11, 12, 13, 15, 16 had to be excluded from the evaluation because of the low wind speed. RUNs 8, 12, 14 were excluded because of high Richardson numbers.

5.1.2 Description of the selected records

This leaves records 2, 3, 4, 5, 6, 9, 10 for further evaluation. Measurements from tower T2/T3 and T2/T4 had to be discounted because mast T2 experienced blockage and wake effects for all wind directions, see fig. 5.1. From the remaining RUNs, 2 and 3 feature predominantly longitudinal, 9 and 10 lateral coherence. The other RUNs show a mixture of longitudinal and lateral coherence properties. RUN-9 has to be treated with care because it includes some variation in the wind speed. The influence of the wind turbine WT1 (Wind Harvester, rotor diameter 14 m, lattice tower 18 m high), which was not operating during the period of data collection, on the coherence between T3 and T4 has been neglected.

<table>
<thead>
<tr>
<th>No</th>
<th>duration (min)</th>
<th>Θ (deg)</th>
<th>u (m/s)</th>
<th>Ri</th>
<th>No</th>
<th>duration (min)</th>
<th>Θ (deg)</th>
<th>u (m/s)</th>
<th>Ri</th>
</tr>
</thead>
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<td>-0.07</td>
<td>9</td>
<td>450</td>
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<tr>
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<td>135</td>
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<td>0.02</td>
<td>10</td>
<td>580</td>
<td>52</td>
<td>7.2</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.03</td>
<td>11</td>
<td>200</td>
<td>48</td>
<td>4.2</td>
<td>-0.03</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>114</td>
<td>6.2</td>
<td>-0.03</td>
<td>12</td>
<td>620</td>
<td>63</td>
<td>4.1</td>
<td>-1.1</td>
</tr>
<tr>
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<td>500</td>
<td>108</td>
<td>7.1</td>
<td>-0.01</td>
<td>13</td>
<td>300</td>
<td>272</td>
<td>4.5</td>
<td>-0.04</td>
</tr>
<tr>
<td>6</td>
<td>400</td>
<td>99</td>
<td>8.9</td>
<td>0.06</td>
<td>14</td>
<td>400</td>
<td>288</td>
<td>6.5</td>
<td>0.08</td>
</tr>
<tr>
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<td>86</td>
<td>4.7</td>
<td>-0.07</td>
<td>15</td>
<td>250</td>
<td>298</td>
<td>4.4</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>500</td>
<td>93</td>
<td>2.4</td>
<td>0.22</td>
<td>16</td>
<td>350</td>
<td>282</td>
<td>5.0</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

Table 5.1: RUNs 1-16 recorded at Rutherford Appleton Wind Test Site. The length of each time series, average wind speed (T4) and wind direction (T2) are given. The temperature and wind speed difference at two levels (T2) were used to calculate the Richardson number.
5.2 Experimental results in flat terrain

The proposed model (eq. 4.13) differs from the standard Davenport one through dependence on the turbulence intensity for both lateral and longitudinal separations and through independence from the wind speed for the lateral separation. Data for longitudinal separation distances will be used to support the dependence of the coherence on turbulence intensity. Data for lateral separation distances will be used to support the dependence of the coherence on turbulence intensity and independence from wind speed. In order to facilitate the comparison of the experimental results with the model function a dimensionless frequency \( \tilde{n} \) is defined:

\[
\tilde{n} = \frac{nd}{u} \left( \left( \frac{a_1}{u} \cos \alpha \right)^2 + \left( \frac{a_2 \sin \alpha}{u} \right)^2 \right)^{\frac{1}{2}}
\]

Substituting this frequency in eq. 4.12 results in a simple exponential function: \( \gamma^2 = e^{-\tilde{n}} \). This function is subsequently compared with experimental data from the Rutherford Appleton Wind Test Site. The Nyquist frequency is indicated as \( \tilde{n}_c \).

5.2.1 Longitudinal separation

Rutherford Appleton data

The experimental results from RUN-2 are compared with the theoretical model in fig. 5.2. The wind direction is roughly longitudinal, along the line from tower T3 to tower T4. The theoretical model is calculated using eq. 4.12 with a relative wind direction angle of 9 degrees. It was found that the parameters \( a_1 = 30 \) (longitudinal) and \( a_2 = 35 \) (lateral) result in a good fit for all experimental RUNs from the Rutherford testsite. The error-bars are obtained as the sum of the standard deviation (square root of eq. B.6) and the bias (eq. B.4), based on the experimental coherence estimate \( \gamma^2 \). It was tried to exclude as far as possible additional errors due to changes in wind direction and wind speed (non-stationarity), obstacles and roughness changes (non-homogeneity). As part of these efforts RUN-2 for example was, for the evaluation of the high frequency part subdivided in three periods of roughly three hours each. The resulting more stationary (in wind speed) periods were evaluated separately and later bin-averaged.
Figure 5.2: Longitudinal coherence. The coherence function for RUN-2 is compared with the theoretical model (relative angle $\alpha = 9$ degrees) ($\bar{n}_c = 3.7$). The parameter $a_1 = 30$ and $a_2 = 35\frac{A}{m}$ are in agreement with the experimental data for RUN-2.

Figure 5.3: Longitudinal coherence. The coherence function for RUN-3 is compared with the theoretical model (relative angle $\alpha = 7$ degrees) ($\bar{n}_c = 3.3$). The parameter $a_1 = 30$ and $a_2 = 35\frac{A}{m}$ result in a good fit for RUN-3.
RUN-3 also features wind directions roughly along the line from tower T3 to tower T4. The relative angle is smaller than for RUN-2. The fitted model is in even better agreement with the data (fig. 5.3). By using different numbers of intervals for low and high frequency, the variance is kept small over the widest possible range (Appendix B.3). The high coherence estimates for low frequencies have low variance, even for the very small numbers of segments (≥ 3) which are used for the low frequency range throughout the analysis.

Published experimental data

The obtained coherence decay parameter is compared with other published values in tab. 5.2. The reference numbers in the table refer to the following publications: L1: [Champagne70, Ropelewski73]; L2: [Panofsky74]; L3: [Berman77]; L4: [Perry78]; L5: [Ropelewski73], L6: [Panofsky75]; L7: [Steinberger-Willms93].

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Location</th>
<th>$I$</th>
<th>$c_1$</th>
<th>$a_1 = c_1/I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>RAL (RUN-2)</td>
<td>0.12</td>
<td>3.6</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>RAL (RUN-3)</td>
<td>0.13</td>
<td>3.9</td>
<td>30</td>
</tr>
<tr>
<td>L1</td>
<td>wind tunnel</td>
<td>0.02</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>L2</td>
<td>Lk Ont. (water)</td>
<td>0.1</td>
<td>2-3</td>
<td>20-30</td>
</tr>
<tr>
<td>L2</td>
<td>Lk Ont. (land)</td>
<td>0.20</td>
<td>6-7</td>
<td>30-35</td>
</tr>
<tr>
<td>L3</td>
<td>Aberdeen</td>
<td>0.2-0.3</td>
<td>6-14</td>
<td>34 (20-50)</td>
</tr>
<tr>
<td>L4</td>
<td>Rock Springs</td>
<td>0.1-0.22</td>
<td>3-8</td>
<td>33 ±5</td>
</tr>
<tr>
<td>L4</td>
<td>O’Neill</td>
<td>0.14</td>
<td>2/4/4/6</td>
<td>15/30/30/43</td>
</tr>
<tr>
<td>L5</td>
<td>O’Neill</td>
<td>0.2</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>L6</td>
<td>O’Neill</td>
<td>0.17</td>
<td>4/6/7/11</td>
<td>24/35/41/64</td>
</tr>
<tr>
<td>L7</td>
<td>Butjadingen</td>
<td>0.21</td>
<td>6.3</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 5.2: Longitudinal decay parameter, experimental data and values taken from the literature. The decay parameter $c_1$ shows a wide scatter of values which is reduced by normalisation with the turbulence intensity.

The value of turbulence intensity for O’Neill (L6) represents the mean of the values of L4 and L5. The turbulence intensity for Lake Ontario (L2), O’Neill and Rock Springs (L4) was assumed to be twice the quoted vertical turbulence intensity.
The scaling of the classical coherence decay factor $c_1$ with the turbulence intensity reduces the scatter of the decay factors significantly. The data from Rutherford Appleton test site fit well into the general pattern. The remaining scatter reflects the difficulties in recovering the turbulence intensity for each experiment and the variation in methods applied in previous studies for fitting the exponential function to the data.

From the results obtained here and the results quoted in the literature, a longitudinal decay parameter of $a_1 = 30 \pm 10$ can be recommended.

### 5.2.2 Lateral separation distance

**Rutherford Appleton data**

The coherence for lateral separation is lower than for longitudinal separations. This requires resolution of lower frequencies, which is achieved by dividing the time series in as few intervals as possible. As a trade off, the variance of the experimental results is increased. Data from lateral directions (RUN-9 and RUN-10) is presented in fig. 5.4 and fig. 5.5. Both coherence functions are compared with the model of eq. 4.12. A lateral decay parameter of $a_2 = 35 \frac{A}{m}$ was found give some agreement with the data.

**Published experimental data**

The experimental result is compared with the published literature in tab. 5.3. The values of turbulence intensity at measurement height $z$ which are given in brackets are estimates derived from the roughness length $z_0$ of the terrain using:

$$I \approx \frac{1}{\ln \frac{z}{z_0}}$$

The points of reference L10 [Bowen83] have been normalised by the turbulence intensity calculated from a roughness length estimate by [Flay82]. The value taken from L8: [Shiotani80] refers to a separation of 45 m. The turbulence intensity is taken as the turbulence intensity quoted for wind coming from the sea. L9: [Kristensen79a] also publishes alternative decay parameters for a fit in the low frequency region.
Figure 5.4: Coherence for lateral separation $\alpha = 85$. The coherence function for RUN-9 is shown as function of the frequency $\tilde{\eta}$ ($\tilde{\eta}_c = 28$). The parameter $a_1 = 30$ and $a_2 = 35 \frac{m}{n}$ are not contradicted by RUN-9.

Figure 5.5: Coherence for lateral separation $\alpha = 74$. The coherence function for RUN-10 is shown as a function of the frequency $\tilde{\eta}$ ($\tilde{\eta}_c = 22$). The parameter $a_1 = 30$ and $a_2 = 35 \frac{m}{n}$ are not contradicted by RUN-10.
Table 5.3: Data for lateral coherence. Experimental results (RUNs 9 and 10) and values from the literature. Scaling with the turbulence intensity reduces the scatter of the data.

Of the values quoted in tab. 5.3, the values from Aberdeen (L3) and Christchurch (L10, second and third line) refer to separations larger than the measurement height and are thus of higher significance for this paper. The evaluations for smaller distances from Christchurch (L10: first line) should probably be dismissed, as a measurement caravan used in that experiment seems to provide an obstacle upwind of the measurement line, see [Flay88] for the experimental arrangement. L11 values are from a series of campaigns in flat terrain (Bouin), above a bridge (St Nazaire) and on top of a hill (Lastours), [Sacré92]. Lastours is a site with complex terrain which is believed to influence the coherence.

The scaling with the turbulence intensity again proves to be effective in reducing the scatter in the decay parameters. The proposed model is supported by the experimental data. Based on our data and tab. 5.3 a value for the decay parameter of \( a_2 = (35 \pm 10) \frac{m}{s} \) is recommended.
Figure 5.6: Coherence for mixed cases. Coherence for $\alpha = 12$ (upper), $\alpha = 18$ (centre) and $\alpha = 18$ (lower) are compared with the model predictions ($\bar{n}_c = 5.2, 6.8, 9.3$). The coherence functions are shown as a function of the frequency $\bar{n}$. The parameter $a_1 = 30$ and $a_2 = 35 \frac{a}{m}$ provide an acceptable fit for all three RUNs.
5.2.3 Angular dependence

If the wind direction is neither longitudinal nor lateral to the two points of interest both the lateral and longitudinal decay parameters are needed to describe the coherence. For increasing angle the lateral coherence quickly becomes the dominant factor. The results of the three data sets from Rutherford Appleton Laboratory which have intermediate wind directions are summarised in tab. 5.4.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Location</th>
<th>$\alpha$</th>
<th>$u$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
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<td>RAL (RUN-4)</td>
<td>12</td>
<td>6.2</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>RAL (RUN-5)</td>
<td>18</td>
<td>7.1</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>RAL (RUN-6)</td>
<td>27</td>
<td>8.9</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 5.4: Experimental data for intermediate directions (RUNs 4,5,6). Wind direction, wind speed and turbulence for the timeseries from intermediate direction.

Using the previously identified decay parameters of $a_1 = 30$ and $a_2 = 35 \frac{\Delta}{m}$ it can be seen (fig. 5.6) that the proposed model is in agreement with the experimentally determined coherence functions.
5.3 Data from Susetter Hill

For a detailed description of the measurement set-up at Susetter Hill see sec. 3.4.1. Wind speed and wind direction records had been recorded simultaneously at 15 m (A), 15 m (B) and 10 m, 45 m (M) height with a sampling frequency of 0.2 Hz. These three masts have been used to calculate the longitudinal and lateral coherence functions \( \gamma_{AB} \), \( \gamma_{AM} \) and \( \gamma_{BM} \) with respective separation distances of A to B: 850 m, A to M: 760 m and B to M: 90 m. The speed up factors for Susetter Hill obtained by the LSD approach are presented in tab. 3.1. As mentioned above, all measurements were made in the 'outer layer' where rapid distortion theory is applicable.

5.3.1 Recorded data

Wind speed (cup anemometer) and wind direction measurements were recorded simultaneously at the three masts during four periods: one in January 1991, two in August 1991 and one in October 1991. The total length of the periods were 89h, 86h, 37h, 34h respectively with a sampling frequency of 0.2 Hz. Several RUNs were selected with little variation in wind speed and wind direction. The main properties of the selected RUNs are listed in tab. 5.5. RUN-2 is the longest which makes it the RUN most suited to obtain information on the larger two separations. The wind directions given are the absolute measured wind direction and the wind

<table>
<thead>
<tr>
<th>RUN</th>
<th>Duration (min)</th>
<th>( \Theta / \alpha ) (deg)</th>
<th>( u (\text{m}) )</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>145/35</td>
<td>9.8</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>189/09</td>
<td>7.7</td>
<td>0.11</td>
</tr>
<tr>
<td>3</td>
<td>375</td>
<td>190/10</td>
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<td>244/64</td>
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<td>0.07</td>
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<td>350</td>
<td>341/19</td>
<td>8.7</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 5.5: RUNs selected for evaluation. The records presented in this table have been selected for this evaluation because they are stationary. RUN-2 is especially suited to evaluate the coherence for the larger two separation distances.
direction relative to the North-South line connecting the measurement masts. The wind direction is taken from mast M at 45 m height. The wind speed is the mean wind speed from mast B (15 m) and mast M (10 m). The turbulence intensity is corrected for the effects of the low sampling frequency assuming a Kaimal spectrum [Olesen84]. All selected RUNs have relatively high wind speeds thus giving the expectation of neutral atmospheric stability.

5.3.2 Presentation

To compare the experimental data with the proposed model for coherence over a hill a normalised frequency \( \tilde{n} \) is defined as follows:

\[
\tilde{n} = \frac{1}{s} \text{Ind} \left( (a_2 \sin(\alpha))^2 + \left( \frac{a_1}{u} \cos(\alpha) \right)^2 \right)^{\frac{1}{2}} \tag{5.1}
\]

Plotting the experimental data as a function of \( \tilde{n} \) should collapse it to a simple exponential function \( \gamma^2 = e^{-\tilde{n}} \), which is compared with experimental results. The Nyquist frequency is indicated as \( \tilde{n}_c \).

5.3.3 Longitudinal Coherence data

The two time series, RUN-2 and RUN-3, feature longitudinal wind directions. The relative wind direction is \( \alpha = 9\text{deg} \) for RUN-2 and \( \alpha = 10\text{deg} \) for RUN-3. The lengths of the time series are 33:30 h and 6:15 h, respectively. RUN-2 has been evaluated for separations of 90, 760 and 850 m and the shorter time series, RUN-3, for 90 m. The proposed model eq. 4.13 is, for given decay parameters \( a_1 = 30 \) and \( a_2 = 35 \frac{f}{m} \), compared to the experimental results.

For a separation distance of 90 m, fig. 5.7 shows a comparison between the proposed model and data from RUN-2. The coherence is plotted as function of the normalised frequency \( \tilde{n} \). The experimental data collapse as intended to an exponential function. Very good agreement between the theoretical model and the experimental data is found for this data. The incorporation of the speed up factor has successfully compensated for the increased coherence. Due to the exceptional length of the RUN, even very low frequencies are resolved and these too are fitted well.
For the shorter time series in RUN-3, (fig. 5.8), the standard deviation error bars at 90 m are slightly larger than for RUN-2. The agreement of the model with the experimental data is very good. The slightly higher coherence obtained for RUN-3 may be caused by non-stationarity, non-resolved terrain features or obstacles to the flow in the vicinity of mast M. The data for a longitudinal separation of 90 m supports the proposed model and especially the use of the speed up factor in eq. 4.13.

The coherence function between masts A (15 m) and M (10, 25 m) with separations of 760 m and 850 m have been calculated only for RUN-2. The other RUNs are too short to resolve the coherence function for large separations (or low frequencies). The experimental data from RUN-2 is compared with the proposed model in fig. 5.8 (centre) and fig. 5.8. (lower). The speed up factor is 1.2 for both pairs of reference points. The data for the separations of 760 m and 850 m are not adequate to confirm in detail the influence of the speed up factor on longitudinal coherence. Nevertheless, for these larger separations some agreement with the proposed model is found.

\[ \gamma^2 \]

\[ \begin{align*}
0.01 & \quad 0.1 & \quad 1 & \quad \bar{n}c & \quad 10 \\
0.8 & \quad 0.6 & \quad 0.4 & \quad 0.2 \\
\end{align*} \]

Figure 5.7: Comparison of RUN-2 (longitudinal, \(\alpha = 9\) deg, \(s = 1.3\), \(\bar{n}c = 4.7\)) with the proposed model. For a separation distance of 90 m a very good agreement between experimental data and proposed model is observed.
Figure 5.8: Longitudinal coherence for 90 m, 760 m and 850 m distances. The longitudinal coherence is compared with the model predictions for three different distances. Upper: (RUN-3, \(\alpha = 10^\circ\), \(s = 1.3\), \(\bar{n}_c = 3.8\)), centre: (RUN-2, \(\alpha = 9^\circ\), \(s = 1.2\), \(\bar{n}_c = 40\)), lower: (RUN-2, \(\alpha = 9^\circ\), \(s = 1.3\), \(\bar{n}_c = 93\))
Figure 5.9: Lateral coherence in complex terrain. The coherence a lateral distance of 90 m is in agreement with the model calculations. Upper: (RUN-1, $\alpha = 35$, $s = 1.4$, $\tilde{n}_c = 9.8$), centre: RUN-4, $\alpha = 64$, $s = 1.5$, $\tilde{n}_c = 12$) and lower: (lateral, $\alpha = 67$, $s = 1.5$, $\tilde{n}_c = 19$)
5.3.4 Lateral coherence data

Three different time series, RUN-1, RUN-4 and RUN-5 are used to compare the proposed model with experimental data for directions other than longitudinal. The RUNs available are much shorter than RUN-2 and the model expectations place the coherence at absolute frequencies an order of magnitude lower than for a purely longitudinal case. The experimental resolution of these low frequencies is difficult. So, an increase in standard deviation and bias of the result has to be accepted. For the larger separation distances a proper interpretation of any results becomes very difficult. This analysis considers results with a lateral separation distance of 90 m only. As before $a_1 = 30$ and $a_2 = 35\frac{\alpha}{m}$ are used to calculate the model predictions.

The first time series, RUN-1, has a relative angle of $\alpha = 35\text{deg}$. fig. 5.9 (upper) shows the coherence between points B and M (90 m) for RUN-1. Taking the above mentioned limitations into account, the agreement between experiment and model predictions is quite good.

RUN-4 and RUN-5 have relative wind directions of $\alpha = 64\text{deg}$ and $\alpha = 67\text{deg}$ respectively. The experimental coherence for a distance of 90 m has been evaluated. The results are compared in fig. 5.9 (centre) and fig. 5.9 (lower) with the proposed model. The statistical error involved is high but the data is not contradicting the model predictions.
5.4 Summary - validation of the coherence model

The proposed coherence model (eq. 4.13) has been compared with experimental data, published by other authors and suitable existing raw data, which has been evaluated for the purpose. From this the following conclusions are drawn:

Coherence decay and turbulence It has been shown that scaling Davenport's coherence decay parameter with turbulence intensity is very helpful in collapsing the data for longitudinal as well as for lateral separations. For neutral stability, it is for practical reasons more straightforward to scale the lateral coherence decay parameter with the turbulence intensity, rather than with the quotient of length scales as previously proposed.

Lateral coherence and wind speed The lateral coherence is a function of the turbulence intensity and the separation distance. There is some experimental evidence that the lateral coherence decay should not be regarded a function of the mean wind speed. Further testing of the model should be carried out to confirm that the decay of lateral coherence does not depend on the mean wind speed.

Complex terrain Experimental results have been used to evaluate the model in complex terrain. The coherence model shows agreement with experimental data recorded in complex terrain at Susetter Hill, Shetlands when speed up effects are taken into account. Further research is needed to support the proposed relationship.

Other factors A dependence of the coherence decay parameter on the separation distance has been discounted. Experimental support for the proposed angular dependency of the coherence has been found.

The new coherence model is felt to be adequate to describe of the stochastic relationship between the wind field and the resulting power production for a group of wind turbines. In the next chapters the response of an individual turbine to the turbulent wind field is discussed before combining the effects by individual turbines to a prediction for the flicker from a wind farm using the new coherence model.
Part II

Wind - turbine - grid interaction
Chapter 6

Wind turbine model

The wind turbine blade perceives the stochastic wind field as described by a rotationally sampled spectrum. The turbine is also subjected to deterministic load changes caused by terrain effects, tower shadow and blade imbalances. The dynamic response of the turbine to the incoming fluctuations is described here.

Figure 6.1: Experiment to obtain rotational sampled data. A time series, as seen by a rotating observer, is created by sampling data from each anemometer of the array in rotational sequence 1..12.
6.1 Rotational Sampling

A blade of a wind turbine cutting through the air will experience on its path longitudinal fluctuations in wind speed which are different from those at a stationary point. A time series sampled from an array of anemometers (fig. 6.1) in rotational sequence can be used to experimentally simulate the 'rotational sampling' of the blade.

6.1.1 Models for rotational sampling

The spectrum of power output and blade loads from the experimental Gedser wind turbine was reported in the late 70s to show some reduction of low frequency components (fig. 6.2) and unexpected peaks at distinct frequencies [Verholek78]. These experimental results have in the following years been found to be a result of the interaction between turbine rotor and wind field.

![Graph showing spectrum comparison](image)

**Figure 6.2:** Filter by lack of coherence. Wind spectrum at a single point (upper) and integrated over the rotor circumference using the spatial coherence information (lower)
Filter by lack of coherence

The reduction of low frequency components was explained by [Lundsager80] and is due to the integration over the rotor area performed by the wind turbine blade. It can be modelled for example using a simple first order low pass filter proposed by [Madsen84]. This process is also called a 'filter by lack of coherence' as it is clear that for a fully coherent wind field the frequency spectrum for a rotating observer would be just the same as for a stationary one, fig. 6.2 shows the effect of such a filter.

The frequency modes

The observed increase for specific higher frequency modes is caused by rotational sampling [Connell80]. The rotating blade sees in its Lagrangian co-ordinate system velocity fluctuations at a higher frequency. An early theoretical model developed by [Rosenbrock55] for a rotating observer was in the early 80s combined with more recent models of atmospheric turbulence to provide a new description of the effect of rotational sampling [Holley81, Connell81, Connell82] and [Kristensen82]. The new model was verified by data from a number of atmospheric experiments (LIDAR, beam mounted Hotfilm anemometer, anemometer array). A review of the research carried out can be found in [Powell86a].

Time domain simulation for a B bladed rotor

An improved approach was proposed by [Veers89] calculating the spectra at each point of the rotor before sampling. The advantage being that more than one point (for example for a two bladed rotor) can be considered at the same time. This time domain model and especially the implementation of turbulence and coherence functions has been discussed in a number of publications [Winkelaar91, Kretz94, Sørensen94, Petersen94]. The step from the frequency spectrum to a time domain simulation (and back) is quite time consuming. Models based in the frequency domain are here the preferred choice.
6.1.2 Models in the frequency domain

The above mentioned studies use a time domain simulation of a rotating blade to obtain an appropriate wind time series. This time series is then used as input to model for example blade loads in wind turbines. In the context of the work described here it is desirable to use models which are based in the frequency domain [Dragt84, Sørensen95b]. This avoids the time consuming change of domains and gives easier access to the physics determining the process. The model used here is described in [Dragt84, Dragt85].

Wind for a rotational observer

For a given wind spectrum $S$, the spectrum $\tilde{S}$ seen by the rotational observer, rotating at a frequency $n_0$, is described by [Dragt85] as Fourier series over $M$ rotational modes

$$\tilde{S} = \sum_{M=-\infty}^{+\infty} k_M (n - M n_0) S(n - M n_0)$$  \hspace{1cm} (6.1)

![Figure 6.3: The rotational observer. The distance between a fixed point (spectrum $S$) and the rotating observer (spectrum $\tilde{S}$) is denoted $d(\psi)$.](image)
with the Fourier coefficients $k_M$:

$$k_M(n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \gamma^2 (d(\beta), n) \cos(M\beta) d\beta$$

(6.2)

where $\beta$ is the angle between observer and the reference position and the distance $d(\beta)$ of a fixed point to a rotating point at a radius $r$ is given by (fig. 6.3):

$$d(\beta) = 2r \left| \sin\left(\frac{1}{2} \beta \right) \right|$$

The spectrum $\tilde{S}$ is the spectrum as seen by an observer rotating with a distance $r$ from the reference point. The so obtained rotationally sampled spectrum is presented in fig. 6.4 together with the stationary spectrum (see Appendix B for presentation of spectra). Turbulent energy is shifted from low frequencies to higher frequency modes, at multiples of the rotational frequency. In the centre of the first rotational mode a very narrow frequency dip, which is characteristic for the Dragt model can be identified if a high frequency resolution is used. This dip has been discussed in [Dragt85] and is a consequence of the mathematical structure of the model but is not found in measurements.

Figure 6.4: Spectrum for a rotational observer. Wind spectrum at a fixed point and as seen by a rotational observer
6.1.3 Shaft power spectrum

The variance of the turbine power and its spectral distribution is needed for a description of the power fluctuations in the frequency domain. This section is concerned only with the spectral distribution of the power fluctuations not with its absolute value. First the relationship between the wind spectrum and the shaft power spectrum is investigated in this section, in a later section the impact of the power train will be studied.

The rotationally sampled spectrum \( \tilde{S} \) represents the impact of the wind fluctuations at one point of the rotor. To obtain the spectral contributions of the wind fluctuations to the total shaft torque the spectra from B blades has to be combined and integrated over the rotor radius.

Adding the spectrum of B blades

For rotating observers on B Blades, with a phase shift due to the rotor position the rotational sampled spectrum is after [Dragt85] in arbitrary units:

\[
\tilde{S}' = B^2 \sum_{M=0, \pm B, \pm 2B, \ldots} k_m (n - Mn_0) S(n - Mn_0)
\]

(6.3)

For a three bladed rotor only contributions for the basic and 3p, 6p ... modes remain. The signal generated from the assumed B observers is not fully coherent. The coherence between the observer on any pair of two blades is \( \gamma_{12}^2 \). The observed spectrum \( \tilde{S}' \) is identical at each blade. The combined spectrum \( \tilde{S}'' \) for a three bladed rotor is then:

\[
\tilde{S}'' = \left( B + \gamma_{12} (B^2 - B) \right) \tilde{S}'
\]

(6.4)
Figure 6.5: Shaft torque spectrum. Shaft torque spectrum derived from the rotational sampled wind spectrum with deterministic components added at one and three times the rotational frequency. The spectrum is for a 3 bladed turbine with 39 m diameter at 7 m/s wind speed and 12% turbulence intensity.

Adding torque from blade segments

To obtain the torque contributions from the whole rotor we have to integrate the spectrum $\tilde{S}_i''$ at distance from the shaft $r_i$ over $N$ segments, the coherence between the blade segment and the rotor centre is called $\gamma_{\omega}$:

$$\tilde{T} = \frac{1}{N} r_i \sum_{i}^{N} \tilde{S}_i '' \gamma_{\omega}$$  \hspace{1cm} (6.5)

An example for a shaft torque spectrum $\tilde{T}$ obtained with eq. 6.5 is presented in fig. 6.5. The forces experienced by a blade along its radius are not coherent for higher frequencies, thus the contribution of higher rotational modes is reduced. This is especially true for large size rotors.
Shaft power

The shaft torque is multiplied by the rotational frequency of the turbine to obtain the shaft power. This rotational frequency is either constant, for a fixed speed turbine, or a function of the wind speed for a 2 speed turbine. The constant rotational speed does not influence the normalised spectrum of fluctuations. The shape of the normalised shaft power spectrum is identical to that of the normalised shaft torque spectrum. Power fluctuations caused by variable speed turbines need to be studied separately and are not considered in this thesis.

6.1.4 Deterministic components

Deterministic components of the power output signal of a wind turbine are generated by elements which break the rotational symmetry of the applied wind turbine model. These signals are a function of the blade positions. The main factors leading to deterministic signals are the influence of the tower, ground effects, yaw misalignment and rotor imbalances.

Experimental observations

The following observations of deterministic effects have been reported in the literature. In some other publications deterministic effects might have been observed but often no distinction is made between peaks caused by rotational sampling and deterministic peaks.

Mod-0: Strong mechanical vibrations caused by the tower shadow have been observed in various downwind turbines. In [Twidell94] the severe tower shadow of the MOD-0 turbine is described. The 2 bladed downwind turbine caused infrasonic vibrations in its neighbourhood.
WT model: The effect of the tower shadow on a wind turbine model for upwind and downwind turbines has been analysed in the wind tunnel by [Graham99].

180 kW stall: Deterministic peaks in the spectrum of output power at 1p and 3p frequencies have been identified by [Thiringer98].

ECN Research: A small deterministic effect is reported from experiments at an experimental two bladed 25 m wind turbine [Dragt84, Dragt86]. The deterministic component reported is less than 1% of the stochastic component.

Vestas 500 kW: The measurement and simulation of two Vestas 39/500 turbines presented in [Thomsen96b, Thomsen96a] show only small deterministic components at 1p in the power output signal. The observed deterministic components of the power output spectrum are limited to very narrow frequency bands. Only in some reported cases does the amplitude of the deterministic peaks reach the order of magnitude of the stochastic signal.

Modelling the deterministic signal

Deterministic signals are generated at multiples of the rotational frequency. They are limited to these frequencies but the peaks are widened by dynamic coupling with the wind turbine structure, especially the wind turbine blades. Yaw misalignment and teetering for a two bladed turbine do also contribute to the widening of the frequency bands.

Amplitude of the spikes: The deterministic components have been modelled as spikes in the torque spectrum. The amplitude of the spikes is estimated following suggestions [Leithead91, Leithead92]. The contribution of the deterministic component is split 1:9 between 1p and 3p and the deterministic 3p component is 1:5 of the rotational sampled component at 3p. The definition of the amplitude of the deterministic signal via the size of the stochastic 3p component implies a functional relationship which might only be adequate for some special cases.
Width of the spikes: The width of the deterministic components is assumed here to be the same as for the rotational sampled components. This is to account for the widening of the spikes due to the effects considered above. The resulting spectrum for a rotational observer and for the shaft torque are presented in fig. 6.5. The power output spectrum obtained using the described method has been compared with published data and seems to be in qualitative agreement with experimental results. From the limited experimental evidence available the assumptions made for the width of the spikes are possibly somewhat conservative and overestimate the impact of deterministic disturbances.

Further research: Further research is needed to improve the modelling of the deterministic components. When improved models for effects like tower shadow, rotor imbalances or yaw misalignment become available these should be used to refine the assumptions made.
6.2 Power train

The effect of rotational sampling in conjunction with a simple drive train model has been studied by [Taylor87a]. Other authors have considered more detailed analysis or expanded models to take into account wind shear or to describe vertical axis wind turbines [Powell86b].

The combination of low speed shaft, gearbox, high speed shaft and generator is called the power train. The power train of a number of turbines has been investigated with respect to their power train characteristic [Leithead91, Leithead92, Leithead93, Leithead96a, Leithead96b]. The results from this investigation allowed the description of a 'typical' turbine and power train. The turbine described is a fictitious turbine, its parameters have been derived from a number of distinct turbines. They are considered to be typical for commercial fixed speed wind turbines. Higher order modelling of for example the generator does only affect frequencies above the range of interest, compare for example [Chedid93]

6.2.1 Combined drive train and Generator

The dynamics of the drive train can be approximated with a transfer function which is often given in Laplace s-space representation [Leithead92, Leithead96b]:

\[ G(s) = \frac{C_0}{C_5 s^4 + C_4 s^3 + C_3 s^2 + C_2 s + C_1} \]

with the parameters \(C_0-5\) being parameters specific to the selected turbine. The six free parameters in the equation eq. 6.6 are determined by the inertia of rotor and generator, damping constants, stiffness of high speed and low speed shaft and the slope of the generator torque/speed curve. Additionally the gearbox ratio has to be taken into account. A power train transfer function (in Laplace notation) for a realistic, but fictitious, turbine would be [Leithead92]:

\[ G(s) = \frac{2123.383}{s^4 + 33.39 s^3 + 7566.13 s^2 + 6421.30 + 80900.23} \]

Of the above mentioned parameters only the gear box ratio is publicly available for most commercial wind turbines. The detailed transfer function eq. 6.6 for a given turbine cannot be determined by a wind farm developer. The function is essential and has to be approximated for a given turbine using a minimal number of (accessible) parameters.
6.2.2 Idealised power train transfer function

The eq. 6.6 represents a system with two resonance frequencies and two damping constants. The first resonance frequency $\omega_1$ with damping constant $\eta_1$ is attributed to the rotor and low speed drive train while the second $\omega_2, \eta_2$ represents the generator and high speed drive train [Leithead92, Leithead96b]:

$$G(s) = \frac{\omega_1^2 \eta_2}{N(s^2 + 2\eta_1 \omega_1 s + \omega_1^2)(s^2 + 2\eta_2 \omega_2 s + \omega_2^2)}$$

The gearbox ratio $N$ is of no further consequence here as eq. 6.8 is only used to determine relative contributions to the variance spectrum of power train torque. Instead of using the set of power train parameters described above we can assume 'reasonable' values for the power train resonance frequencies and damping factors in eq. 6.8. Reasonable means here that the turbine has been designed to the criteria outlined in [Leithead93, Leithead96a, Leithead96b].

The value for the first power train resonance is of special importance here. A design aim for the first resonance frequency for a turbine is to reduce torque transients induced by fast disturbances in the drive train. This leads to the

![Figure 6.6: Power train transfer function.](image)

Figure 6.6: Power train transfer function. Example for the power train transfer function using a 'sensible turbine design'.

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following assumptions for a well designed first drive train mode of a B bladed turbine with a rotational frequency $\omega_r$:

$$\omega_1 \approx \frac{1}{2} B \omega_r \quad \text{and} \quad \eta_1 \gg \frac{1}{2}$$

The values for $\eta_2$ and $\omega_2$ depend on the specific turbine design, however for this study we use $\eta_2 = 0.15$ and $\omega_2 = 20 \cdot \omega_1$ as reasonable values. The transfer function of such a power train is presented in fig. 6.6.

The resulting normalised power output spectrum $S_p$ is given in fig. 6.7. The low frequency contributions, the deterministic peak at $1p$ (the rotational frequency) and at $3p$ (three times the rotational frequency) introduced by rotational sampling are dominant. Frequencies above $3p$ are damped due to the applied power train transfer function.

![spectrum of power output](image)

**Figure 6.7: Spectrum of power output.** Visible are the slow fluctuations, 1p and 3p components. Frequencies above 3p are damped due to the applied power train transfer function.
6.2.3 Control specific properties

The basic power train design is now combined with elements specific to the individual turbines under consideration. If available, turbine specific parameters should be used to specify the power train transfer function in more detail. Of the three main turbine concepts, pitch control, stall control and variable speed, only pitch and stall control are considered in this thesis.

Stall controlled turbine

The basic power train design is used for stall regulated turbines without any additional modifications.

Pitch controlled turbine

Any change of the pitch angle by the pitch actuator will lead to a new flow field around the turbine. The wind field will not adjust instantly to the new conditions but with a time delay. Due to this 'induction lag' between wind flow and pitch angle, increased forces have been observed on the NIBE turbines.

A simple Induction Lag filter is suggested in [Leithead92] following measurements by [Stig-Oye86] on the NIBE turbines:

\[ IL = \frac{1 + 11.25s}{1 + 7.5s} \]

A more generalised formulation for this induction lag filter is suggested here:

\[ IL = \frac{1 + t_1 s}{1 + t_2 s} \]  \hspace{1cm} (6.9)

with: \( t_2 = 2 \cdot \frac{D}{u} \) proportional to the wake core length (the time in which the old wake is replaced) and \( t_1 = t_2 \cdot 1.5 \cdot \frac{D}{50} \) proportional to the width of the wake. High frequency contributions in the spectrum are amplified relative to the low frequency content of the spectrum. A transfer function for the induction lag is shown in fig. 6.8. A similar generalisation and experimental data are published in [Snel91, Snel95, Schepers95]. The suggested generalisation extends the experimental results from NIBE to other turbine types. Further experimental data is needed to confirm the magnitude of the induction lag effect.
It was important for this thesis to take the induction lag into account as it amplifies the flicker for one specific wind turbine type, the pitch regulated turbines. It has to be stressed however that models and model verification are not well advanced and more research - which is outside the scope of this thesis - is needed in this area to further the understanding of the induction lag effect.

![Induction lag filter](image)

**Figure 6.8: Induction lag filter.** Temporal misalignments between the wind flow and the pitch angle do increase the torque fluctuations for a pitch controlled turbine.
6.3 Linearisation

The wind field is described by its mean wind speed, spectrum and spatial coherence. For each (stationary) mean wind speed a mean power output is given by the power curve. To connect the fluctuations in power output with the fluctuations in the wind field an approximate expression for small disturbances is used.

6.3.1 Linearisation via sensitivity coefficients

The relationship between shaft torque and wind speed can be approximated by a linear relation, such that for given mean conditions (denoted with the index \(i\)) of wind speed, rotational speed etc. variations in wind speed \(\delta u\) are translated into variations of torque \(\delta T\):

\[
\delta T = g_i \delta u
\]

The coefficients \(g_i\) can be calculated for a known blade symmetry at each point of the rotor with a blade element code and stored in look up tables [Leithead91]. This type of linearisation is not realistic under the given constraints for this work as it requires the precise knowledge of blade geometry and properties which are generally not publicly available.

6.3.2 Linearisation via power output

The linearisation via the power output can be based on the turbine power curve and does not require any additional knowledge about the turbine design. This method of linearisation via the power curve is chosen in this thesis. The linearisation has been applied to slow fluctuations and in a slightly modified way also to fast fluctuations.
Linearisation for low frequencies

A local linear relationship between small fluctuations of power and wind speed around their respective mean values $P$ and $u$ is assumed (see [Albers95] for an alternative formulation).

\[
\delta_p = \left. \frac{\partial P}{\partial u} \right|_u \delta_u \quad (6.10)
\]

Using the slope of the power curve $\frac{\partial P}{\partial u}$ implies that for every fluctuation the turbine power output follows the fluctuation in the wind speed. The power limiting control of the wind turbine (stall or pitch) will for high wind speeds reduce the slope of the power curve and can thus nearly eliminate the impact of wind fluctuations on the power output. This is true for sufficiently slow changes which are represented by low frequencies in the variance spectrum. In practice though low frequency fluctuations remain, which would indicate the need for $\frac{\partial P}{\partial u}$ to be non-zero although small. The value of this non-zero offset needs to be subject of further investigation. It is approximated by zero in this thesis.

The linearisation via power output has been applied in this thesis for low frequencies of the variance spectrum. For changes faster than $1/2$ of the rotational frequency a modified approach has been chosen.

Linearisation for higher frequencies

For higher frequencies a constant slope $\frac{\partial P}{\partial u}$ is assumed. This slope is obtained as the maximum slope of the stationary power curve:

\[
\delta_p = \max \left( \frac{\partial P}{\partial u} \right) \delta_u \quad (6.11)
\]

This reflects that high frequency fluctuations can not be reduced by the power limiting control mechanism of the wind turbine. The high frequency components of the variance spectrum result in significant power fluctuations below as well as above rated power.
6.4 Summary - wind turbine model

The wind turbine model used in this thesis consists of the following components.

**Rotational sampling**

FD model of the wind a rotational wind turbine blade will experience (eq. 6.1). The spectrum is integrated over the rotor area yielding a shaft torque spectrum.

**Deterministic component**

Assumptions for the magnitude of deterministic effects have been made.

**Power train**

A power train is modelled with a simplified representation of its transfer function. This has been achieved by assuming a representative 'reasonable' design (eq. 6.8).

**Control**

The induction lag of pitch regulated turbines is modelled using a generalised induction lag filter (eq. 6.9). Variable speed turbines are treated as pitch regulated turbines but with all high frequency fluctuations eliminated.

**Linearisation**

The linearisation via the power curve supplies the necessary connection between power and wind speed fluctuations (eq. 6.10 and eq. 6.11).

Some simplifications had to be introduced for the purpose of this thesis. The turbine model does for example not consider the modification of the transfer function due to the tower dynamics, pitch control dynamics or their interaction. The implemented model uses for practical reasons the smallest possible number of parameters to model the turbine.

The wind turbine model converts the description of mean wind speed and stochastic wind field into mean power and power fluctuations ($P_{W}$).

\[
P_{st}^2 = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) \, dn
\]

The next two chapters will look at the voltage fluctuations and flicker resulting from the power fluctuations derived in this chapter.
Chapter 7

Flicker measurement

Voltage fluctuations lead to changes in the light intensity (flicker) of an incandescent light bulb. The nature of these disturbances can lead to significant discomfort in people. Rectangular voltage changes of equal flicker severity ($P_{st} = 1$) are shown in fig. 7.1 as function of the repetition rate [IEC91].

![Fig. 7.1: Points of equal flicker severity. IEC-curve for rectangular voltage changes which correspond to the same level of human annoyance as function of the repetition rate.](image)

Figure 7.1: Points of equal flicker severity. IEC-curve for rectangular voltage changes which correspond to the same level of human annoyance as function of the repetition rate.
7.1 Flicker severity

Guidelines exist describing the impact of flicker on humans and means to measure this effect for a rational assessment. This specifies limits to be set and measures to be implemented to reduce the voltage variations to an acceptable level.

7.1.1 Flicker perception

People will perceive the flicker at a certain level of relative voltage changes, but tend to complain however, only when the disturbance is sufficiently persistent or severe. The disturbance from flickering light bulbs is a function of the amplitude, frequency and wave-form of the disturbances. The disturbance from non sinusoidal wave-forms has been analysed in [Mombauer87].

7.1.2 International limits

Flicker severity has been studied in laboratory experiments and by evaluation of the number of complaints received by utilities. Based on these experiments, limit curves have been derived, defining a level of disturbance which is acceptable to a sufficiently large number of customers. A number of examples for such limit curves, applied by utilities are compared in [Walker79].

The current standard relationship between rectangular voltage changes of a certain repetition rate resulting in unity flicker severity is given in [IEC91], fig. 7.1. The standard follows the definition given by the International Union for Electroheat (UIE) [Mirra88] and is referenced by the International Electrotechnical Commission (IEC) standards on Electromagnetic Compatibility (EMC) [IEC94] and more relevant for commercial wind turbines in [IEC96a] and [IEC96b]. The flicker severity limit is defined for repetition rates from $10^{-3}$ Hz to 24 Hz with a maximum at 17 Hz (corresponding frequencies are given by repetition rate divided by 2).
7.1.3 Flicker measurement

Flicker measurement on wind turbines is problematic as other sources of flicker can mask the flicker produced by a wind turbine. For example [Craig95b] reports the impact of an operating quarry masking during weekdays the flicker from a wind turbine.

The measurement of the 'voltage' flicker, i.e. the fluctuation of voltage seen by a load connected to the point of common connection (PCC) is not performed by direct measurement of the voltage as the relative changes in voltage are very small. Instead the currents injected by the wind turbine are measured. The current fluctuations take place relative to a cleaned, idealised and simulated reference voltage signal. This simulated voltage signal is used to avoid pollution of the measurement from fluctuations which are not caused by the wind turbine. The measured current flicker is then converted to an equivalent voltage flicker and evaluated by a flickermeter.

The problems associated with external sources of voltage fluctuation have no bearing on the prediction methodology developed in this thesis, details can be found in [Hansen96] and [Ehlers95].

7.2 The flickermeter

A Flickermeter is a specialised instrument to measure the severity of voltage fluctuations. Various implementations exist, which have been built to conform with the requirements for flickermeters laid out in the current IEC standard.

7.2.1 Standard Flickermeter

Several pre-standard flickermeter designs are presented in [Lavers86] and compared with the flickermeter of the UIE [Mirra88]. The current IEC standard for flicker severity and flickermeter design [IEC90] (replacing [IEC86]) is based on the UIE flickermeter. The IEC standard for the flickermeter has been implemented as a European and British standard. The IEC standard flickermeter is recommended for inclusion into the IEEE recommended practices [IIalpin99].
7.2.2 Flickermeter specifications

The requirements of the IEC standard [IEC90] flickermeter are to reproduce the flicker severity with better than 5 % accuracy over the whole definition range. To define the flickermeter response, a number of test signals and the required response of the instrument are specified. A test signal for the flickermeter can be described by a carrier signal and a disturbance. A carrier signal with amplitude $a_c$ and frequency $\omega_c$ is disturbed by a rectangular signal with amplitude $a_m$ and frequency $\omega_m$:

\[ Y(t) = a_c \sin (\omega_c t) \left[ 1 + a_m \text{sign} (\sin (\omega_m t)) \right] \]  

(7.1)

Using the tabulated relative voltage changes of fig. 7.1 the rectangular test signals should yield $P_{st} = 1$ for all frequencies with no more than 5 % deviation allowed.

7.2.3 Linearity

The specification for the standard flickermeter requires linearity: The flicker severity increases linearly with the relative voltage changes $\frac{\Delta V}{V_0}$, or if the voltage changes increase by a factor $\xi$ the flicker severity increases by the same factor:

\[ \frac{\Delta V'}{V_0} = \xi \frac{\Delta V}{V_0} \Rightarrow P'_{st} = \xi P_{st} \]  

(7.2)

This linearity condition is required for a wide range of voltage changes and over the complete frequency interval.

7.2.4 Implementation

The flickermeter specifications do not prescribe the method of implementation. The standard contains implicit references to a variety of possible implementations. These options can be divided into analog and digital implementations. The digital implementation can either be an algorithm in time domain or an implementation in the frequency domain. Flicker measurements at wind turbines to date have been based on digital, time domain based flickermeters.
7.3 Digital flickermeter

The standard for flickermeters suggests a number of modular components for the design of a (time domain) flickermeter in order to meet the requirements of the standard.

7.3.1 Suggested standard implementation

The flickermeter assumes that a sinusoidal signal is polluted with relatively small voltage fluctuations (sec. 7.2.2). The analysis of this pollution is performed by first eliminating the carrier signal with a demodulator, weighting the remainder according to the human perceptibility to such fluctuations, modelling the transfer function of the human brain and finally calculating the flicker severity from the weighted signal.

Demodulation

The signal is amplified to a reference level at which the carrier frequency and the disturbance are separated by a demodulator. A square demodulator has been suggested in the standard. The demodulation is achieved by squaring the signal and subsequent band pass filtering. For a detailed description of the square demodulator and an alternative demodulator see [Ehlers95].

The output of the demodulator is, for small voltage changes, a linear function of the amplitude of the input voltage changes. The band-pass filter suggested in the standard to demodulate the signal consists of one 6th order low-pass filter at 35 Hz together with a 1st order high-pass filter at 0.05 Hz eliminates both, DC and high frequency components.

Weighting function

The norm [IEC90] describes the flicker perceptibility to rectangular voltage changes (fig. 7.1). It also describes a weighting function (fig. 7.3) based on a table of sinusoidal test signals. These are given for frequencies between 0.5 Hz to 25 Hz, a transfer function is suggested which approximates the tabled values:
\[
F_\omega(s) = \frac{\lambda_1 \omega_1 s}{s^2 + 2\lambda_2 s + \omega_1^2 (1 + \frac{s}{\omega_2})(1 + \frac{s}{\omega_4})} \frac{1 + \frac{s}{\omega_2}}{1 + \frac{s}{\omega_4}}
\]

\[
\lambda_1 = 1.74802 \quad \omega_1 = 2\pi 9.15494 \quad \omega_2 = 2\pi 2.27979
\]
\[
\lambda_2 = 2\pi 4.05981 \quad \omega_3 = 2\pi 1.22535 \quad \omega_4 = 2\pi 21.9
\]

Using this transfer function reproduces results between 1 Hz and 24 Hz with satisfactory accuracy. In order to cover lower frequencies at which larger voltage fluctuations are required to reach flicker perceptibility, the flickermeter specifications include a range selector. The standard does not specify the transfer function in the low frequency range.

**Human response**

The human response to voltage fluctuations is modelled by squaring the signal and the subsequent application of a first order lowpass with a time constant of 300ms. This is in the frequency domain equivalent to an integration over the high frequency part of the spectrum. And the result can be interpreted as the variance of high frequency components.

The resulting signal is called the instantaneous flicker sensation \(P_{ft}\) (not to be confused with the flicker severity \(P_{st}\)). For the low frequency variations which are present in this signal a statistical evaluation is performed. The flicker severity \(P_{st}\) is calculated as the square root of a percentile-weighted mean value of the distribution of sensation levels. This is roughly equivalent to the standard deviation of the distribution or the square root of the integral of the voltage spectrum over all frequencies.

**7.3.2 Time domain flickermeter**

The suggestions made in the standard have been used in a number of implementations, for example in the algorithm developed in [Hansen96] and [Ehlers95] in co-operation with WINDTEST, Kaiser Wilhelm Koog, Germany. Other implementations are described in [Mirra88] or are applied for example in [Sørensen96b].
Test implementation

For this thesis a digital, time domain flickermeter was programmed using the description of [Hansen96]. A good agreement with the standard was achieved for test frequencies ranging from 3 to 20 Hz. The weighting filter uses the values tabled in the IEC standard for 0.5 to 20 Hz (fig. 7.3) for the response to sinusoidal high frequency disturbances. Flickermeters based on the above mentioned table alone, do not meet the standard requirements at lower frequencies \((n < 1Hz)\).

The low frequency behaviour of the time domain implementation of the digital flickermeter can be improved by using values from an expanded table for response to sinusoidal disturbances. These have been used to derive for each frequency decade a separate transfer function. The digital flickermeter with range selector has been tested and was found to give sufficiently accurate readings for low \((n > 0.01Hz)\) and high frequencies \((n < 25Hz)\).

The digital flickermeter can be used to assess voltage flicker related to wind turbines and wind farms if the low frequency response is modelled accurately. A more convenient way however is to use a standard flickermeter which works exclusively in the frequency domain.

Band-width limitation

Published measurements have sometimes been carried out with limited bandwidth. This limitation in bandwidth can be caused by:

- A measurement period smaller than the specified 10 minutes has been chosen, this procedure is equivalent to introducing a high pass filter which eliminates some or all the effects of low frequency fluctuations [Weinel90, Möller94, Gerdes96].

- The implementation of the IEC flickermeter algorithm is inadequate and does not consider voltage fluctuations below 1 Hz. This is not helped by the fact that the prescribed test for the flickermeter only features one frequency below 1 Hz at 1/60 Hz, and tables of sinusoidal test signals in the standard are only provided for 0.5 to 25 Hz in [IEC90]. More information on the required low frequency characteristics is only available in [IEC91].
Figure 7.2: Voltage spectrum and Flicker spectrum with limited and full bandwidth. Voltage spectrum leading to flicker (upper). Limited bandwidth flickermeter ($P_{st} = 0.035$, centre) and full bandwidth flickermeter ($P_{st} = 0.05$, lower).
To model the characteristics of a limited bandwidth flicker assessment a flickermeter has been used which operates only on the frequency band 0.1 Hz to 25 Hz. The flicker spectrum of limited bandwidth and full bandwidth calculations are compared in fig. 7.2 (the presentation of spectra is described in Appendix B). The total flicker \( P_{st} \) is equivalent to the integral over the spectra shown. The limited bandwidth flicker spectrum shows contributions to the total flicker only for frequencies created by rotational sampling and deterministic effects. The spectrum from the full bandwidth flickermeter reveals the significance of the low frequency components.

**7.3.3 Frequency domain flickermeter**

An alternative flickermeter which operates entirely in the frequency domain (FD-flickermeter) has been developed. The possibility of such a design is mentioned in the standard, also been suggested in [Srinivasan91] and partially implemented by [Chen97] (0.1 .. 25 Hz). The design is rather straightforward and valid provided

![Unity Flicker Severity for Sine Waves](image)

**Figure 7.3: FD-flickermeter response to sinusoidal voltage variations.**

The relative, sinusoidal voltage changes which lead to \( P_{st} = 1 \) are compared with the values suggested for the UIE flickermeter for the high frequency band.
the flicker severity $P_{st}$ is a linear function of the voltage changes. The linear relationship is part of the requirements laid out in the IEC standard and thus assumed valid.

Assuming that $S_v(n)$ represents the spectrum of voltage fluctuations and with the above assumption of linearity (sec. 7.2.3), $G_v(n)$ can be defined so that:

$$P_{st}^2 = \int_0^\infty S(n) \circ G_v(n) dn$$

(7.3)

The function $G_v(n)$ (tab. 7.1, fig. 7.3) has been found by an iterative process. This function is constructed so that the flickermeter response matches the functional description of the standard flickermeter.

<table>
<thead>
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<th>$Hz$</th>
<th>$\frac{AV}{V}$</th>
<th>$Hz$</th>
<th>$\frac{AV}{V}$</th>
<th>$Hz$</th>
<th>$\frac{AV}{V}$</th>
<th>$Hz$</th>
<th>$\frac{AV}{V}$</th>
<th>$Hz$</th>
<th>$\frac{AV}{V}$</th>
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</thead>
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<td>3.05</td>
<td>0.2</td>
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<td>0.545</td>
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<td>0.3</td>
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<td>10.5</td>
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<td>2.5</td>
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<td>25.0</td>
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<td>1.25</td>
<td>3.0</td>
<td>0.605</td>
<td></td>
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</tbody>
</table>

Table 7.1: The required frequency response of the FD-flickermeter. Frequency response of the flickermeter for the full frequency range. Using these values for the flickermeter design reproduces the functionality of the standard flickermeter.
The FD-flickermeter has been tested with the rectangular voltage changes leading to unity flicker (fig. 7.1) which are tabled in the standard. The error of the FD-flickermeter is shown in fig. 7.4. It is well below the required 5%.

Figure 7.4: FD-flickermeter response. Flickermeter response to rectangular voltage fluctuations which should give $P_t = 1$. The flickermeter response is very accurate obtaining unity flicker values for the range of interest between 0.001 Hz and 25 Hz.
7.4 Summary – flicker measurement

This chapter has discussed the relationship between voltage fluctuations, flicker severity and flicker. This relationship ($H_{VF}$) is evaluated by a flickermeter. Some problems with current implementations of the standard flickermeter have been pointed out. They are mainly caused by the lack of a defined transfer function in the frequencies below 1 Hz. The necessity of a range selector and different specifications for the lower frequencies is hinted at but not clearly prescribed in the standard. A standard flickermeter has been implemented and tested to be accurate from 0.01 Hz to 25 Hz.

A new algorithm for the evaluation of voltage fluctuations has been proposed. This new FD-flickermeter is faster and better suited for the purpose of flicker prediction. The FD-flickermeter meets the requirements for a standard flickermeter. The new FD flickermeter performs accurately over the whole frequency range. The new flickermeter avoids time consuming conversion of data between the time and frequency domain. It allows a direct calculation of a flicker spectrum from the spectrum of voltage fluctuations. The flicker spectrum can then be integrated to obtain the flicker short term value $P_{st}$.

$$P_{st}^2 = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) \, dn$$

The flickermeter needs as input the spectrum of voltage fluctuations. How this is obtained from the spectrum of power fluctuations (chap. 6) will be discussed in the next chapter together with alternative, semiempirical methods of to predict a value for the flicker severity.
Chapter 8

Prediction of flicker

It is desirable to be able to predict the flicker caused by voltage fluctuations in the design phase of a wind turbine development. Such a prediction can be based on experimental data gained from measurements on other wind turbines. A number of such empirical and semi-empirical methods have been proposed. Based on a discussion of these proposals a new prediction method for voltage fluctuations is suggested which allows an accurate prediction of the flicker for a new wind turbine development.

![Diagram](image)

Figure 8.1: Equivalent single phase diagram. The wind turbine (WTG) is represented by a source of active and reactive power connected to the distribution network with reference impedance $Z_{sc}$, after [Ballard84].
8.1 Network representation

The electric coupling of wind turbine generator and distribution network is best represented by an equivalent circuit like in fig. 8.1. A wind turbine generator is connected via a reference impedance $Z_{sc}$ to the point of common coupling (Pcc). The impedance $Z_{sc}$ represents the combined impedances of the generator and the network impedance. The flicker is analysed at the Pcc where a customer might be connected, experiencing the voltage fluctuations caused by the wind turbine.

8.1.1 Exact solution

The voltage at PCC derived from this formulation is given by the solution of the quadratic equation [Ballard84]:

$$V^4 + V^2 \left(2(QX_{sc} - PR_{sc}) - V_0^2\right) + (QX_{sc} - PR_{sc})^2 + (PX_{sc} + QR_{sc})^2 = 0 \quad (8.1)$$

with:

- $V_0$: nominal source line voltage
- $P$: active power
- $Q$: reactive power
- $V$: $V_0 + \Delta V$, line voltage at the PCC

The quadratic equation eq. 8.1 gives one real solution [Larsson97, Lorenzo97]. An example for the sensitivity of the voltage changes to the generated power is given in [Ballard84], fig. 8.2:

$$V = \sqrt{a + \sqrt{a^2 - b}}$$

$$a = \frac{V_0^2}{2} - (R_{sc}P - X_{sc}Q)$$

$$b = (P^2 + Q^2)Z_{sc}^2 \quad (8.2)$$

The sensitivity $c_e(P)$ of voltage variations $V$ resulting from small relative changes to the power in eq. 8.2 at a given power level can be calculated for any known $Q(P)$ characteristic for a given network. The relative voltage variation is then:

$$\frac{\Delta V}{V_0} = c_e(P) \frac{\Delta P}{P_{sc}} \quad (8.3)$$
8.1.2 Approximate solution

The exact solution of eq. 8.1 requires knowledge of the Q(P) characteristic of the wind turbine generator. This information is not generally available, so that it becomes necessary to use an approximate solution instead. The error made in using an approximation to eq. 8.1 is discussed in [Bossanyi97]. The following approximation is quoted by [VDEW92], [Klosse97a], [Sørensen96b] and [Ehlers95] for the calculation of wind turbine flicker:

\[ V - V_0 = \frac{R_{sc} P}{V_0} + \frac{X_{sc} Q}{V_0} \]  \hspace{1cm} (8.4)

This approximation neglects the change of the phase angle due to the connection of the wind turbine. From eq. 8.4 follows:
\[
\frac{dV}{dP} = \frac{R_{sc}}{V_0} + \frac{X_{sc}}{V_0} \frac{dQ}{dP}
\] (8.5)

The resistance \( R_{sc} \) and reactance \( X_{sc} \) can be expressed in term of the absolute value of the impedance \( Z_{sc} \) and the phase angle \( \psi \) by: \( X_{sc} = |Z_{sc}| \sin(\psi) \) and \( R_{sc} = |Z_{sc}| \cos(\psi) \). Reordering eq. 8.5 and introducing the impedance yields:

\[
\frac{1}{V_0} \frac{dV}{dP} = \frac{\cos(\psi)}{P_{sc}} + \frac{\sin(\psi)}{P_{sc}} \frac{dQ}{dP}
\] (8.6)

The term \( \frac{dQ}{dP} \) is given by the active power to reactive power slope which is a characteristic of the generator used.

### 8.2 Induction generator characteristic

The dynamic properties of the generator have been modelled as part of the power train. The voltage fluctuation on a network also depends on the electrical properties of the generator and the properties of the network the generator is connected

![Figure 8.3: Characteristic dependence between active and reactive power for an induction generator. The function can be approximated by a second order polynomial or in the region of \( \frac{P}{P_{rated}} > 0.5 \) by a linear relationship. (from [Klosse97b])](image)
to. The generator is best described by the characteristic function of active power versus reactive power.

Many wind turbines use induction generators. A characteristic functional relation \( P(Q) \) between active (P) and reactive power (Q) can be found for these turbines. The characteristic can be approximated by a first or second order polynomial.

### 8.2.1 Second order polynomial

A second order polynomial is used for example in [Sørensen95a] with the two parameters \( \zeta_1 \) and \( \zeta_2 \):

\[
Q = \zeta_1 + \zeta_2 P^2
\]  

(8.7)

Assuming capacitive compensation reduces \( \zeta_1 \) to zero, the derivative is:

\[
\frac{dQ}{dP} = 2\zeta_2 P
\]  

(8.8)

### 8.2.2 First order polynomial

For a limited region \( (P/P_{\text{nominal}} = 0.5..1.0) \) it is also possible to approximate the characteristic with a simpler first order polynomial with parameters \( \zeta_3 \) and \( \zeta_4 \), the 'VDEW approximation', [VDEW94, Klosse97a]:

\[
Q = \zeta_3 + \zeta_4 P
\]  

(8.9)

which yields:

\[
\frac{dQ}{dP} = \zeta_4
\]

### 8.2.3 Generator specific angle

The gradient of the characteristic function \( Q(P) \) is then used to define as a generator specific parameter, the flicker relevant angle \( \phi_D \):

\[
\phi_D = -\arctan \frac{dQ}{dP}
\]  

(8.10)

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A simple functional relationship cannot be given when the slip of the generator varies over a wide range or if the induction generator is decoupled from the grid by means of an AC-DC-AC link [Klosse97a].

8.3 Prediction methods

A number of empirical and semi-empirical models have been developed to predict voltage fluctuations and flicker caused by wind turbines. The models discussed here use a simplified description of the wind turbine generator and the network characteristic as parameter.

8.3.1 Prediction with the linear approximation

A semi-empirical method has been proposed by VDEW [VDEW94] for flicker assessment of wind turbines in Germany. The simplified wind turbine generator is represented by its rated power and a linear approximation for the PQ characteristic of the generator (sec. 8.2). Inserting eq. 8.10 in eq. 8.6 yields for incremental changes of power:

\[
\frac{\Delta V}{V_0} = \cos(-\phi_D)^{-1} \frac{\Delta P}{P_{sc}} (\cos(\psi) \cos(-\phi_D) + \sin(\psi) \sin(-\phi_D)) \\
= \cos(\phi_D)^{-1} \frac{\Delta P}{P_{sc}} \cos(\psi + \phi_D) \\
\approx \frac{\Delta P}{P_{sc}} \cos(\psi + \phi_D)
\]

Following the definition of flicker severity in [IEC91] the short term flicker severity \( P_{st} \) is proportional to the relative voltage fluctuations. Using the flicker coefficient \( c_D \) as proportionality factor the VDEW approximation leads to ([Klosse97a], and similar [IEC96b, VDEW94],):

\[ P_{st} = c_D \frac{P_{rated}}{P_{sc}} \cos(\psi + \phi_D) \] (8.11)

\( \psi \) : grid impedance angle
\( \phi_D \) : flicker relevant angle
\( P_{sc} \) : short circuit power of the grid
\( P_{rated} \) : rated power of turbine

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The turbine specific parameters $\phi_D$ and $c_D$ have been measured by DEWI, Windtest and Windconsult for a variety of turbines and can be obtained from DEWI [DEWI94], Windtest or the manufacturers directly. The relationship between flicker and the impedance angle $\psi$ (eq. 8.11) has a minimum at $\psi + \phi_D = \frac{\pi}{2}$. Experimental evidence suggests that in this minimum the flicker severity is underestimated, so that the term $\cos(\phi_D + \psi)$ should be limited to a minimum value of 0.1 [VDEW94].

The VDEW method is not purely empirical as it takes turbine properties (PQ-slope) into account. However, the measured flicker coefficient is a function of the wind field properties. A flicker coefficient derived with this method is only valid, if the measurement location is representative for the design location of a wind turbine and the wind turbine does not operate in a wind farm.

8.3.2 Prediction with second order polynomial

Inserting the second order polynomial eq. 8.7 with $a_1 = 0$ in the turbine specific relation between active and reactive power defined as $\tan\phi_{wr} = \frac{Q}{P}$ yields

$$\zeta_2 P = \tan\phi_{wr}$$

and with eq. 8.8 [Sørensen95a] obtains:

$$\frac{dQ}{dP} = -2\tan\phi_{wr}$$

The angle $\phi_{wr}$ is here not a constant but a function of the wind speed. Inserting this relation in eq. 8.6 yields for increments in power, (similar in [Tande96]):

$$\frac{\Delta V_{sc}}{V_0} = \frac{\Delta P}{P_{sc}} |\cos(\psi) - 2\tan(\phi_{wr})\sin(\psi)|$$

Following the definition of flicker severity in [IEC91] the short term flicker severity $P_{st}$ is proportional to the relative voltage fluctuations. From experimental evidence it is suggested that $P_{st}$ is proportional to the turbulence intensity $I$ and the rated power $P_{rated}$. Using the proportionality factor $c_{tsn}$ [Sørensen96a] yields:

$$P_{st} = c_{tsn} I \frac{P_{rated}}{P_{sc}} |\cos(\psi) - 2\tan(\phi_{wr})\sin(\psi)|$$

(8.12)
The factor $c_{tn}$ is suggested by [Sørensen95a, Sørensen96a] not to be constant but a function of the wind conditions. To use eq. 8.12 requires knowledge of the active/reactive power characteristic and the factor $c_{tn}$.

### 8.3.3 Other models

The current efforts to improve models of the voltage fluctuations caused by wind turbines concentrate on three different areas. Detailed time domain modelling of the wind turbine dynamics and the generator, improved semi-empirical description or a purely empirical approach.

**Time domain turbine modelling**

Time domain simulation of the voltage fluctuations from wind turbines is in principle an alternative to the models employed in this thesis. A typical time domain model has for example been used in [Vegazones99] to model the grid impact of a wind farm. The simulation contains a detailed wind turbine and generator model. As mentioned in sec. 6.2 the parameters for such a model are generally not available. Important statistical parameters like turbulence spectrum and coherence function of the wind field describing it as a function of the turbine location are on the other hand usually not considered.

**Expanded VDEW model**

The linear approximation model (sec. 8.3.1) can be improved by introducing the turbulence intensity into the model. A semi-empirical method to represent this effect is proposed in [Amaris99]. The importance of including a distinction between deterministic and stochastic component of fluctuations is stressed. The stochastic component being dependent on the turbulence intensity whereas the deterministic signal is modelled as a constant. No consideration is given to changes in the turbulence spectrum or a more specific turbine modelling.
Empirical method

Measurements of voltage fluctuations can be transferred from one wind turbine to another if the second turbine would operate to the same characteristic set of parameters. It is currently suggested [IEC99] to measure the flicker for each wind turbine at a given wind speed range, within a prescribed turbulence band and for different simulated network conditions. This set of flicker measurements is then used if the same turbine type is subsequently installed at another location.

This procedure requires costly, time consuming measurements which do not seem to be justified by the result. The proposed empirical method is based on measurement only and not on a description of the underlying processes. Changing a component in the turbine design would necessitate a new set of measurements to be carried out. The method does not take into account that the wind field depends to a high degree on the terrain and the position of the turbine in a wind farm. For locations with the same turbulence intensity and mean wind speed the voltage flicker can deviate significantly from the predicted result.

8.4 New flicker assessment method

The models presented in the last sections have been studied and elements have been identified which are useful for a new flicker assessment method.

The measurements carried out by DEWI [DEWI94], Windtest and others provide a valuable source of information about the electrical properties of wind turbines. The parameters $\phi_D$ and $c_D$ have either been published or can be obtained from manufacturers.

The second order polynomial approximation to a generator curve is more realistic than a linear approximation, but it is necessary, as for the exact equation in eq. 8.3 to know or measure the P(Q) characteristic of the wind turbine. A linear description is sufficiently accurate as maximum flicker is expected for high wind speeds at which the generator characteristic can be represented using a linear approximation.

The proportionality of flicker to the power output eq. 8.8 is now combined with eq. 8.6.
\[ \frac{1}{V_0} \frac{dV}{dP} = \frac{\cos(\psi)}{P_{\text{sc}}} - \frac{\sin(\psi)}{P_{\text{sc}}} \frac{\Delta P}{P_{\text{rated}}} \]  

(8.13)

\[ \zeta_2 P = \zeta_2 \frac{P}{P_{\text{rated}}} \]

The slope \( \frac{\Delta Q}{\Delta P} \) is not known for the whole characteristic but for the range which has been linearly approximated by DEWI and published in form of the flicker relevant angle \( \phi_D \). It seems reasonable to assume that the value is also representative at \( \frac{P}{P_{\text{rated}}} = 1 \). It is then possible to replace:

\[ a'_2 = -\frac{\tan(\phi_D)}{2} \]

which yields the voltage fluctuation as function of the variation in power output \( \Delta P \):

\[ \frac{\Delta V}{V_0} \approx \left( \cos(\psi) + \tan(\phi_D) \sin(\psi) \right) \frac{P}{P_{\text{rated}}} \frac{\Delta P}{P_{\text{sc}}} \]  

(8.14)

This approximation to eq. 8.3 calculates the voltage variations with only the turbine specific \( \phi_D \) and the network specific \( \psi \) required as input. This approximation is used to convert a spectrum of power fluctuations into a spectrum of voltage fluctuations which can then be used as input the FD-flickermeter chap. 7.

The new model represents a compromise between theoretical model and required experimental data. The only additional turbine specific parameter, apart from the parameters already used to derive the power fluctuations, is the slope of the \( P(Q) \) characteristic which can be obtained from the turbine manufacturers.
8.5 Summary - prediction of flicker

A number of methods has been studied with respect to their suitability to predict voltage fluctuations and flicker from a wind turbine. Based on this analysis a new method has been proposed for prediction of voltage fluctuations (eq. 8.14). This method uses the turbine characteristic generator slope, the grid impedance angle, the network and turbine rating and the power fluctuations (chap. 6) to predict the flicker caused by a wind turbine. All these parameters are readily available to the wind farm developer and allow a prediction of flicker for any planned development. The result of the prediction is a voltage spectrum which is best evaluated with an FD-flickermeter (chap. 7).

\[ P_{st}^2 = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) dn \]

The link of the chain \( (H_{PV}) \) discussed in this chapter concludes the elements needed to represent the flicker from a single wind turbine. The next chapter will discuss what influence the grouping of wind turbines in a wind farm has on the flicker \( (H_{CW} \) and \( H_{FT}) \).
Chapter 9

Wind farm model

The wind speed is reduced in the wake of a wind farm and the turbulence intensity is increased (see fig. 9.1). The level and spectral shape of turbulence is influenced by the position of a turbine in a wind farm thus influencing the power generation and voltage variation (flicker). It is discussed here, how best to model this effect.

Figure 9.1: Wake of a wind turbine. One wind turbine is downwind of another. The area of reduced velocity is called the 'wake' of a wind turbine.
9.1 Reduction in mean wind speed

A number of research groups have analysed the problem of wind turbine wakes and suggested methods to model the wake effect.

**Jensen** presented in 1983 the PARK model which is surprising in that it, despite its simplicity, proves sufficiently accurate to predict the power yield of wind farms ([Jensen83b, Katič86]).

**Lissaman** presented a model [Lissaman77, Lissaman79] and [Lissaman82] which is based on the description of turbulent jets by [Abramovich63]. A similar model has been proposed by [Vermeulen81a].

**Ainslie** used in 1985 a theoretical approach by solving the equations of motion with an experimental profile as initial conditions. ([Ainslie85, Ainslie88, Smith91].

All models use a cylindrical co-ordinate system and assume rotational symmetry around the centre-line. A review of different models can be found in [Luken87], [Taylor90] and [Waldl97]. The summary given here is based on the detailed discussion in [Schlez92] and covers the models of relevance to this thesis.

9.1.1 The wake model proposed by Jensen

The model was presented in the publication [Jensen83b]. A more recent publication [Katič86] improves the description but discusses also its limitation. Possible modifications to the model are discussed in [Waldl97].

Initial value and profile

Jensen assumes a linear expansion of the wake from the rotor \((x=0)\) as a function of the distance from the turbine (fig. 9.2): \(R_w(x) = c_\omega x + R_r\). The rotor radius is \(R_r\) whereas \(R_w\) is the radius of the wake. The velocity deficit profile is a rectangular one for the whole wake and the wake expansion coefficient is \(c_\omega\). The velocity outside the wake is \(u_0\). \(u_2\) is, using the actuator disk model, the velocity after the expansion of the streamtube.
The wind velocity is assumed in the model to be reduced to $u_2$ in the rotor plane and the conservation of mass is used as follows:

$$\pi R_r^2 u_2 + \pi (R_w(x)^2 - R_r^2) u_0 = \pi R_w^2(x) u(x, 0)$$

and solved for a wind velocity $u(x, 0)$ at a distance $x$ on the centerline, using the thrust coefficient $c_t$ and the relation $u_2 = u_0 \sqrt{1 - c_t}$ [Katic'86]:

$$u(x, 0) = u_0 \left( 1 + \frac{R_r}{R_r + c_w x} \right)^2$$

The factor $c_w$ has to be determined by experiments.

Discussion

The following points have to be considered when using the model:

- The conservation of momentum is violated for every point but one in the wake.
- The use of conservation of mass does not take into account any radial velocity components.
- The authors of this model recommend its use only for inter turbine spacing of more than 4 rotor diameter [Katic'86].
The model proposed by Jensen has been implemented in the PARK code. From a theoretical point of view its use cannot be recommended, however for a range of applications it does produce sufficiently accurate results. Adjustments to the wake expansion angle have been discussed in [Waldl97].

9.1.2 Lissaman wake model

[Lissaman77] used the universal profile shape derived in [Abramovich63] for turbulent jets to describe the wake profile. The model used elements of the laminar flow theory and was adapted to experimental results in [Lissaman79] and [Lissaman82].

**Initial value**

Lissaman subdivides the wake into three zones near, transient and far wake (fig. 9.3). The velocity $u_2$ is used as initial velocity deficit directly downwind of the rotor. Using mass conservation and the Froude theorem [Froude89] the

![Diagram of wake model](image)

**Figure 9.3: Sections of the streamtube.** The wake is divided in 'near', 'transition' and 'far' region after [Lissaman77]
expansion of the streamtube is:

\[
\frac{R_2^2}{R_1^2} = \frac{u_r}{u_2} = m \frac{u_0 + u_2}{u_0} = \frac{m}{2} \left( 1 + \frac{1}{m} \right) = \frac{m + 1}{2}
\]  

(9.1)

with:

- \( m \): axial induction factor
- \( u_r \): windspeed at turbine rotor
- \( u_0 \): undisturbed wind velocity
- \( R'_r \): expanded radius

The radius of the wake at a downstream location 'X' in a distance \( x \) from the turbine is called \( R_w(x) \). The expanded radius \( R'_r = R_r \sqrt{\frac{m + 1}{2}} \) replaces the rotor radius \( R_r \) as initial radius for the wake.

Wake profiles

The 'near' region is characterised by a zone of constant wind speed, the core, which is eroded by the shear generated turbulence at its boundary. A rectangular shaped profile is used to describe the wake in the rotor plane \((x = 0, \text{eq. 9.2, fig. 9.4})\).

![Figure 9.4: Profile of the wake according to Lissaman. The velocity deficit in the wake (as a ratio of the core deficit) is plotted as a function of the radius (normalised with the local wake radius). The three different shapes used by Lissaman are shown.](image)
Profile 0 ($x = 0$): (Box shaped profile)

\[
\frac{u(0,r)}{u(0,0)} = \begin{cases} 
1 - \frac{m-1}{m} & \text{for } r \leq R_w(x) \\
1 & \text{for } r > R_w(x)
\end{cases}
\]  \hspace{1cm} (9.2)

this rectangular profile is used to describe the wind speed in the core region of the wake. The core is slowly eroded and at the end of the near wake a second profile is used ($x = x_n$, eq. 9.3) to describe the wake:

Profile1 ($x = x_n$): (Wide bell shaped profile)

\[
\frac{u(x_n,r)}{u(x_n,0)} = \begin{cases} 
1 - \frac{m-1}{m} \left(1 - \left(1 - \frac{R_w(x)}{R_w(x)} \frac{r}{R_w(x)} \right)^{\frac{3}{2}} \right) & \text{for } r \leq R_w(x) \\
1 & \text{for } r > R_w(x)
\end{cases}
\]  \hspace{1cm} (9.3)

this profile of the transitional wake zone slowly relaxes to the 'universal' wake profile (eq. 9.4, fig. 9.4) used for distances larger than $x_f$, the far wake.

Profile2 ($x > x_f$): (Regular bell shaped profile)

\[
\frac{u(x_f,r)}{u(x_f,0)} = \begin{cases} 
1 - \frac{m-1}{m} \left(1 - \left( \frac{r}{R_w(x)} \right)^{\frac{3}{2}} \right) & \text{for } r \leq R_w(x) \\
1 & \text{for } r > R_w(x)
\end{cases}
\]  \hspace{1cm} (9.4)

Parameters of the profile are the wake radius and the velocity deficit. The wake radius is defined in a way that at each point of the streamtube the momentum of the flux is conserved [Schlez92]:

\begin{align*}
(x = 0) : & \quad R_w^2(0) = R_f^2 \\
(x = x_n) : & \quad \frac{R_w^2(x_n)}{R_f^2} \approx (0.212 + 0.145m)^{-1} := l_d^{-2} \quad (9.5) \\
(x = x_f) : & \quad \frac{R_w^2(x_f)}{R_f^2} \approx (0.134 + 0.124m)^{-1} := l_e^{-2}
\end{align*}

The central velocity deficit in the wake and the rate of wake expansion is assumed to be constant in the transition region between $x_n$ and $x_f$. With the abbreviations $l_d$ and $l_e$ as defined above $x_f$ can be expressed as a function of $x_n$:

\[
x_f = x_n \frac{l_d(1 - l_e)}{l_e(1 - l_d)}
\]
Further downwind, the expansion of the stream-tube becomes a linear function on the (ambient) turbulence intensity.

\[ \frac{dR_w}{dx} = \frac{I_{amb}}{0.36} \]  

(9.6)

The parameter 0.36 has been derived in analogy to the expansion of 'plumes' [Abramovich63].

**Discussion**

The model transfers theories that have been developed for a different purpose to wind turbine wakes. However critical simplifications have been used: the assumption of a rectangular profile in the rotor plane (or close to it) with a wind velocity of \( u_2 \) derived from the laminar theory for large distances; the expansion of the wake is assumed to be linear, other functions for this expansion have been derived by [Abramovich63].

The main weakness of the model is the description of the 'near' region. The assumptions made for the wind velocity and profile in this region are then carried on into the description of the transition and far region. However the introduction of universal profiles and the derivation of the model on the basis of the conservation of momentum make this model attractive.

**9.1.3 Ainslie's Model**

The wake model of [Ainslie88] starts with the assumption of a universal profile [Abramovich63] at a distance of 2 rotor diameter. The two parameters of this bell shaped profile are determined empirically. The equations of motion sec. 2.1.2 are solved numerically for the remainder of the wake using an eddy viscosity closure scheme.

**Initial values**

The model avoids the difficult region directly behind the rotor and starts at 2D distance with the initial velocity deficit as a function of the ambient turbulence \( \alpha \) and the thrust coefficient \( c_t \).
Figure 9.5: The wake according to the Ainslie model. The wake is derived at a distance of 2 rotor diameter (D) from experimental results. The remainder of the wake is calculated by a numerical approach.

\[ 1 - \frac{u(2D,0)}{u_0} = D_m = c_t - 0.05 - (16c_t - 0.5) \frac{I}{10} \]

A profile by [Abramovich63] is used for wake at \( x = 2D \):

\[ \frac{u(x,r)}{u(x,0)} = 1 - D_m e^{-\left(\sqrt{3.56R_b}\right)^2} \]  

(9.7)

The width of the profile \( R_b \) is calculated at 2D via conservation of momentum:

\[ \frac{R_b^2}{4R_r^2} = \frac{3.56c_t}{8D_m(1 - \frac{1}{2}D_m)} \]  

(9.8)

Where the wake radius \( R_b \) is the half width of the profile divided by the arbitrary number \( \sqrt{3.56} \).

Discussion

The model avoids the problems of the description of the flux near the rotor by using an experimental profile at 2D. The numerical solution of the differential equations requires more computing power. Given the steady increase in available computer performance this is only a weak argument against such a model. The model is therefore recommended for further use. Variations of the model are discussed in [Smith91], [Taylor90], [Luken86] and [Voutsinas91].
Wake models have been implemented in a number of commercially available wind farm layout programs. For optimisation of large wind farms the fast PARK model or a modified version of this model is recommended, while for an exact assessment the Ainslie eddy viscosity model should be preferred.

9.2 Turbulence in a wind farm

The turbulence intensity in the wake of a wind turbine is increased. Different causes account for this increase:

- The interaction of the rotor with the air results in 'rotor generated turbulence'
- Shear stresses in the wake cause turbulent eddies the 'shear stress related turbulence'
- The reduction in mean wind speed at the rotor leads at constant standard deviation to increased turbulence intensity.

The turbulence intensity can be obtained from empirical formulae or as a result of the flow models used. An overview is given in [Waldl197].

9.2.1 Experimental evidence

The turbulence intensity in the wake of a wind turbine is increased due to the reduced wind speed. Measurements from a number of sites confirm that the increase is not only due to the reduced wind speed but that the standard deviation is increased [Rasmussen90, Schlez92, Højstrup93].

A double peak structures of the standard deviation can be observed in the near and transitional region. This is because the turbulence generation from wind shear and blade tip vortices takes place on the edges of the wake. A model for the profile of the turbulence intensity in the wake does not exist. Empirical formulae have been developed to model the centre line level of turbulence intensity in a wake.
9.2.2 Empirical formula for added turbulence

The increase of the turbulence intensity $I_{add}$ is defined as:

$$I_{add} := \sqrt{\sigma_u(x, r < R_w)^2 - \sigma_{u_0}^2}$$

A variety of empirical formulae have been proposed to predict the added turbulence intensity in the wake. Comparisons of some of the suggested approximations have been published in [Waldl97] and [Thomsen99]. A brief overview of the different approaches is given in the following sections.

Crespo formula

In the publication [Crespo93] the following empirical formula is presented:

$$I_{add} = 0.73a_x^{0.8325} I_{amb}^{-0.0325} \left( \frac{x}{D} \right)^{-0.32}$$

where $a_x$ is the axial induction factor $a_x = \frac{1}{2} \left( 1 - \sqrt{1 - c_t} \right)$. This formula is based on numerical calculations with the code UPMWAKE.

Larsen formula

The authors of [Larsen96] assume that for a sufficient large inter turbine spacing only the wind shear generated turbulence is of relevance and derive with this assumption:

$$I_{add} = 0.93x^{-\frac{1}{3}} \left( 1 - (1 - c_t)^{\frac{1}{3}} \right)^{\frac{1}{2}}$$

Quarton formula

For a mean value of the turbulence intensity in the wake Quarton gives an empirical formula ([Quarton89], [Quarton90], [Hassan92]):

$$I_{add} = 0.057 \ c_t^{0.7} \ (100 I_{amb})^{0.68} \left( \frac{x}{x_n} \right)^{-0.57} \quad (9.9)$$

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with:

\( I_{\text{add}} \): added turbulence intensity \( I_{\text{amb}} \): ambient turbulence intensity

This formula is derived from the scatter diagram of a number of wake measurements. By the introduction of empirical parameters the scatter points were forced to collapse to one point. The model is based on a larger number of wind tunnel and field measurements.

The extension of the near wake \( x_n \) is given by [Lissaman82, Vermeulen81b]:

\[
\frac{x_n}{R^*} = \left[ \left( \frac{dR_k}{dx} \right)_a^2 + \left( \frac{dR_k}{dx} \right)_m^2 + \left( \frac{dR_k}{dx} \right)_\lambda^2 \right]^{-\frac{1}{2}}
\]

where the erosion of the core is a function of:

- Ambient turbulence \( \left( \frac{dR_k}{dx} \right)_a = \frac{I_{\text{amb}}}{0.36} \)
- Shear stress generated \( \left( \frac{dR_k}{dx} \right)_m = 0.27d^\frac{m-1}{m+1} \)
- Rotor generated \( \left( \frac{dR_k}{dx} \right)_\lambda = \sqrt{\frac{c_d \sigma \lambda^2}{6(m+1)}} \)
- Vermeulen rotor generated \( \left( \frac{dR_k}{dx} \right)_\lambda = 0.012B \lambda \)

In this formula \( R_k \) is the radius of the core region, \( \alpha \) the ambient turbulence, \( \sigma \) the relative area covered by the rotor, \( c_d \) the drag coefficient of a rotor blade and \( \lambda \) the tip speed ratio of the rotor. For details see [Pahlke90]. The Quarton formula has been validated against a variety of experimental data and is recommended here.

### 9.2.3 Spectrum in a wind farm

Measurements of turbulence spectra in wind farms have been presented in [Højstrup93, Kelley93, Høstrup99]. They do not allow any decisive conclusion, if a change in the shape of the spectrum takes place in the wake of a wind turbine.

It is assumed for this assessment that the spectrum is shifted with mean wind speed in analogy to the situation of flow over a hill (sec. 3.3.2).
9.3 Flicker from a wind farm

If a number of wind turbines is connected to a common point of connection (fig. 9.6) the lumped voltage fluctuations caused by these turbines have to be determined. This should be done in accordance with the standard [IEC96b]. The lumped flicker can be obtained for correlated and not correlated signals. The spatial coherence between power fluctuations at different points in a wind farm provides additional information. This information can be used when adding the flicker contributions from wind turbines in a wind farm.

\[ P_{t}' = \sqrt{\sum_{i} P_{st}^{\omega}} \]  

(9.11)

with

Figure 9.6: Grid connection of wind turbines. The flicker at the point of common connection PCC depends on wind regime, the properties of the wind turbine generators \( WT_i \) and the impedance \( Z_i \) the turbine is connected to the PCC.

9.3.1 Adding different sources of voltage flicker

The lumped flicker \( P_{st} \) from several sources with flicker \( P_{sti} \) is reduced compared to that of a single source, provided they are not fully coherent. For a number of flicker sources the IEC suggest the following differentiation [IEC96b]:

With
\( \varpi = 4 \) : coincidence impossible \quad \varpi = 2 \) : stochastic fluctuations

\( \varpi = 3 \) : coincidence unlikely \quad \varpi = 1 \) : fully correlated

The values of \( \varpi \) can be a function of the flicker frequency. As a general rule it is suggested to use \( \varpi = 3 \). For wind farm situations it is recommended by [Klosse97b] and [Tande96] to use \( \varpi = 2 \) so that for \( N \) equal \( P_{st} \) sources the total flicker value is \( P_{st} = \sqrt{NP_{st}} \). This suggests to represent the sum of voltage fluctuations as an addition of \( N \) stochastic, uncorrelated signals. \( \varpi = 1 \) is used for correlated sources. This would yield for \( N \) turbines with equal flicker severity level \( P_{st} = NP_{st} \).

This relationship will be discussed in the following sections and a suggestion, how best to add the flicker from different wind turbines in a wind farm will be made.

9.3.2 Spatially distributed wind turbines

The following argument is analogous to the analysis of power output from spatially distributed wind turbines in [Beyer90a] and [Steinberger-Willms93]. The spectrum of power output is here replaced by the flicker spectrum (\( S_F \)). The flicker severity (\( P_{st} \)) and the flicker spectrum are related by \( P_{st} = \int S_F(n)dn \).

The flicker spectrum is the sum of flicker spectra caused by the individual turbines \( S_{Fi} \) at the PCC:

\[
S_{Fij} = \sqrt{(coh_{ij}(n)S_{Fi}(n)S_{Fj}(n)e^{i\Phi_{ij}(n))}}
\]

with

- \( coh_{ij} \) : coherence between a pair turbines
- \( \Phi_{ij} \) : phase relationship between a pair of turbines

For a wind farm consisting of \( N \) turbines and assuming a random phase relationship between the individual flicker spectra yields:

\[
S_F = \sum_i \sum_j \sqrt{(coh_{ij}S_{Fi}(n)S_{Fj}(n))}
\]

(9.12)

For illustration of equation eq. 9.12 the two extreme cases will be considered:

- For \( coh_{ij}(n) = 0 \) and \( S_i = S_j \) (independent signals):

\[
S_F = NS_{Fi}(n) \Rightarrow P_{st} = \sqrt{NP_{sti}}
\]
In this case the flicker from \( N \) turbines is \( \sqrt{N} \) times the flicker from one turbine. This is equivalent to \( m = 2 \) in eq. 9.11.

- For \( \text{coh}_{ij}(n) = 1 \) and \( S_{Fi} = S_{Fj} \) (coherent signals):

\[
S_F = N^2 S_{Fi}(n) \Rightarrow P_{st} = NP_{sti}
\]

In this worst case scenario the wind farm acts like a single generator with the lumped power output of the individual turbines. The flicker level increases linearly with \( N \) the number of turbines.

The coherence function will be close to 1 for low frequency and close to 0 for high frequencies. In the following it is investigated how this frequency dependence of the coherence in a wind farm affects the voltage fluctuations caused by wind turbines.

![Wind farm frequency response](image)

**Figure 9.7: Wind farm frequency response.** Wind farm frequency response for a turbine spacing of 2 D (dotted line) and 4 D (thick line). Frequencies higher than 0.1 Hz are reduced due to averaging in a wind farm.
Wind farm

In order to model the effect of a wind farm on the flicker spectrum the following example is used: A linear wind farm with 4 turbines, the wind blowing along the line, 2D/4D spacing, D=40m, v=10m/s, I=10%, $S_i(n) = 1$ for all frequencies:

Separation distances ($d$) in a wind farm are generally larger than the turbine height. The two point coherence in eq. 9.12 is then best represented by the expression developed in chap. 4:

$$\gamma^2(\alpha) = e^{-\frac{2\pi d}{u} \left( \left( \frac{a_1}{u} \cos \alpha \right)^2 + \left( \frac{a_2}{u} \sin \alpha \right)^2 \right)}$$

(9.13)

with:

- $u$: mean wind speed
- $d$: distance between two points
- $a_1 \approx 30$: longitudinal decay constant
- $a_2 \approx 35$: lateral decay constant
- $\alpha$: angle between wind speed and separation line

An experimental study of the coherence function in wind turbine wakes has been published by [Høstrup99]. The results indicate lower coherence in the near wake region a wind turbine, so that the result derived here may be only valid for sufficiently large turbine spacing.

The resulting reduction due to wind farm averaging is presented in fig. 9.7 for a wind turbine spacing of 2D and 4D. The relative spectrum of the voltage fluctuations in a wind farm of 4 turbines shows that the result approximates for high frequencies the result expected for time series with zero coherence (statistically independent signals) and for low frequencies the result expected for unity coherence. Thus a wind farm will generally reduce the relative voltage fluctuations for high frequencies but not to the same degree for frequencies below approximately $\frac{1}{10}$ of a Hz.
9.4 Summary - wind farm effects

The flicker of wind turbines is modified by wake effects and the coherence between the turbines. Both aspects have been discussed in this chapter and the following conclusions have been drawn:

**Wake effect** The reduction in wind speed is best described by an eddy-viscosity type model. The turbulence level in a wind farm is increased due to the reduced wind speed. The further increase of turbulence due to wind shear and rotor induced turbulence is best modelled using the Quarton formula. These results can be made available as output from current wind farm layout packages.

**Spectrum and Coherence** The effect of the wake on the spectrum and coherence is assumed to be equivalent to that of the increased wind speed over a hill (chap. 3, chap. 4).

**Lumped flicker** The relative voltage fluctuations (flicker) are reduced in a wind farm as compared to the sum of flicker from single wind turbines in a wind farm. This is especially true for frequencies above 1 Hz. Large voltage fluctuations in a wind farm situation are expected to be observed only at low frequencies or in transient conditions.

As in the last chapter the importance of the low frequency voltage variations has again been underlined by this analysis. Low frequency relative voltage variations are less reduced in a wind farm as high frequency variations. The flicker level in a wind farm is due to the decrease in mean wind speed decreased but can also increase due to the increased turbulence intensity generated by turbines and wind shear.

\[
P^2_{st} = \int (S \circ H_{KC} \circ H_{CW} \circ H_{WP} \circ H_{PV} \circ H_{VF} \circ H_{FT}) \, dn
\]

This chapter has described the last two chain links, the influence of the wind farm on the wind field \(H_{CW}\) and the wind field on the lumped flicker \(H_{FT}\). Predictions made with the new flicker prediction model will now be compared with experimental data.
Part III

Model results and validation
Chapter 10

Model Validation

It is possible at this point to apply the full flicker prediction model to test cases and compare the results with measurements. The model developed in the last chapters allows the prediction based on a small number of basic assumptions and turbine parameters. The flicker prediction model for a wind turbine consists of the following components.

- Turbulence and turbulence spectrum
- Description of the influence of the terrain
- Effects of a wind farm on the wind field
- Description of the turbine
- Modelling of the voltage fluctuations on a network
- Wake and turbulence in a wind farm
- Description of the coherence between the power production in a wind farm

The key parameters for each of these model components can be adjusted to best represent the specific circumstances of a particular wind development. The predictions made by the model are validated in this chapter against measurements.
10.1 Experimental data

Experimental data suitable for the validation of the flicker prediction model was not measured within the context of this thesis. Instead published measurements from a number of wind turbines in Denmark have been used. Details of this experiment are given below and in [Sørensen95a, Sørensen96b, Sørensen96a] and [Tande96]. Suitable data representing and validating a wind farm situation was not available.

10.1.1 Experimental set-up

The measurements consisted of power and current measurements which were used to simulate the voltage fluctuations on exemplary networks. The voltage fluctuations were then evaluated with a digital flickermeter algorithm. The flicker severity was published as function of wind speed and turbulence intensity. Two of the example networks used are network 1 with a short circuit power $P_{sc} = 30\text{ MVA}$ and network 2 with $P_{sc} = 17\text{ MVA}$, for details see [Sørensen96b].

10.1.2 Flickermeter program

A flickermeter program was used to evaluate the voltage fluctuations. This program was tested against the flickermeter standard and a hardware implementation of a standard flickermeter.

The digital flickermeter program (DEFU) used in the evaluation was tested using sinusoidal fluctuations defined in [IEC90] for a range of 0.5 to 25 Hz [Sørensen96b]. Unfortunately this is not sufficient to prove that the flickermeter program conforms with the flickermeter standard [IEC90, IEC91]. If a flickermeter is to be used for the assessment of flicker from wind turbine the full range low frequency fluctuations have to be considered.

10.1.3 Hardware flickermeter

A commercial flickermeter was compared in [Sørensen96b] with the flickermeter program used. The available documentation on commercial flickermeter has been
studied. Three manufacturers have been asked to supply information on the
compliance of their instrument with the standard [IEC90, IEC91].
Information supplied by the manufacturer of the flickermeter used in the pub-
lication shows that the company has tested the flickermeter for 0.5 to 25 Hz
[Voltech99d, Voltech99b, Voltech99c, Voltech99a]. Again, this is not sufficient to
prove that the instrument or the flickermeter program conforms with the flicker-
meter standard.
A flickermeter of the type Siemens P513 which is also sold as SIMEAS-N has also
been available for the flicker evaluation but was not used because of technical
problems [Sørensen96b]. It was not possible to obtain information about the
compliance of this flickermeter with the standard.
An alternative flickermeter which has been developed by Boconsult in Italy is
sold under the name Panasonic by Matsushi, Japan. The flickermeter complies
with the standard for all frequencies according to the manufacturer. Test results
have not been obtained. Deviations of flickermeter output from the standard are
according to this manufacturer not at all unusual [Amato99], deviations for high
frequencies are discussed in [Boconsult99].
None of the manufacturers supplied conclusive evidence of the performance of
the instruments for low frequency fluctuations. Thus it is essential that the user
performs their own checks on this aspects before using a commercial flickermeter
for the assessment of wind turbine flicker. The DEFU flickermeter program was
not verified for frequencies below 0.5 Hz.

10.1.4 Turbine parameter

The flicker predictions and measurements from three different turbines on two
different model networks were compared. The turbines used to represent typical
flicker sources are two stall (Nordtank 300kW, Nordtank 500kW) and one pitch
regulated wind turbine (Vestas 500kW).
Information on the power characteristic, flicker coefficient $c_D$ and the flicker rele-
vant angle $\phi_D$ was obtained directly from the turbine manufacturers. The turbine
parameters used for the analysis are summarised in Appendix D

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10.2 Flicker as function of the wind field

The published measurements are used to study the dependency of flicker on variations in mean wind speed and turbulence intensity. The turbine is situated in flat terrain (Norrekaer Enge).

10.2.1 Flicker and wind speed

The flicker prediction is compared with flicker measurement in fig. 10.1 and fig. 10.2, using model networks 1 and 2 respectively. The measured flicker values increase nearly linear with wind speed over the whole range of wind speeds observed. Deviations from this linear relationship correspond with an above average turbulence intensity.

The graph fig. 10.1 compares the measurements with the simple model (sec. 8.3.1) using a flicker coefficient $c_D$ (straight line). The prediction with a flicker coefficient does not model the dependency of the flicker severity on the wind speed, but $\frac{\Delta P}{P}$.

![Figure 10.1](image)

**Figure 10.1:** Flicker measurements at a 300 kW stall regulated turbine in model network 1. Flicker measurements are compared with the predictions from a semiempirical model and the new prediction model.
could be a function of the wind speed. The second prediction (lower line) was obtained using the new algorithm (sec. 8.4) at a turbulence intensity of 12%. The absolute value of the flicker and the dependence on wind speed show general agreement between prediction and measurement.

The second, lower line in fig. 10.2 represents a prediction made with a limited bandwidth flickermeter. The qualitative agreement between prediction and measurement appears to be better when a bandwidth limited flickermeter is used. This indicates that the measurement might have contained a high pass filter such as is introduced by using recorded intervals of less than 10 minutes to evaluate the data. This would explain the observed difference between predictions using a full bandwidth flickermeter and measurement. Other possible explanations, for example that the new flicker prediction algorithm might not be adequate or that the flickermeter used to evaluate the measured data does not reflect accurately the low frequency fluctuations, will be discussed in the following sections.

![Figure 10.2: Flicker measurements at a 300 kW stall regulated turbine in model network 2. Flicker measurements are compared with the predictions from the new model using a limited bandwidth flickermeter (lower) and alternative a full bandwidth flickermeter (upper).](image-url)
10.2.2 Flicker and turbulence

In this section the dependence of the flicker on the turbulence intensity is studied. Flicker as function of the turbulence intensity has been measured and presented in [Sørensen96b]. The turbine used was a Nordtank 28/300 turbine at model network 1. The measurement has been performed at a constant mean wind speed of 7 m/s.

The new model predicts that the flicker is a linear function of the turbulence intensity, as long as the stochastic components of the flicker spectrum are dominant. This is expected for a wind speed of 7 m/s. The predictions are shown together with the measurements in fig. 10.3 showing a linear dependence of the flicker severity from the turbulence intensity. The model prediction is in good agreement with the measurement. The use of a constant flicker coefficient is not adequate. The turbulence intensity should be considered as an important parameter in the flicker assessment.

Figure 10.3: Flicker and turbulence intensity. Voltage flicker as function of the turbulence intensity at a wind speed of 7 m/s for a Nordtank 28/300 turbine. The model predictions (lower line) are compared with measurements and simple coefficient method (upper line).
10.2.3 Comparison with flicker coefficients

Simple flicker coefficients can be used to predict the flicker caused by a wind turbine. This requires the wind speed and turbulence intensity during the measurement to be representative for the later application of the coefficient. Effects due to terrain or wind park influences can not be considered with this method. Examples of predictions from such coefficients are presented in fig. 10.3 and fig. 10.4. The flicker coefficients used here have been obtained directly from the manufacturers (Appendix D), coefficients are also published in [DEW194]. The agreement with the measured data and the model predictions for the simple test cases used here are satisfactory.

10.3 Flicker and the wind turbine

The flicker of a grid connected wind turbine depends not only on the wind field but also depends on its size and type, and in particular the fundamental control

![Diagram](image)

**Figure 10.4:** Flicker and turbine size. Flicker prediction for a NTL 37/500 on network 1. The magnitude of flicker from the larger turbine is predicted (line) as accurate as for the smaller turbine from the same manufacturer.
strategy used for the turbine. Two main types of turbines, stall controlled and pitch controlled turbines, have to be considered in this analysis. Hybrid turbines which combine elements of more than one of the basic types are not considered in this stage.

A variable speed turbine, which is connected to the grid via a power conditioning unit is in principle able to deliver its power without causing any voltage fluctuations. Variable speed turbines are not considered in this thesis.

10.3.1 The size of the wind turbine

A flicker prediction algorithm should predict the flicker from turbines of different kind and size. Increasing the turbine size influences the voltage flicker through three different processes:

- Larger generators operate generally at a lower slip. The resulting change in the flicker relevant angle results in increased voltage flicker.
- The coherence over a larger rotor is reduced. This increases the low frequencies and reduces the 3p components (filter by lack of coherence).
- The lower rotational speed shifts the rotational sampled spectrum towards lower frequencies.

The figure fig. 10.4 shows the prediction for a Nordtank 37/500 turbine. The predicted voltage fluctuations on example network 1 are increased if compared to the Nordtank 28/300 and in quantitative agreement with the measured values.

10.3.2 Stall control turbines

The turbines modelled so far have been stall controlled turbines. Above rated power the low frequency components can be dampened by the stall mechanism. Such damping is not expected for below rated power operation. The comparison of data and prediction shows evidence that the predicted values are high at wind speeds around 12 m/s. The low frequency components are expected to contribute significantly to the flicker at these wind speeds and have
been observed to do so [Lundsager80]. The reason for the difference between measured and predicted flicker values at these wind speeds is not known. One reason for the observed discrepancy may be faulty data. Another possibility that cannot be discounted at this moment is that the modelling of the stall controlled turbines is not complete. More experimental data from stall regulated turbines is needed to confirm the validity of the model.

10.3.3 Pitch control turbines

Above rated power the low frequency components can be damped by the power limiting mechanism for pitch regulated turbines. This stagnation of the fluctuations can be observed in fig. 10.5 and fig. 10.6 for the measured flicker values between 13 and 17 m/s wind speed.

At wind speeds between 17 and 20 m/s the measured flicker values exceed the predicted values. This is likely due to transitional effects - turbines being disconnected from the network - which are not considered here.

The measurement and predictions made for the Vestas 39/500 at model network 1 are in qualitative and quantitative agreement. The predicted and measured flicker of the turbine is larger than for the Nordtank 37/500, this is expected. Pitch regulated turbines produce more flicker than stall regulated turbines of the same size. This is due to the induction lag effect described in sec. 6.2.3 and the limitations of the pitch control system.
Figure 10.5: Flicker from a pitch regulated turbine. Measurements from a Vestas 39/500 turbine are compared with the prediction for model network 1. The flicker from the pitch controlled turbine is predicted accurately by the model.

Figure 10.6: Flicker from a pitch controlled turbine. Measurements from a Vestas 39/500 turbine are compared with the prediction at model network 2. The flicker from the pitch controlled turbine is predicted accurately by the model. Also shown is the prediction using a flicker coefficient.
10.4 Example Calculations

To demonstrate the effect of a wind farm configuration on the predicted flicker values a number of wind farms is studied. These are linear wind farms and arrays of MxN turbines. M is the number of turbines along wind direction N is the number of turbines across wind. The distance between the turbines is given in multiples of the turbine diameter. The example turbine used for all example wind farms is the Vestas 39/500, wind speed is 10.5 m/s at hub height and turbulence intensity 12%.

10.4.1 Linear wind farm

Two special cases of a linear wind farm, Example 1 (N=1, M=5, d=4D) and Example 2 (N=5, M=1, d=4D) are discussed here. The configurations for both wind farms are presented in the sketch below, turbines are numbered from left to right and top to bottom. Details are given in tab. 10.1.

![Example 1 Configuration](image1)

Example 1 (N=1, M=5, d=4D)

![Example 2 Configuration](image2)

Example 2 (N=5, M=1, d=4D)

Wake effect in the example wind farms

The wake effects in the wind farms Example 1 and Example 2 detailed above has been calculated using the WindFarmer program (Appendix C). The eddy viscosity model proposed by Ainslie (sec. 9.1.3) was used to calculate the wake deficit. The turbulence intensity in the wind farm is calculated from the Quarton formula (sec. 9.2.2).
Table 10.1: The four example wind farms. Summary of parameters used to demonstrate the model predictions

<table>
<thead>
<tr>
<th></th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
<th>Example 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbines alongwind N</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Turbines crosswind M</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Inter turbine spacing (diam)</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Turbulence, amb. at hub (%)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>Wind velocity (m/s)</td>
<td>10.5</td>
<td>10.5</td>
<td>10.5</td>
<td>10.5</td>
</tr>
<tr>
<td>Turbine rated power (kW)</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Turbine control</td>
<td>pitch</td>
<td>pitch</td>
<td>pitch</td>
<td>pitch</td>
</tr>
<tr>
<td>Rotor diameter (m)</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>Terrain</td>
<td>flat</td>
<td>flat</td>
<td>flat</td>
<td>ridge</td>
</tr>
</tbody>
</table>

Table 10.2: Wake effect to Example 1 and Example 2 wind farms. In wind farm Example 2 the turbines are aligned across the wind direction and no wake effect is observed. For the Example 1 wind farm, the wind speed in the wake of the first is reduced. The turbulence intensity at these turbines is increased.
Results of the wake and turbulence calculations are given in tab. 10.2. The wind speed and power production for the wake affected wind turbines 2-5 in Example 1 wind farm are reduced. The turbulence intensity increases for the same turbines due to additional turbulence created. Example 2 wind farm is not wake affected. The modification of the turbulence spectrum is assumed to be equivalent to the changes observed due to the speedup over a hill, but with the speedup factor smaller than 1 (sec. 9.2.3).

Flicker from Example 1 and 2 wind farms

The predicted flicker spectrum from a single wind turbine is given in fig. 10.7. Apart from the low frequency fluctuations only the deterministic $1p$ signal contributes significantly to the voltage flicker.

![Flicker Spectrum](image)

**Figure 10.7:** Flicker spectrum for one turbine. Flicker spectrum for turbine number 5 in Example 1 wind farm
Figure 10.8: Flicker spectrum for Example 1 wind farm. Combined flicker spectrum for the 5 turbines of Example 1 wind farm. The low frequency fluctuations are of increased importance in a wind farm. The relative high coherence along the row of wind turbines leads to a flicker value of $F_{st} = 0.30$.

Figure 10.9: Flicker spectrum for Example 2 wind farm. Combined flicker spectrum of the 5 turbines of Example 2 wind farm. The low coherence across the row of wind turbines results in a low overall flicker value of $F_{st} = 0.28$. 

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The highest flicker is expected for fully coherent output signals. The flicker from Example 1 wind farm would then be $P_{st} = 0.51$ and from Example 2 wind farm $P_{st} = 0.55$. Such coherence values could be observed for a synchronisation of turbine rotors in a wind farm which has been observed to happen [Gerdes94]. This synchronisation is considered to be an extraordinary event.

Generally it can be assumed that the deterministic signals in a wind farm are statistically independent. The stochastic signals at $3p$ are independent because the coherence function even in extreme cases does not show a significant coherence at such a frequency. A fully incoherent addition of the signals the predicted values would yield $P_{st} = 0.23$ and from Example 2 wind farm $P_{st} = 0.25$.

The low frequency fluctuations from wind turbines in a wind farm are not independent. The coherence function at such frequencies and separation distances is significantly different from 0. The combined spectrum from 5 turbines of the same type in Example 1 wind farm is presented in fig. 10.8. The overall predicted flicker value is $P_{st} = 0.30$ and thus exceeds the flicker predicted for incoherent signals.

The same calculation for Example 2, fig. 10.9 wind farm with crosswind configuration results as expected in a lower flicker value of only $P_{st} = 0.26$. The results are also summarised in tab. 10.3.

<table>
<thead>
<tr>
<th>Flicker $P_{st}$</th>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{ij}^2 = 0$</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>Predicted</td>
<td>0.30</td>
<td>0.26</td>
</tr>
<tr>
<td>$\gamma_{ij}^2 = 1$</td>
<td>0.51</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 10.3: Predicted flicker values for the test cases. Predicted flicker for Example 1 and Example 2 wind farm are compared with the maximum and minimum for correlated, not correlated.
10.4.2 Wind turbine array

As a next case a array of wind turbines, Example 3 (N=3, M=3, d=3D) is considered. The example turbine used is a Vestas 39/500, wind speed is 10.5 m/s and ambient turbulence intensity 12%.

Example 3 (N=3, M=3, d=3D)

Example 4 (N=5, M=1, d=3D, terrain)

The results of the wake calculation and single turbine flicker predictions are given in tab. 10.4. The worst case overall flicker for fully coherent signals is \( P_{st} = 0.94 \).

\[ \frac{nS_F(n)}{P_{st}^2} \]

![Flicker spectrum](image)

**Figure 10.10:** Flicker spectrum for Example 3 wind farm. The predicted spectrum for 3x3 array of wind turbines with regular 3 D spacing shows the importance of low frequency fluctuations.
For incoherent signals $P_{st} = 0.31$ is expected.

The predicted flicker spectrum can be seen in fig. 10.10. Deterministic high frequency fluctuations are present at $1P$ and a small peak at $3P$. The relative low frequency fluctuations are increased if compared to the single turbine spectrum. The maximum for these fluctuations is around $1/1\min$. For winds from west at $10.5 \, \text{m/s}$ a flicker of $P_{st} = 0.45$ is predicted for the Example 3 wind farm. This is significantly higher than the $P_{st} = 0.31$ expected for incoherent signals. The results are also summarised in tab. 10.3.

<table>
<thead>
<tr>
<th>Example 3</th>
<th>power output (kW)</th>
<th>wind speed (m/s)</th>
<th>turbulence (%)</th>
<th>$P_{st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine 1</td>
<td>326</td>
<td>10.5</td>
<td>12</td>
<td>0.11</td>
</tr>
<tr>
<td>Turbine 2</td>
<td>326</td>
<td>10.5</td>
<td>12</td>
<td>0.11</td>
</tr>
<tr>
<td>Turbine 3</td>
<td>326</td>
<td>10.5</td>
<td>12</td>
<td>0.11</td>
</tr>
<tr>
<td>Turbine 4</td>
<td>142</td>
<td>7.5</td>
<td>25</td>
<td>0.10</td>
</tr>
<tr>
<td>Turbine 5</td>
<td>142</td>
<td>7.5</td>
<td>25</td>
<td>0.10</td>
</tr>
<tr>
<td>Turbine 6</td>
<td>142</td>
<td>7.5</td>
<td>25</td>
<td>0.10</td>
</tr>
<tr>
<td>Turbine 7</td>
<td>156</td>
<td>7.7</td>
<td>23</td>
<td>0.09</td>
</tr>
<tr>
<td>Turbine 8</td>
<td>156</td>
<td>7.7</td>
<td>23</td>
<td>0.09</td>
</tr>
<tr>
<td>Turbine 9</td>
<td>156</td>
<td>7.7</td>
<td>23</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Table 10.4: Wake effect to Example 3 wind farm.** The example wind farm forms an array, towards the rear of the array a decrease in flicker due to lower wind velocities is observed.

<table>
<thead>
<tr>
<th>Flicker $P_{st}$</th>
<th>Example 3</th>
<th>Example 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_i^2 = 0$</td>
<td>0.31</td>
<td>0.38</td>
</tr>
<tr>
<td>Predicted</td>
<td>0.45</td>
<td>0.39</td>
</tr>
<tr>
<td>$\gamma_i^2 = 1$</td>
<td>0.94</td>
<td>0.86</td>
</tr>
</tbody>
</table>

**Table 10.5: Predicted flicker values for the test cases.** Predicted flicker for Example 3 and Example 4 wind farms is compared with the maximum and minimum for correlated, not correlated.
10.4.3 Linear wind farm on a ridge

The last example investigated is Example 4 wind farm. This linear wind farm (N=5, M=1, d=3D) is located on a ridge. The speed up across the ridge is 1.5, the turbulence intensity measured on the ridge at hub height is 12%. The wind speed considered is again 10.5 m/s at hub height. The wind blows across the ridge. Wake effects are not expected in this situation. Turbulence intensity and wind speed remain unchanged. The power output for all turbines is 326 kW.

The 5 individual turbines in this configuration produce a flicker of $P_{st} = 0.17$, (fig. 10.11). The increase in flicker per turbine compared to Example 2 wind farm is due to the terrain effect. A maximum of $P_{st} = 0.86$ would be reached for a fully coherent signal, an incoherent signal would result in $P_{st} = 0.38$. The predicted value of $P_{st} = 0.39$ is only slightly above this value. The results are also summarised in tab. 10.3 together with the other example wind farms.

![Flicker spectrum](image)

**Figure 10.11:** Flicker spectrum for Example 4 wind farm. The predicted spectrum for linear wind farm along a ridge shows an increased flicker due to the effect of the terrain on the wind fluctuations.
10.5 Sensitivity to parameter variation

The flicker from a wind farm depends on a number of parameters. The following table indicates the significance of the main input parameters. The values are given relative to a reference case also shown in tab. 10.6.

<table>
<thead>
<tr>
<th>Value</th>
<th>Symbol</th>
<th>reference value</th>
<th>typical variation</th>
<th>effect on flicker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind speed</td>
<td>$u$</td>
<td>10</td>
<td>5 ... 20</td>
<td>0.5 ... 2</td>
</tr>
<tr>
<td>Turbulence</td>
<td>$I$</td>
<td>0.12</td>
<td>0.08 ... 0.25</td>
<td>0.7 ... 1.5</td>
</tr>
<tr>
<td>Topography</td>
<td>$s$</td>
<td>1.0</td>
<td>0.6 ... 1.6</td>
<td>0.6 ... 1.6</td>
</tr>
<tr>
<td>Wake</td>
<td>$s$</td>
<td>1.0</td>
<td>0.6 ... 1.0</td>
<td>0.6 ... 1.0</td>
</tr>
<tr>
<td>Diameter</td>
<td>$D$</td>
<td>40</td>
<td>30 ... 70</td>
<td>0.75 ... 1.75</td>
</tr>
<tr>
<td>Frequency</td>
<td>$n_0$</td>
<td>35</td>
<td>20 ... 40</td>
<td>0.9 ... 1.1</td>
</tr>
<tr>
<td>Generator slope</td>
<td>$\phi_D$</td>
<td>25</td>
<td>10 ... 40</td>
<td>0.85 ... 1.3</td>
</tr>
<tr>
<td>Impedance</td>
<td>$X_{sc}/R_{sc}$</td>
<td>1</td>
<td>1 ... 2</td>
<td>1 ... 0.7</td>
</tr>
<tr>
<td>Short circuit power</td>
<td>$P_{sc}$</td>
<td>20</td>
<td>20 ... 100</td>
<td>1 ... 0.2</td>
</tr>
</tbody>
</table>

Table 10.6: Sensitivity to parameter variation. The typical variation and importance of different parameters on the voltage flicker is presented. Of dominant influence is the short circuit power of the network at the point of common connection.

Clearly the short circuit power of the connecting network is the most important factor influencing flicker. For a given network the turbine parameters influence the flicker to the same degree as the site specific parameters. For given network and turbine parameters the site specific parameters and wind farm layout can still lead to significant variations of the flicker level caused. Thought should also be given to avoid producing lp power fluctuations and to reduce remaining levels by active filter systems. This makes it worthwhile to consider the flicker caused by a wind farm as a design criterion.
10.6 Summary - model validation

The proposed flicker prediction model has been compared to the measured flicker for three single turbines. The model has been tested in this chapter against published experimental data for a pitch and two stall regulated turbines and shown general agreement for all wind speeds. Very good agreement between the model predictions and the measurements has been observed for the pitch regulated turbine up to 15 m/s wind speed.

The following observations have been made:

**Low frequencies** Slow wind fluctuations are important to the flicker prediction.

**Coherence** The coherence has to be considered when adding flicker contributions from a group of wind turbines.

**Deterministic components** A second important factor is the magnitude of deterministic components at 1P - one times the rotational frequency.

**Turbulence** The flicker is proportional to the turbulence intensity.

**Wind speed** Voltage fluctuations and flicker increase with wind speed.

The model allows an easy prediction of the flicker potentially caused by a wind turbine with only few, publicly accessible basic parameters. More data and information on the turbines is needed to improve the prediction of the flicker in some areas, especially for wind speeds above 15 m/s. The influence of rotor imbalances and low frequency fluctuations needs to be modelled in more detail to improve the predictions.

Predictions of the flicker of several example wind farms have been made. The presented model predicts, depending on the wind farm configuration, flicker values which are larger than one would expect for incoherent signals. While high frequency components of the flicker spectrum are reduced in a wind farm, low frequency fluctuations are of increased importance.
Chapter 11

Conclusions and outlook

Apart from producing energy, wind turbines can have a positive influence on the voltage levels and stabilise weak networks. On the other hand wind turbines can, even in normal operation, cause voltage flicker.

Flicker is caused by the turbulent nature of the wind resource and its interaction with the wind turbines. Flicker is also caused by deterministic components in the power output of a wind turbine. Flicker can also be caused by repeated transient events, like the connecting or disconnecting of generators. Flicker from transient changes can be avoided, using for example soft start units of sufficient quality.

In this thesis only the flicker originating from turbines during normal operation has been considered. The flicker is modified by terrain, turbine design, wind farm layout and the grid structure.

The aim of the research presented here was to develop a computational framework that allows one to predict the voltage flicker while the wind farm is still on the drawing board. To be useful in practice this prediction has to be based on a set of parameters that can realistically be expected to be available to a wind farm developer.

This chapter summarises the framework proposed, presents conclusions drawn from the analysis and gives recommendations for further development of a flicker prediction tool.
11.1 Wind field modelling

A standard wind turbulence spectrum has been adapted to be applicable to complex terrain. It is proposed that the spectrum over a hill is modified by a frequency shift towards lower normalised frequencies. The model predictions are supported by experimental observations but more data is needed to confirm the observed shift of the spectrum. It is suggested that the turbulence spectrum for a wind turbine in complex terrain be approximated by using standard wind flow modelling software to derive the speed up and subsequently use this information to generate a modified turbulence spectrum.

A new coherence model has been developed and validated which improves the description of the wind field coherence for large separation distances. The new description takes account of the relationship between the coherence and the turbulence intensity, which the standard Davenport model does not. The lateral coherence in this new model no longer depends on the wind velocity. The coherence model can be modified to include the effect of the topography represented by a speed up factor, in a manner similar to the turbulence spectrum. Experimental data has been presented to support the new coherence model but further experimental data is needed, especially to confirm the lack of dependence of the lateral coherence on the wind speed and to support the proposed adaption of the coherence function for complex terrain.

11.2 Wind turbine model

The flicker caused by a group of wind turbines depends not only on the wind fluctuations but also on the individual turbine characteristic. It is suggested that these can be satisfactorily modelled in a simplified fashion.

The influence of the rotor can be described using a frequency domain based model for the rotational sampling. Standard, isotropic coherence models have been compared and are found to be generally adequate for this purpose. An approximation to a standard coherence model was used in this thesis to improve computational efficiency whilst retaining sufficient accuracy.

Deterministic components at the blade passing frequency of the rotor have been
modelled as a fixed percentage of the 3p component introduced by the rotational sampling of the wind field. The modelling of the amplitude and width of these deterministic peaks is thus approximate and needs to be refined when better models and more experimental data on the deterministic components become available.

The power train for both pitch and stall regulated turbines has been modelled by a 'reasonable' design based on published work. The first resonance frequency of this power train is above the first rotational frequency, the second at twenty times the rotational frequency. Feasible values have been chosen for the frequencies and damping constants. These constants should be reviewed if detailed knowledge on the turbine under investigation is available to the analyst. The power train model does currently not distinguish between the power trains of stall and pitch regulated turbines. This is a subject deserving further attention.

Pitch regulated turbines are generally subject to higher torque fluctuations and thus cause higher voltage flicker than otherwise equivalent stall regulated turbines. It is proposed to model this effect by the inclusion of a simple induction lag filter. Experimental data confirming the existence and magnitude of this induction lag were not available. The accuracy of the induction lag model and alternative models need to be subject of further investigation.

11.3 Grid integration of a group of wind turbines

The measurement of voltage flicker requires the use of a flickermeter conforming to the relevant international standard. A fast flickermeter algorithm (FD-flickermeter) has been developed in this research which works in the frequency domain. The FD-flickermeter avoids time consuming conversion of data between the time and frequency domain. It allows a direct calculation of the $P_{f}$ value from the spectrum of the voltage fluctuations. The performance of this flickermeter algorithm has been assessed and found to be accurate.

Models for wind turbine wakes and turbulence levels within wind farms have been reviewed and suitable models recommended. Little is known about turbulence
spectra in wake situations. It is suggested to treat the modification of the turbulence spectrum in the area of reduced wind speed downstream of a turbine in a manner equivalent to that of increased wind speed on top of a hill. This is a pragmatic approach and further research is needed to assess what changes in the spectrum can be expected within a wind farm and its consequences for the voltage flicker.

11.4 Conclusions

The aim of this work was to develop a method to assess the overall flicker effect of a group of wind turbines before it has been constructed. This would give utilities and wind farm developers a valuable tool to detect flicker issues caused by the integration of wind farms into the utility network before they become apparent. A frequency domain model has been developed to enable the user to efficiently predict the voltage flicker. The theoretical models of wind turbulence spectra and spatial coherence have been expanded for this purpose and a flickermeter representation in the frequency domain was developed. In other areas simplifications have been made for computational efficiency and to keep the number of parameters necessary to predict the flicker levels to a minimum. Individual components and the overall model have been, where possible, validated against experimental data and the results are encouraging.

The resulting model allows a fast prediction of the flicker caused by wind turbines with only few parameters that are accessible to a typical wind farm developer. The principle of flicker assessment in the frequency domain has been demonstrated with this model. Further work is needed to validate and refine the model.

11.5 Outlook

The important factors leading to flicker from wind turbines have been considered in this thesis. However, some of the suggested models need further evaluation and some simplifications should be replaced by more complete models, once such models become available. Further research should be carried out to achieve a
refinement of the present model. This research should focus especially on the
description of the wind turbine.
A detailed comparison of the model predictions with the measured spectrum of the
power output from a number of turbines should give valuable information. This
may help to verify and improve the models for the amplitude and width of the
deterministic peaks in the spectrum. Control specific differences between stall and
pitch controlled turbines should be investigated and modelled in greater detail.
Results and performance of the frequency domain model should be compared with
those of a time domain model.
It should also be attempted to obtain experimental data to improve the under-
standing and modelling of wind turbulence spectra and coherence spectra in a
wind farm environment and in complex terrain. Detailed data from a wind farm
should be obtained, that allow both, a detailed validation of the model equations
for each turbine and a validation of the complete wind farm model.
The modelling has been carried out in MATLAB. Some specific Matlab functions
have been used throughout to process measured data and compare them with the
models developed. The flicker model itself however does not require any in depth
Matlab programming and can easily be transferred to any other programming
environment. It is proposed that the model could be incorporated into one of the
standard wind farm design codes.
Appendix A

List of Publications

Some of the results presented in this thesis have been published as contributions to conferences or articles in journals, they are listed below.

Journal: [Schlez96a] This article presents the basic idea and motivation for this thesis as well as some experimental data.

Conference: [Schlez96b] The paper presents experimental results for spectra and coherence functions in complex terrain.

Conference: [Schlez96c] The (oral) paper describes the modified Kaimal spectrum for flat terrain and the influence of the terrain on the turbulence spectrum (chap. 2 and chap. 3).

Conference: [Schlez97] The (oral) paper discusses models and experimental results for the coherence in flat and complex terrain (chap. 4 and chap. 5).

Journal: [Schlez98] The article contains a detailed motivation, discussion and validation of the coherence model for large separation distances and flat terrain. A condensed version of this discussion is part of chap. 4 and chap. 5.

Conference: [Schlez99b] The paper describes the FD flickermeter (chap. 7).

Conference: [Schlez99a] The paper presents the application of the coherence to calculate the lumped power output in a wind farm (chap. 9).

A copy of the journal publications is attached below. Further publications are being prepared.
Appendix B

Evaluation of experimental data

In many chapters of this thesis experimental data is evaluated and compared with the developed theoretical models. The raw data need to undergo a process of selection and cleaning. The problems and steps taken to obtain the maximum amount of reliable information are described here.

![Hanning Filter](image)

**Figure B.1: Hanning window.** A Hanning window of 10 min length is used as a low pass filter to identify and remove trends in the time domain.
Figure B.2: Demonstration of the detrending of a time series. The original signal, sampled at 2 Hz, was filtered by a low pass filter to obtain the underlying trend (smooth curve).

B.1 Wind speed and direction

Requirements for wind speed and direction data are the removal of erroneous measurements, stationarity and trend removal. Dynamic effects, like sudden changes of wind direction and wind speed are excluded from the analysis and have to be considered separately.

B.1.1 Trend removal

The data files have been searched manually for records with reasonably stationary wind speed time series. The trend of each of the selected time series has been calculated using a low pass filter in the time domain. A Hanning window as shown in fig. B.1 has been used throughout. The fig. B.2 shows an example time series and the resulting low pass filtered time series.
Figure B.3: Detrending of a time series. The smoothed time series (see figure above) is centred and subtracted from the original time series. The picture presents the detrended timeseries as used for further evaluations.

The width of the Hanning window was generally set to 10 min. For demonstration purposes a window length of 3 min has been used in the example presented here. The smooth trend obtained is first centred and then subtracted from the original time series, fig. B.3 shows the time series with the trend removed.

B.1.2 Wind direction

The method used to obtain mean and variance from a circular distribution has been described in detail in [Fisher83a, Fisher83b, Fisher87]. The wind direction data have then been classified in wind direction bins and faulty data were removed. These can consist of data recorded during an instrument failure or data which are likely to be influenced by obstacles e.g. by the measurement tower, the guys or the mounting of the measurement equipment. The criterion for selection of a time series was reasonable stationarity in wind direction, ideally with a deviation from the mean wind direction of less than 10 degrees.
B.2 Spectral representation

Autospectra or short spectra are a representation of the variance of a time series in the frequency domain. In the evaluation of spectra, special care has been taken to avoid wrong results as effect of the limited length (bandwidth) of the measured signal. The pollution of the results through frequency leakage and aliasing has been reduced as far as possible. All spectral representations in this thesis are presented using the same methodology. This enables the reader to easily compare spectra of torque, power, voltage, flicker or wind speed.

B.2.1 Aliasing

The sampling of high frequency variations with a low sampling frequency results in 'aliasing', that is folding of the high frequency part of the spectrum to lower frequencies. To avoid aliasing a suitable low pass filter is applied to the original signal when it is recorded. Once the sampling of the data is completed no distinction between the original and the aliased frequencies can be made.

B.2.2 Frequency leakage

Analysing a finite time series via Fast Fourier Transformation (FFT) is equivalent to cutting one interval out of a infinite time series using a (rectangular) boxcar window. This implicitly applied boxcar function influences the result of the FFT by adding its contributions to all frequencies; this effect is called frequency 'leakage'. Leakage effects can be reduced by tapering the data e.g. with a Hanning Window (fig. B.1) which was used in this thesis.

B.2.3 Bandwidth correction

Often spectra are normalised by their measured variance and then compared with the normalised theoretical models, for example the (modified) Kaimal spectrum (eq. 2.18). This use of the 'integral criterion' is very useful in practice, although [Olesen84] warns that its use causes difficulties if the bandwidth of the measurement is not properly accounted for.
This is only true if the integration is performed over the full range of frequencies from 0 to infinity. In practice the length of a time series is always limited. This results in a limited bandwidth for the spectral representation of the time series. If such an experimental spectrum is normalised by its variance, the integral of the spectrum over the evaluated bandwidth becomes unity, whereas the theoretical spectrum integrated over the same bandwidth is less than unity. The error made has been studied by [Olesen84] for different types of spectral representations. The graph fig. B.4 represents the error made using the integral criterion for the Kaimal spectrum (eq. 2.18) at 15 m height for different low and high cut off frequencies. For example, the error obtained with an upper (normalised) frequency limit of \( f = 1 \) is 9%.

To compare experimental spectra with theoretical models a correction is introduced to account for the limited bandwidth. Each spectrum is normalised by its variance which is represented by the integral over the spectrum. The normalised

![Figure B.4: Error caused by limitation of the bandwidth. The spectral estimate is erroneous, the dependency of this error on low and high cut off frequencies is shown in the figure for the Kaimal spectrum used in this thesis \((h=15m)\).](image)

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spectra are then multiplied with the integral of the theoretical spectrum over the experimental bandwidth. For a time series of 10 min length, sampling rate 0.2 s, wind velocity 10 m/s and height=15 m e.g. this correction is approx. 0.73 and this is far from insignificant.

B.2.4 Area true representation

The integral over a spectrum gives the total variance of the analysed time series:

\[ \sigma^2 = \int_0^{\infty} S(n)dn = \int_0^{\infty} n \cdot S(n)d\ln(n) \]  

(B.1)

We present all spectra as \( \frac{nS(n)}{\sigma^2} \), over a logarithmic frequency scale, so that equal areas represent equal shares of the total variance. Different methods of presentation are compared in [Stull88], chap. 8.6.

Turbulence spectra are presented in straightforward application of eq. B.1. Each unit area under the spectrum shows the contribution to the total variance at the frequency it is displayed.

Flicker spectrum

Flicker spectra show the contributions at each frequency to the square of the short term flicker severity \( P_{st} \). Equivalent to eq. B.1 we can write:

\[ P_{st}^2 = \int_0^{\infty} S(n)dn = \int_0^{\infty} n \cdot S(n)d\ln(n) \]  

(B.2)

The short term flicker severity \( P_{st} \) has the dimension of a standard deviation.

Other spectra

Spectra of voltage, torque or power output are presented equivalent to eq. B.1. Some of these spectra have not been normalised and present \( nS(n) \), over a logarithmic frequency scale, so that equal areas represent equal relative contributions to the variance.
B.3 Coherence

The quality of coherence information is critically dependent on the quality of data, but also depends on the algorithm used to extract the information. Bias and Variance of the coherence are sometimes presented as results rather than being reduced to the least possible values and then interpreted as numerical uncertainties. Special care has been taken in this thesis to reduce the numerical uncertainties as far as this is possible.

B.3.1 Coherence estimate

The coherence was calculated by dividing the two time series \(Y\) and \(Z\) into \(s = 1..N\) segments. With the estimators \(S_{YY}, S_{ZZ}, Q_{YZ}, C_{YZ}\) for spectral densities, quad and co spectra respectively, we can express the coherence estimator for discrete frequencies \(n_i\) [Carter71]:

\[
|\langle \gamma(n_i) \rangle|^2 = \frac{\left[ \frac{1}{N} \sum_{j=1}^{N} C_{YZ,j}(n_i) \right]^2 + \left[ \frac{1}{N} \sum_{j=1}^{N} Q_{YZ,j}(n_i) \right]^2}{\left[ \frac{1}{N} \sum_{j=1}^{N} S_{XX,j}(n_i) \right] \left[ \frac{1}{N} \sum_{j=1}^{N} S_{YY,j}(n_i) \right]}
\]  

(B.3)

It has to be stressed here, that if the coherence estimate is calculated without subdividing the time series into \(N\) segments and averaging the spectra and cross spectra over these segments, then the coherence estimator will always be unity for all frequencies [Bendat71]. This follows as a direct consequence of the definition of the coherence function and the representation of spectra, co- and quad spectra with discrete Fourier coefficients [Carter72, Kristensen86, Handwerker93].

The coherence estimate obtained by eq. B.3 deviates from the true coherence of the underlying process. The error of such estimates can be described by its variance and a bias. The work of [Goodman57] and [Amos63] derives bias and variance of the coherence estimator for two random processes that follow an identical gaussian statistic. This work was expanded on by [Carter72], its applicability to the coherence estimates of wind measurements is suggested in [Kristensen86].

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B.3.2 Bias

The bias $B_c$ and the variance $V_c$ of the coherence estimate are functions of the finite number of segments $N$ a finite time series can be split into. For non overlapped segments the bias is given by [Carter72] as:

$$B_0 = \frac{1}{N} - \frac{2}{N+1} \gamma^2 + \frac{1}{N(N+1)(N+2)} \gamma^2$$

$$+ \frac{2}{N(N+1)(N+2)(N+3)} \gamma^3$$

$$B_c = \begin{cases} B_0 & \text{if } B_0 > 0 \\ 0 & \text{otherwise} \end{cases}$$

(B.4)

which can be approximated for large $N$ by

$$B_c \approx \frac{1}{N} (1 - \gamma^2)^2$$

(B.5)

When a time series is for example subdivided into 10 intervals, the coherence estimate will always be greater than 0.1 for all frequencies. Subdividing a time series into 100 intervals will give a minimum coherence of 0.01 but the length of the time series and thus low frequencies may not be resolved with sufficient precision.

B.3.3 Variance

For the variance $V_c$ of non overlapped segments [Carter72] obtains:

$$V_0 \approx \frac{(N-1)}{N(N+1)} \left( \frac{1}{N} + \frac{2}{N+2} \gamma^2 - 2 \frac{2N^2 - N^3 - 2N + 3}{(N+1)(N+2)(N+3)} \gamma^2 \right)$$

$$+ 2 \frac{N^2 - 6N^3 - N^4 + 10N - 6}{(N+1)(N+2)(N+3)(N+4)} \gamma^3$$

$$+ \frac{13N^5 - 15N^4 - 113N^3 + 27N^2 + 136N - 120}{(N+1)(N+2)^2(N+3)(N+4)(N+5)} \gamma^4$$

(B.6)

$$V_c = \begin{cases} V_0 & \text{if } V_0 > 0 \\ 0 & \text{otherwise} \end{cases}$$

This formula for the variance can be approximated for large $N$ by:
Using this approximation for a pair of time series with a true coherence of $\gamma^2 = 0.33$ and $N = 10$ gives a variance of 0.017 for the coherence estimate.

The graphs in chap. 5 present the coherence estimate and the variance and bias calculated on the basis of this estimate. The variance and bias are presented using a single set of error-bars. For example a coherence estimate of $\gamma^2 = 0.33$ with $B_c = 0.08$ and $V_c = 0.017$ is presented as $0.33 \pm 0.13$. The combination of both bias and variance in the graphs is used to simplify the presentation, but it should be kept in mind that the indicated error is composed of two effects of different nature.

**B.3.4 Numerical application**

In order to obtain a good estimate for the coherence, it is desirable to split the time series in as many intervals as possible. This minimises the bias and variance of the estimate. However, the greater the number of segments, the smaller are the individual segments which limits the frequency resolution of the Fourier transformed data.

In order to retain maximum spectral resolution and reduce the bias and variance, an algorithm was developed which calculates for the same time series the coherence estimate with different, large and small numbers of segments and then combines the results. As $N$ is not necessarily large the variance of the experimental results is calculated using eq. B.6 instead of eq. B.7.

The coherence has been calculated with the function cohere.m from the MATLAB signal processing toolbox, according to eq. B.3. The data are split into a number of overlapping segments (50% overlap) which are individually weighted with a Hanning window. The choice of the number and overlapping of segments is based on the theoretical derivations and recommendations in [Carter72].

Overlapping of the segments prohibits the exact determination of the occurring bias and variance as the derivation of the above quoted formulae (eq. B.4 and eq. B.6) are no longer valid. However the equations can be used to provide as upper bound to the actual bias and variance in the coherence estimate.
Appendix C

Software packages

The following software packages have been used to prepare this thesis.

MATLAB is the computing environment sold by The Math Works Inc. Most of the calculations in this thesis have been carried out using MATLAB Version 4, including the signal processing and the signals and systems toolbox [MATLAB94, MATLAB95].

WASp is the wind flow model sold by Risø National Laboratories Denmark. WASp version 4.0 has been used in this thesis for wind flow modelling [Troen88].

GNUPLOT the command-driven interactive function and data plotting program is free software, Copyright (C) 1986 - 1993, 1998 Thomas Williams. The program has been used in most parts of this thesis for function plots.

TextPad is a file editor by Helios Software Solutions.

WindFarmer is a wind farm layout and optimisation software developed and sold by Garrad Hassan and Partners Ltd. Wind farm wake loss and turbulence intensity in wind farms was calculated with WindFarmer.
\LaTeX\ The layout of this thesis was designed using the text layout package \LaTeX. The main advantages of the \LaTeX\ software are:

- free
- available for nearly all platforms
- high flexibility
- portability between all platforms
- high quality layout
- designed to handle large scientific documents
- printer independent layout
- style file for many scientific journals available

\LaTeX\ is a text layout system, expanding the software package TeX. Distributions of \LaTeX\ like the EM\LaTeX\ distribution used in this thesis combine the core program with a number of useful tools. Information about TeX are available from:

\textit{CTAN, the Comprehensive TeX Archive Network.}

\url{http://www.ctan.org}

or for the UK

\url{http://www.tex.ac.uk}

Distributions of \LaTeX\ are available for nearly all computer systems from a network of public servers (CTAN). Most programs and macro packages are available in source code or designed in a way to allow the user to adjust the system. This open system works extremely well and through co-operation and contributions of a large number of TeX users the program is constantly updated and improved. I would like to express my sincere thanks to all individuals participating in this project.
Appendix D

Turbine characteristics

This chapter presents the turbine parameters used in chap. 10. Turbines in this thesis are identified by rotor diameter and nominal rated power. A fictitious turbine 'Manu 30/400' is a turbine of the manufacturer 'Manu' has got a rotor diameter of 30 m and a nominal rated power of 400 kW. The turbines used in this thesis are Vestas 39/500, Nordtank 28/300 and Nordtank 37/500.

Nordtank 28/300 The Nordtank 300 kW turbine was located in the Norrekaer Enge wind farm [Taylor90]. Unfortunately this is not the 300 kW model with 31 m diameter which has been widely sold and is well documented. Thus the power curve which is independently measured and published for the 31 m diameter model had to be scaled to the smaller diameter turbine.

Nordtank 37/500 The Nordtank 37/500 was in the early nineties a standard turbine. The manufacturer Nordtank is now part of NEG Micon. Information on both the Nordtank 30/300 and the Nordtank 37/500 was obtained from NEG Micon.

Vestas 39/500 The Vestas 39/500 was in the nineties a standard turbine. Data on the turbine has been received directly from the manufacturer. The power curve and electrical properties have been independently measured by Windtest Kaiser Wilhelm Koog GmbH, 1994.
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* Maximum of power curve

** Given for Nordtank 30/300
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Publications
Meteorological and topological factors influencing the power quality of wind turbine electricity generation from wind farms in complex terrain

W. Schlez* and D. G. Infield**

SYNOPSIS
Many wind turbine sites in Europe are located in remote areas, many of these also in complex terrain, such as hill tops or mountain ridges. In these areas, often supplied by weak grids, the power quality resulting from the interconnection of wind farms will be an important factor for the design of wind farms. As studies to date have concentrated on power quality for wind farms located in flat terrain, the effect of complex terrain on the power quality of wind turbines has, up to now, not been thoroughly investigated.

Spectral density functions calculated from wind speed data collected in moderately complex terrain are presented. These give important information required for modelling the impact of embedded wind turbine generation, and also suggest the direction of future work. The indications are that calculations of the power quality should take into account the effects of complex terrain. This is true especially for one of the important parameters related to power quality: the dynamic voltage variation known as flicker.

INTRODUCTION
Concern has already been raised about impact of wind farms on the voltage quality of the grid near the point of interconnection. Good wind sites are often located away from strong sections of the electricity grid. As the sites situated close to strong grids are used up, there will be an increasing pressure to use sites which are less attractive from the point of view of electrical connection, and therefore a need will develop for assessment of the impact of wind turbines and wind farms on the grid locally.

Grid connected wind turbines. The impact of wind turbines on the electrical grid in general has been investigated by different authors [1, 2]. For wind farms interconnected to the grid three main problem areas can be identified: power fluctuations, harmonics and voltage variations.

As wind energy is a decentralised source of energy the generators are more likely to be placed on the far end of existing grid extensions. At these points the grid is quite often "weak" in the sense that the voltage is sensitive to power variations caused by loads or generators. These short term (0.005–25 Hz) voltage fluctuations known as flicker may cause people disturbance. This paper concentrates on the meteorological factors which are considered to influence the flicker effect.

Flicker spectrum. Dynamic voltage fluctuations on the grid cause variations of the light intensity in incandescent light bulbs. These variations disturb human visual perception, resulting in headaches etc. The human perceptibility for light
Variations has been tested and from the results a standard has been developed which aims to avoid any significant disturbance to people. The human perceptibility for voltage variations increases from low frequencies to higher frequencies with a maximum at about 17 Hz. This perceptibility is reflected in the flicker standard as shown in Figure 1, taken from [3].

Wind spectrum. Flicker caused by wind turbines can result from different causes such as the variation in wind speed, aerodynamical influence of the wind turbine tower or the drive train dynamics. Here we concentrate on the meteorological aspects. The power spectral density function (spectrum) is a representation of a time series in the frequency domain. The Kaimal spectrum equation (1) is widely accepted to be representative of sites situated in flat terrain [4].

\[
\frac{n \cdot S(n)}{\sigma^2} = \frac{0.164(f_{\text{ref}})}{1 + 0.164(f_{\text{ref}})^{2/3}}
\]

Where:

- \( S \): power spectral density
- \( n \): frequency
- \( \sigma^2 \): variance of time series
- \( f \): normalised dimensionless frequency \( n \cdot z/u \)
- \( z \): height above ground
- \( u \): mean wind speed
- \( f_{\text{ref}} \): reference normalised dimensionless frequency.

All Kaimal spectra shown here are calculated with:

\[ f_{\text{ref}} = 0.0144 \text{Hz} \left( \frac{z}{30 \text{ m}} \right)^{0.78} \]
as proposed by Frost [5].

Turbulence and turbulence spectra. There are not many measurements of wind spectra available for complex terrain. Most wind measurements concentrate on speed up effects, and theoretical models have been derived for such purposes. A good summary of measurements in complex terrain can be found in [7].

Speed up effects. The power output of a wind turbine depends on characteristics of the wind, such as speed and turbulence. Furthermore the mean wind speed at the rotor determines the operational state of the wind turbine (i.e. its control mode) as well as the reduced wind velocities in the flow behind the turbine (wake) influencing the state of downwind turbines. The nature of power control obviously influences level and quality of the power output from a turbine and so influences the voltage variation.

The increase of wind speeds over hilltops has been measured (e.g. at Askervein hill [8]) and...
Meteorological and topological factors influencing power quality from wind farms in complex terrain

Schizel, In-field

also modelled by different groups. Complex fluid dynamics models and simple approximations have both been applied to this problem. Predictions have been found to be of uncertain accuracy. The expected error is in the range of 2 to 3 m/s, which is not acceptable for commercial wind power applications (e.g. see [9]). Nevertheless as flicker can generally be interpreted as a consequence of a disturbance to the steady state conditions, the absolute value of the mean wind speed (steady state) is less important than its variance (disturbance). Therefore the precision of the predictions gained by available models seems to be adequate for flicker analysis.

Increased turbulence in complex terrain. Considering a single hill located in otherwise flat terrain, the variance of the wind stays constant over the hill and only the mean wind speed increases, therefore the turbulence intensity over the hill decreases. This approach is supported by the Engineering Sciences Data Unit (ESDU) [10] for slopes with less than 5%, see also Panofsky et al. [7].

For rolling terrain an increase in variance and turbulence intensity has been reported for low frequencies [7, 11] and it has been suggested that the Kaimal spectrum is not valid in this region. However, similar deviations from spectra have also been measured in wind originating directly from the sea [12]. Furthermore, the low frequency region of the spectra is strongly dependent on the atmospheric stability [4, 13]. Nevertheless it is at least plausible to expect an increased roughness length for the rolling terrain to explain the increased turbulence (see [5] pp. 404-409).

Modified Kaimal spectra. Different authors have tried to modify the Kaimal spectrum for complex terrain. One possibility is to introduce additional variables into the equation in order to fit the Kaimal shaped form to the existing data [11, 14]. This empirical approach is not applicable for our purpose as we need to predict spectra rather than analyse existing data.

As a parametrisation of the power spectral density function with terrain characteristics is yet to be developed, further work has to be based on the original Kaimal form. Thus contrary to the experimental results mentioned above, it may be acceptable to use a simple approach for complex terrain. Rather than varying the spectral shape, a uniform scaling factor is proposed, to take into account the increased variance. This is elaborated on in the next section, and supported by experimental evidence.

Experimental data. Three different sets of data supplied by National Wind Power have been used for evaluation. Two sets from Cold Northcott (Cornwall) and a third from Cemmaes (Central Wales). Both locations are wind farm sites in the UK. They are both sites of complex terrain. Cold Northcott less so than Cemmaes.

Data from Cold Northcott mast. The first data set from Cold Northcott consists of time series collected with a sampling rate of 2 Hz from a cup anemometer mounted at 25 m height on a meteorological mast. For these data the power spectral density function has been calculated and is shown in Figure 2 together with the theoretical Kaimal spectrum given by equation 1.

The presented graph is an ensemble average of 4 spectral representations of measured time

![Figure 2](norma.png)

**Figure 2** Normalised Cold Northcott wind spectrum and Kaimal spectrum, ensemble of 4 time series, averaged over frequency. The loglinear part of the spectrum can be seen. An instrument response drop off occurs from approximately 0.5 Hz.
series. The ensemble is subsequently averaged within frequency intervals. The average wind speed for the data set was 9.3 m/s. From about 0.5 Hz the instrument response drop-off can be identified, the range from 0.05 to 0.5 Hz shows the expected $-5/3$ power law characteristic expected in the inertial subrange [15, 16].

Data from Cold Northcott wind turbine. The second data set from Cold Northcott are time series measured with a cup anemometer mounted on the nacelle of a wind turbine at 25 m hub height and consists of data recorded for a different purpose with a sample frequency of 50 Hz, see Figure 3. The data represent an ensemble of 2 time series, averaged over frequency intervals. The log-linear part of the spectrum can be identified, and the roll-off occurs from approximately 0.7 Hz.

The data from Cold Northcott wind turbine. The second data set from Cold Northcott are time series measured with a cup anemometer mounted on the nacelle of a wind turbine at 25 m hub height and consists of data recorded for a different purpose with a sample frequency of 50 Hz, see Figure 3. The data represent an ensemble of 2 time series, averaged over frequency intervals. The log-linear part of the spectrum can be identified, and the roll-off occurs from approximately 0.7 Hz.

Data from Cemmaes. The third data set was recorded at Cemmaes with a sampling rate of 1 Hz from a cup anemometer mounted at 25 m height on a meteorological mast. The average wind speed for the data shown is about 8 m/s. Shown is the normalised spectrum for one data set of 4096 sec. The data is averaged in frequency bins. The spectrum shown in Figure 4 is in agreement with the one proposed by Kaimal.
is again in relative agreement with the shape proposed by Kaimal and does not support the conclusions reported in [7].

Experimental results. Despite the difficulties in measuring appropriate wind time series for their spectral evaluation, the results seem to suggest that the use of a Kaimal shaped function as approximation for the spectrum in complex terrain seems to be valid in the frequency range of interest, i.e. from 0.01 to 2 Hz. Above 2 Hz the level of wind speed variance given by the power spectral density is not significant. For lower frequencies than 0.01 Hz a levelling off of the spectra is found whereas simultaneously the human perceptibility for flicker decreases (see Figure 1). Thus further modelling work will concentrate on frequencies in the range 0.01 to 2 Hz.

FURTHER WORK

Wind statistics. Future evaluation of wind time series are planned to confirm the use of the Kaimal function as a useful approximation for the power spectral density even in complex terrain.

For calculating the flicker effects of a wind farm it is also necessary to obtain information about the coherence of the wind field in complex terrain.

Modelling. For the complete study other effects such as the increased turbulence in wind farms, tower shadow, the drive train dynamics and, last but not least, the grid characteristics have to be included.

In order to provide a practical modelling tool accessible to wind farm developers, simplifications in the wind farm modelling will have to be made. Nevertheless, previous work [17] suggests that a suitable model can be developed which will predict the induced flicker on a weak network for a perspective wind farm site.

CONCLUSIONS

As the first part of a study on the flicker effect due to wind farms, theoretical models for wind fluctuations have been compared with measurements. The spectral distribution of the wind fluctuations in complex terrain is different from that in flat terrain. Nevertheless it seems possible to use the flat terrain Kaimal spectrum as the basis for an acceptable approximation. The main effect would be taken into account simply by increasing the turbulence level. Further related work will include the spatial coherence of wind time series and wake effects within the wind farm.

ACKNOWLEDGEMENTS

This study is funded by National Wind Power. The authors thank BMT Fluid Dynamics and the Wind Energy Group for providing measured data through National Wind Power.

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Abstract. Wind speed measurements from the test site at Rutherford Appleton Laboratory have been evaluated with respect to the spatial coherence function. The experimental arrangement provides coherence information for separation distances of 62.80 and 102 m. These are at least three times greater than the measurement heights of 18 m and 18.7 m. Based on these experimental data and data published in the literature, different theoretical formulations are compared and a new, but simple, model for longitudinal and lateral coherence is proposed. At large separations the turbulent wind field is not isotropic, and theoretical models to describe the coherence function for such distances are not available. The new model we propose builds on the classical exponential approach. It takes into account the influence of turbulence intensity, and models the angular dependence of horizontal coherence. It is found that, for constant turbulence intensity, the lateral coherence decay becomes independent of the mean wind speed.

Key words: Coherence model. Spatial coherence. Turbulence

1. Introduction

This study is motivated by the need for reliable models of coherence in wind energy applications. The power production from multiple turbines, driven by stochastic, weakly coherent wind signals is advantageous in terms of electrical power quality in comparison to a single turbine. For a weak network, the power quality depends on the spatial coherence (Beyer et al., 1990a; Paulsen, 1989).

The (two point) coherence function \( \gamma^2 \) at a frequency \( n \) is defined as the ratio between the cross spectral density \( |S_{12}(n)| \) and the product of the power spectral densities \( S_1(n), S_2(n) \) of two time series \( x_1(t), x_2(t) \) measured at the locations \( X_1 \) and \( X_2 \) (Bendat and Piersol, 1971):

\[
\gamma_{12}^2(n) = \frac{|S_{12}(n)|^2}{S_1(n)S_2(n)}.
\]

Contrary to the definition used here, some other authors have defined coherence as \( \gamma \), rather than \( \gamma^2 \), e.g. Frost et al. (1978), Frost and Aspliden (1994). Depending on the point of view \( \gamma^2 \) and \( \gamma \) are referred to as 'squared coherence' and 'root coherence' respectively (Panofsky and Dutton, 1984; ESDL, 1991).
We speak about vertical coherence for two points separated vertically (e.g. at different levels of a meteorological mast). Longitudinal coherence, for two points in line with the (horizontal) wind direction vector and lateral coherence for separations that are horizontal and perpendicular to the wind direction vector (Figure 1).

Separation distances of wind turbines in a wind farm will always be greater than the height of the turbines above ground. For our purpose we are mainly interested in the coherence in near-neutral atmospheric conditions i.e. high wind speeds. Heights from 20 m to 100 m with separation distances from 20 m up to some 10 km were considered. Very few authors have published data for horizontal two point coherence which fit these criteria. This is believed to be due primarily to the non standard requirement to erect several meteorological towers and record wind speed data from them simultaneously. Based on published data and data recorded at the Rutherford Appleton Laboratory in Oxfordshire, United Kingdom, models proposed by different authors are compared in order to find one that fits the data over a wide range of parameters.
2. Davenport's Coherence Model

An exponential model for the coherence function was first proposed by Davenport (1961) for vertical coherence and was later also applied to lateral and longitudinal coherence (Davenport, 1967; Pielke and Panofsky, 1970):

\[ \gamma^2(n) = e^{-\frac{cn}{\ell}}. \]  

(2)

where \( \ell \) is the average wind speed at measurement height \( z \), and \( R \) is separation distance; the 'decay parameters' \( c_i \) have to be determined by experiment. We use the index \( i = 1 \) for longitudinal, \( i = 2 \) for lateral and \( i = 3 \) for vertical separation. The quoted decay parameters vary by a factor of 2, due to the above mentioned inconsistency in the literature with respect to the definition of coherence. All quoted decay parameters in the text presented here relate to Equation 2 and have been adjusted where necessary. In the original publication (Davenport, 1961) the wind speed \( U_{10} \) at 10 m height is used. In subsequent publications e.g. Pielke and Panofsky (1970), \( U_{10} \) was replaced by the wind speed at measurement height. The older definition has, however, been used since then in several publications (Shiotani and Iwata, 1971; Duchêne-Maruillaz, 1975; Soucy et al., 1982; Frost and Aspliden, 1994). The normalization with \( U_{10} \) leads to an unwanted dependence of the decay factor on the measurement height, as has been pointed out by Kristensen and Jensen (1979).

When using the exponential model, Equation (2), for lateral coherence, a wide spread of the coherence decay parameter \( c_2 \) is observed. The values quoted in the literature range from 10 to 60. The experimental results for the longitudinal decay parameter \( c_1 \) range from 0.4 to 1.0 for wind-tunnel experiments, 2 to 3 over sea and 4 to 10 over land. Different models have been developed to reduce this apparent scatter. These will be discussed in this paper and compared to the new model proposed.

3. Coherence Model for Small Separations

The theoretical description of turbulence and of spatial coherence is incomplete. Because of the complexity of the problem, theoretical models have to include far reaching approximations and are often valid only over a very limited region. Besides incompressibility and homogeneity we assume for the moment isotropy of turbulence, which is a valid assumption for small separations. Small is to be understood as small relative to the length scale of turbulence, which can be approximated by the height under consideration (Mann et al., 1991).
3.1. ISOTROPIC TURBULENCE

High spatial coherence is obtained when the separation between the reference points is small compared to the length scale of turbulence. For such small separations isotropy can be assumed and the following simplified theoretical approach describes the spatial coherence well.

The spectral tensor \( \Phi_{ij} \) is a function of the wave number \( k = 2\pi r \) and the energy spectrum \( E(k) \):

\[
\Phi_{ij} = \frac{E(k)}{4\pi k^2} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right). \tag{3}
\]

where \( \delta_{ij} \) is the Kronecker delta. For a lateral separation \( r \), we can express the auto- and cross-spectra as a function of the spectral tensor:

\[
S_{ij} = \int_{-\infty}^{\infty} dk_z \int_{-\infty}^{\infty} dk_z \Phi_{ij} e^{i k_2 r}. \tag{4}
\]

The evaluation of this integral with a chosen energy spectrum, e.g. Kolmogorov spectrum or von Kármán spectrum, results in an expression for the coherence in terms of Bessel functions of the second kind (Roberts and Surry, 1973; Irwin, 1979; Kristensen and Jensen, 1979). Possible alternatives to the integral over the total energy spectrum \( E(k) \) in Equation (4) have been published by Roberts and Surry (1973) and Mann (1994). It is argued that, of the eddies forming the energy spectrum \( E(k) \), only the smaller eddies contribute to the destruction of a larger eddy and thus the integral in Equation (3) should only be performed over the high frequency part of the spectrum. These models can also be approximated by exponential forms, for example Mann et al. (1991):

\[
\gamma^2 = p_2 e^{-p_1 k R}
\]

where \( p_1 \) and \( p_2 \) are simple linear polynomials of \( \frac{R}{r} \). The polynomial \( p_2 \) tends to zero for large distances.

3.2. NON-ISOTROPIC EXPANSIONS

For separation distances comparable to the turbulent length scale, the isotropy of turbulence is disturbed by the shear stress in the surface layer. A model that assumes uniform shear stress in the vertical direction is discussed by Mann (1994) and a formulation was proposed that takes account of blockage by the surface. An alternative approach is to use the two-dimensional horizontal energy spectrum in Equation (3). This spectrum has been proposed by Peltier et al. (1996) and recently compared with experimental data (small separations) by Tong and Wyngaard (1996).
Despite these recent advances in the description of turbulent spectra, the findings of Kristensen et al. still apply, that models based on Equation (3) are not valid for separation distances larger than the appropriate turbulent length scale. This is due to the lack of isotropy in the neutral boundary layer (Kristensen et al., 1981; Kristensen et al., 1983; Kristensen et al., 1989; Henjes, 1997).

To give some idea of the difficulties encountered when applying such models for large separations, we refer to the results of Kristensen et al. (1981) and Beyer et al. (1993) reporting high experimental coherences for separation distances up to and in excess of 10 times the measurement height. From any of the above mentioned models we would expect a coherence very close to 0 for all frequencies. Similarly, a high cross-correlation as has been reported for large separations by Hanna and Chang (1992) would contradict \( \gamma^2(k) = 0 \).

In the light of the current state of theoretical models, we will in the following paragraphs discuss rather simple semi-empirical approaches, which do in practice describe the coherence function for large separations with sufficient accuracy. We assume flat terrain and neutral stratification: parametrisations that include stability can be found in Pielke and Panofsky (1980), Ropelewski et al. (1973), Berman and Stearns (1977), and Soucy et al. (1982).

4. Coherence Model for Large Separations

Past experimental results confirm that an exponential model is well suited to approximate the coherence function of the streamwise velocity component. The exponential decay of the coherence depends on the separation distance \( D \), standard deviation of wind-speed fluctuations \( \sigma \) and the measured frequency \( n \). In a coordinate system that is moving with the mean wind speed, an appropriate form of the exponential model with arbitrary decay constants \( b \) is:

\[
\gamma^2 = e^{-b \sigma n}.
\]  

(5)

We consider now a relation describing this dependency in a coordinate system where the wind field moves relative to the reference points with speed \( U' \). As we are concerned with non-isotropic turbulence, different decay constants are expected for each direction. For longitudinal separation we assume:

\[
\gamma^2 = e^{-a_1 U'}
\]  

(6)

where \( a_1 \) is a longitudinal decay constant. The formulation of Equation (6) is guided by the argument that the measured frequency is increased as the wind field is shifted across the reference points. The distance any eddy travels before it decays is increased with mean wind speed. Both effects have been compensated for in the model by dividing by the mean wind speed \( U' \).
For a lateral separation the picture is different as the shift of the wind field in longitudinal direction will affect the measured frequency but not lead to a shift of eddies from one reference point towards the next. We postulate for the lateral case:

\[ \gamma_z = e^{-\alpha z} \rho. \]  

(7)

The empirical decay constant \( \alpha \) has now unfortunately a dimension \( \text{m}^{-1} \). The equations for longitudinal and lateral coherence can be combined to provide an expression including \( \alpha \) the angle between the wind direction and the line connecting the two reference points. Such a formulation is supported by empirical evidence of correlation coefficients measured in the horizontal plane (Shiotani and Iwatani, 1971; Hayashi, 1991):

\[ \gamma^2(\alpha) = e^{-\alpha^2 \rho^2 \sin^2 \alpha \cos^2 \alpha} \]  

(8)

This model is now compared with models proposed by other authors for longitudinal and lateral coherence. Experimental data reported in the literature and some new experimental evidence. We will focus on the comparison of models for the special cases of longitudinal (Equation 16) and lateral coherence (Equation 7).

5. Discussion of the Proposed Coherence Model

Our model (Equation 8) differs from Davenport's model (Equation 2) in two major ways. That is in the dependence on wind speed and standard deviation. Davenport formulated his model for vertical separations and it was only later applied to longitudinal and lateral separations. It was not recognized at the time that the shift of the wind field in longitudinal direction would require a separate treatment of longitudinal and lateral coherence. In addition to this modification we include the turbulence in our considerations and will show this to be empirically justified.

5.1. Discussion for Longitudinal Coherence

The turbulence intensity has previously been used as a scaling factor by Ropelewski et al. (1973). This scaling successfully collapses data from wind-tunnel experiments, experiments over water and experiments over land. The formula for longitudinal coherence proposed by Ropelewski et al. is identical to Equation 6 and supports the parametrization using the standard deviation.

It should be noted that in an experimental validation, values of coherence lower than those predicted by Equation 8 can occur. This is due to the effect of transverse fluctuations and variations in wind direction, which we have not included in our analysis. A functional dependence of the decay parameter \( \alpha \) on the separation distance \( R \) has been proposed by several authors (Panofsky and Mizuno,
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1975: Perry et al., 1978; Kristen sen, 1979) to account for this effect. A more recent experimental study for distances from 210 m to 25 km in Lower Saxony, Germany, does not support such a parametrisation (Beyer et al., 1990b; Beyer et al., 1993).

For large longitudinal distances (greater than 300 m) Taylor’s hypothesis cannot easily be applied, as all original eddies will have been destroyed whilst being transported over such an extended time period. Consequently, it has been argued that the scaling of the distance travelled by an eddy with the wind speed may no longer be appropriate (Handwerker, 1993). We feel that the assumption of ‘frozen eddies’ can be replaced by a more statistical approach, as suggested by Fordham (1985), so that there remains a strong physical argument for the scaling in Equation (6) with the mean wind speed.

5.2. DISCUSSION FOR LATERAL COHERENCE

Opinions regarding the dependence of the decay parameter on the separation distance are split between the different models. The majority of measurements do not reveal any dependency of the decay parameter on the separation distance. However, Bowen et al. (1983) find $c_2 = c_2(R)$ and Shiotani and Iwatani (1980) $c_2 = c_2(R^{0.5})$. A recommendation which attempts to combine the different models was made by Kristensen and Jensen (1979). It is considered here that the experimental evidence is generally against such a dependency, so that as in Equation (8), it is not included.

The dependency of the lateral coherence on the standard deviation $\sigma$ has been used in Roberts and Surry (1973) and Kristensen et al. (1989). As in the case of longitudinal coherence, a reduction of the scatter of experimental decay constants can be observed if the decay constant is assumed to be proportional to the standard deviation $\sigma$. In contrast to the approach chosen for the longitudinal model, Ropplewski et al. (1973) recommend the use of the ratio between the longitudinal and lateral length scale as parametrisation of the decay parameter (also in Berman and Stearns, 1977). These length scales are themselves a function of the atmospheric stability and are problematic to identify. A more pragmatic approach is taken here. We assume that both lateral and longitudinal decay depends linearly on the standard deviation.

Equation (7) gives a coherence function which, for constant turbulence intensity $u^*$, is independent of the mean wind speed. This somewhat unusual proposal is supported by an experimental campaign from Butjadingen, Northern Germany. These data show no evidence of a dependence of the coherence on the wind speed as would be expected from Equation (2) (Beyer et al., 1993; Handwerker, 1993). However, the experiments do not conclusively prove that there is no such dependency. Experimental data published in Shiotani and Iwatani (1980) show a slight dependence of the decay parameter $c_2$ on the wind speed, which suggests again that for lateral coherence the scaling with wind speed in Equation (2) is inappropriate. The scatter of decay parameters $c_2$ taken from the literature is unaffected by removal of the scaling. Thus, from experimental data alone, it is difficult to
Figure 2. Sketch of the Rutherford Appleton Laboratory test site. Auxiliary buildings (single storey, flat roof) are marked with 'A', wind turbines (not operating) with 'WT-1-4'. Wind speed measurements have been performed at 18 m height at the three towers marked 'T2', 'T3' and 'T4'.

6. Experimental Setup

Data from the Rutherford Appleton Laboratory Wind Test Site were obtained to test the proposed coherence model. The data were recorded on three meteorological...
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TABLE I

Runs 1-16 recorded at Rutherford Appleton Wind Test Site. The length of each time series, average wind speed (T2) and wind direction (T2) are given. The temperature and wind-speed difference at two levels (T2) were used to calculate the Richardson number.

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<th>$\Phi$ (deg)</th>
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towers (T2, T3, and T4). See Figure 2. T2 is equipped with 2 cup anemometers at heights 7 m and 18.7 m, 2 temperature sensors at 2 m and 18.7 m and a wind vane at 17 m height. T3 has a cup anemometer and a wind vane installed at 18 m height. The tower T4 is equipped with only one anemometer at 18 m height. Data from all three masts and the air pressure were recorded simultaneously in March 1996 at a sampling frequency of 0.2 Hz. Because of the low sampling frequency, turbulence intensities were corrected assuming a Kaimal spectrum. The data were split manually into runs exhibiting stationary conditions in terms of wind direction and wind speed. We have used the temperature and wind speed measurements from mast T2 to calculate the Richardson number. It should be noted that, due to the lack of homogeneity in the terrain upwind of T2, the calculation of the Richardson number is not considered to be very reliable. Table I summarizes the records. The wind speed is taken from T4 and the wind direction from T2 because these masts were least affected by obstacles and instrument failures respectively. Runs numbered 1, 7, 8, 11, 12, 13, 15, 16 had to be excluded from the evaluation because of the low wind speed. Runs 8, 12, 14 were excluded because of high Richardson numbers. This leaves records 2, 3, 4, 5, 6, 9, 10 for further evaluation. Measurements from towers T2/T3 and T2/T4 had to be discounted because mast T2 experienced blockage and wake effects for all wind directions. See Figure 2. From the remaining runs, 2 and 3 feature predominantly longitudinal, 9 and 10 lateral coherence. The other runs show a mixture of longitudinal and lateral coherence properties. Run 9 has to be treated with care because it includes some variation in the wind speed. The influence of the wind turbine WT1 (Wind Harvester, 18 m lattice towers), which was not operating during the period of data collection, on the coherence between T3 and T4 has been neglected.
7. Data Preparation

All time series have been corrected for single erroneous measurement points i.e. records with negative values. Detrending was performed for each run as a whole, using a phase preserving filter function 'filtfilt.m' from the MATLAB Signal Processing Toolbox (MathWorks, 1994) with a Hanning window of 10 minutes length. The window length is a compromise in order to remove the trend without losing low frequency information. The Hanning window was chosen to limit frequency leakage. An example for the computation of the trend of run 2 (first 20 min) is given in Figure 3. The start of the trend series is obtained by mirroring the time series at the origin, which leads to reasonable but not perfect results for the first few minutes. The remaining trend line fits the data very well. The trend line minus its mean value was subtracted from the original data to obtain the trend free time series.

The coherence was calculated by dividing the two time series $A$ and $B$ into $N$ segments. With the estimators $S_{1A}$, $S_{2A}$, $Q_{1B}$, $C_{1B}$ for spectral densities, quad and co-spectra respectively, we can express the coherence estimator.
It has to be stressed here, that if the coherence estimate is calculated without subdividing the time series into segments and averaging the spectra and cross spectra over these segments, then the coherence estimator will always be unity for all frequencies. This function follows as a direct consequence of the definition of the coherence function and the representation of spectra, co- and quad spectra with discrete fourier coefficients (Carter, 1972; Kristensen and Kirkegaard, 1986).

The coherence has been calculated with the function cohere.m from the MATLAB signal processing toolbox, according to Equation 9. The data are split into a number of overlapping segments, which are individually weighted with a Hanning window. The choice of the number of segments and the percentage overlapping is based on the theoretical derivations by Carter (1972). Parts of this derivation have independently been published by Kristensen and Kirkegaard (1986). Both publications refer to Goodman (1957) and the tables of Amos and Koopmans (1963).

The bias $B$ and the variance $V$ of the coherence estimate are functions of the number of segments $N$ a time series is split into. For non overlapped segments the bias is given as (Carter, 1972):

$$B = \frac{1}{N} \left( \frac{2}{N+1} \gamma^2 \right) + \frac{1! (N-1)}{\prod_{i=1}^{\gamma^2} (N-i)} (\gamma^2)^2 + \frac{2! (N-1)}{\prod_{i=1}^{\gamma^2} (N-i)} (\gamma^2)^3. \tag{10}$$

which can be approximated for large $N$ by:

$$B \approx \frac{1}{N} (1 - \gamma^2)^2. \tag{11}$$
For the variance of non-overlapped segments Carter (1972) obtains:

\[
V \approx \frac{(1 - N)}{N(N - 1)^2} \frac{1}{N} \\
+ \frac{2N^2 - 2N - 3}{(N - 1)(N - 2)(N - 3)} (\gamma^2)^2 \\
+ \frac{2}{(N - 1)(N - 2)(N - 3)(N - 4)} (\gamma^2)^3 \\
+ \frac{13N^5 - 15N^4 - 113N^3 + 27N^2 - 136N - 120}{(N + 1)(N + 2)(N + 3)(N + 4)(N + 5)} (|\gamma|^2)^4.
\]

This formula for the variance can be approximated for large \( N \) by:

\[
V \approx \frac{2}{N} \gamma^2 (1 - \gamma^2)^2 
\]

In order to obtain a good estimate for the coherence, it is desirable to split the time series into as many intervals as possible. This minimizes the bias and variance of the estimate. However, the greater the number of segments, the smaller are the individual segments, which limits the frequency resolution of the fourier transformed data. In order to retain maximum spectral resolution and reduce the bias and variance, an algorithm was derived that calculates for one time series the coherence estimate for different numbers of segments and then combines the results. As \( N \) is not necessarily large we calculate the variance of the experimental results using Equation (12) instead of Equation (13).

The bias and variance were further reduced by overlapping the segments by 50% as recommended by Carter (1972). Overlapping of the segments, however, prohibits the exact determination of the occurring bias and variance as the derivation of the above quoted formulae (Equations (10) and (12)) are no longer valid. However, the equations can be used to provide as upper bound to the actual bias and variance in the coherence estimate.

8. Experimental Results (Longitudinal)

In order to facilitate the comparison of the experimental results with our model function we define a dimensionless frequency \( \hat{n} \):

\[
\hat{n} = \frac{n R}{c} \left( \frac{u}{c} \cos \theta \right)^2 - \frac{u z \sin \theta}{c} \right)^2.
\]
Substituting this frequency in Equation (8) we obtain a simple exponential function: \( y^2 = e^{-\phi} \). This function is subsequently compared with experimental data from the Rutherford Appleton Wind Test Site.

Figure 4 compares the experimental results from RUN-2 with the theoretical model. The wind direction is roughly longitudinal along the line from tower T3 to tower T4. The theoretical model is calculated using Equation (8) with a relative wind direction angle of 9 degrees. It was found that the parameters \( a_1 = 30 \) (longitudinal) and \( a_2 = 35 \text{ m}^{-1} \) (lateral) result in a good fit for all experimental runs from the Rutherford test site. The error bars are obtained as the sum of the standard deviation (square root of Equation (12)) and the bias (Equation (10)), based on the experimental coherence estimate \( y^2 \). We have tried to exclude as far as possible additional errors due to changes in wind direction and wind speed (non-stationarity), obstacles and roughness changes (non-homogeneity). As part of these efforts RUN-2, for example, was for the evaluation of the high frequency part subdivided in three periods of roughly three hours each. The resulting, more stationary (in wind speed), periods were evaluated separately and later bin-averaged.

RUN-3 also features wind directions roughly along the line from tower T3 to tower T4; the relative angle is smaller than for RUN-2. The fitted model is in even better agreement with the data (Figure 5). By using different numbers of intervals for low and high frequency, the variance is kept small over the widest possible range. The high coherence estimates for low frequencies have low variance, even for the very small numbers of segments \( u \geq 3 \) that are used for the low frequency range throughout the analysis.
Coherence

\[ \text{RUN-3} \]

\[ \begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
\hline
\text{Frequency } f & 0.01 & 0.1 & 1 & 10 \\
\hline
\text{Coherence} & 0.8 & 0.6 & 0.4 & 0.2 \end{array} \]

We compare the obtained coherence decay parameter with other published values in Table II. The reference numbers in the table refer to the following publications: L1 (Champagne et al., 1970; Ropelewski et al., 1973); L2 (Panofsky et al., 1974); L3 (Berman and Stearns, 1977); L4 (Perry et al., 1978); L5 (Ropelewski et al., 1973); L6 (Panofsky and Mizuno, 1975); L7 (Steinberger-Willms, 1993).

The value of turbulence intensity for O’Neill (L6) represents the mean of the values of L4 and L5. The turbulence intensity for Lake Ontario (L2), O’Neill and Rock Springs (L4) was assumed to be twice the quoted vertical turbulence intensity.

The scaling of the classical coherence decay factor \( c \) with the turbulence intensity reduces the scatter of the decay factors significantly. The data from Rutherford Appleton test site fit well into the general pattern. The remaining scatter reflects the difficulties in recovering the turbulence intensity for each experiment and the variation in methods applied in previous studies for fitting the exponential function to the data.

From our results and the results quoted in the literature, we recommend a longitudinal decay parameter of \( a_1 = 30 \pm 10 \).

9. Experimental Results (Lateral)

The coherence for lateral separation is lower than for longitudinal separation. This requires a resolution of lower frequencies, which is achieved by dividing the time series in as few intervals as possible. As a trade off, the variance of the experimental
### TABLE II

Longitudinal decay parameter, experimental data and values taken from the literature. The decay parameter $c_1$ shows a wide scatter of values which is reduced by normalization with the turbulence intensity.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Location</th>
<th>$I$</th>
<th>$c_1$</th>
<th>$a_1 = c_1 / I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>RAL (RUN-2)</td>
<td>0.12</td>
<td>3.6</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>RAL (RUN-3)</td>
<td>0.13</td>
<td>3.9</td>
<td>30</td>
</tr>
<tr>
<td>L1</td>
<td>wind tunnel</td>
<td>0.02</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>L2</td>
<td>Lk Ont. (water)</td>
<td>0.1</td>
<td>2-3</td>
<td>20-30</td>
</tr>
<tr>
<td>L2</td>
<td>Lk Ont. (land)</td>
<td>0.20</td>
<td>6-7</td>
<td>30-35</td>
</tr>
<tr>
<td>L3</td>
<td>Aberdeen</td>
<td>0.2-0.3</td>
<td>6-14</td>
<td>34 (20-50)</td>
</tr>
<tr>
<td>L4</td>
<td>Rock Springs</td>
<td>0.1-0.22</td>
<td>3-8</td>
<td>33 ±5</td>
</tr>
<tr>
<td>L4</td>
<td>O’Neill</td>
<td>0.14</td>
<td>2/4/4/6</td>
<td>15/30/20/43</td>
</tr>
<tr>
<td>L5</td>
<td>O’Neill</td>
<td>0.2</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>L6</td>
<td>O’Neill</td>
<td>0.17</td>
<td>4/6/7/11</td>
<td>24/35/41/64</td>
</tr>
<tr>
<td>L7</td>
<td>Butjadingen</td>
<td>0.21</td>
<td>6.3</td>
<td>30</td>
</tr>
</tbody>
</table>

### TABLE III

Experimental data for lateral coherence (Runs 9 and 10) and values from the literature. Scaling with the turbulence intensity reduces the scatter of the data.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Location</th>
<th>$u$ (m s$^{-1}$)</th>
<th>$I$</th>
<th>$c_2$</th>
<th>$\frac{c_2}{I}$</th>
<th>$a_2$ (m s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>09</td>
<td>RAL</td>
<td>7.4</td>
<td>0.19</td>
<td>56</td>
<td>296</td>
<td>35</td>
</tr>
<tr>
<td>10</td>
<td>RAL</td>
<td>7.2</td>
<td>0.17</td>
<td>49</td>
<td>288</td>
<td>35</td>
</tr>
<tr>
<td>L8</td>
<td>Satoura</td>
<td>16</td>
<td>0.07</td>
<td>28</td>
<td>400</td>
<td>17</td>
</tr>
<tr>
<td>L9</td>
<td>Sotra bridge</td>
<td>-</td>
<td>0.1</td>
<td>22</td>
<td>220</td>
<td>-</td>
</tr>
<tr>
<td>L10</td>
<td>Christchurch</td>
<td>9 (0.1&quot;)</td>
<td>20/42</td>
<td>120/250</td>
<td>13/27</td>
<td></td>
</tr>
<tr>
<td>L10</td>
<td>Christchurch</td>
<td>9 (0.1&quot;)</td>
<td>52.52</td>
<td>310/310</td>
<td>34/34</td>
<td></td>
</tr>
<tr>
<td>L10</td>
<td>Christchurch</td>
<td>9 (0.1&quot;)</td>
<td>52.63</td>
<td>310/370</td>
<td>34/41</td>
<td></td>
</tr>
<tr>
<td>L11</td>
<td>LaTour</td>
<td>14</td>
<td>0.18/0.16</td>
<td>20/22</td>
<td>182/140</td>
<td>13/10</td>
</tr>
<tr>
<td>L11</td>
<td>Bouin</td>
<td>12.5</td>
<td>0.11</td>
<td>25</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>L11</td>
<td>St Nazaire</td>
<td>13.5</td>
<td>0.062</td>
<td>11</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>L3</td>
<td>Aberdeen</td>
<td>2.2-4.4</td>
<td>0.2-0.3</td>
<td>19.53</td>
<td>65-194</td>
<td>37 (16-58)</td>
</tr>
</tbody>
</table>
Figure 6. The coherence function for RUN-9 (lateral: α = 85°) is shown as a function of the frequency $\hat{f}$. It was found that the parameters $a_1 = 30$ and $a_2 = 35$ m$^{-1}$ result in a good fit for all runs.

Figure 7. The coherence function for RUN-10 (lateral: α = 4°) is shown as a function of the frequency $\hat{f}$. It was found that the parameters $a_1 = 30$ and $a_2 = 35$ m$^{-1}$ result in a good fit for all runs.
The scaling with the turbulence intensity again proves to be effective in reducing the scatter in the decay parameters. The proposed model is in good agreement with the experimental data. Based on our data and Table III, a value for the decay parameter of $a_z = (35 \pm 10) \text{ s m}^{-1}$ is recommended.

10. Angular Dependence of Horizontal Coherence

If the wind direction is neither longitudinal nor lateral to the two points of interest both the lateral and longitudinal decay parameters are needed to describe the coherence. For increasing angle the lateral coherence quickly becomes the dominant factor. Table IV summarizes the results of the three data sets from Rutherford Appleton Laboratory that have intermediate wind directions.

Using the previously identified decay parameters of $a_z = 30$ and $a_z = 55 \text{ s m}^{-1}$, we find that the proposed model is in good agreement with the experimentally determined coherence functions. The results are shown in Figures 8, 9 and 10.
Figure 8. The coherence function for RUN-4 (mixed: $\alpha = 12$) is shown as a function of the frequency $\bar{n}$. The low number of intervals (degrees of freedom) used leads to high variance. It was found that the parameters $\alpha_1 = 30$ and $\alpha_2 = 35 \text{ m}^{-1}$ result in a good fit for all runs.

Figure 9. The coherence function for RUN-5 (mixed: $\alpha = 18$) is shown as a function of the frequency $\bar{n}$. It was found that the parameters $\alpha_1 = 30$ and $\alpha_2 = 35 \text{ m}^{-1}$ result in a good fit for all runs.
TABLE IV
Experimental data for intermediate directions (Runs 4, 5, 6).

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Location</th>
<th>α</th>
<th>μ</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>RAL (RUN-4)</td>
<td>12</td>
<td>6.2</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>RAL (RUN-5)</td>
<td>18</td>
<td>7.1</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>RAL (RUN-6)</td>
<td>27</td>
<td>8.9</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Figure 10. The coherence function for RUN-6 (mixed: α = 271) is shown as a function of the frequency $\tilde{f}$. It was found that the parameters $\alpha_1 = 30$ and $\alpha_2 = 35$ m$^{-1}$ result in a good fit for all runs.

11. Conclusions

Our own experimental data and published data of horizontal coherence have been compared with different theoretical approaches.

It has been shown that scaling Davenport's coherence decay parameter with turbulence intensity is very helpful in collapsing the data for longitudinal as well as for lateral separations. For neutral stability, it is for practical reasons more straightforward to scale the lateral coherence decay parameter with the turbulence intensity, rather than with the quotient of length scales as previously proposed. A dependence of the coherence decay parameter on the separation distance has been discounted.
On the basis of theoretical considerations and some experimental evidence it is suggested that the lateral coherence decay should not be regarded as a function of the mean wind speed. The lateral coherence is a function of the turbulence intensity and the separation distance.

A new model has been proposed taking these arguments into account (Equation 18). Good experimental agreement with this model has been found for the angular dependence of the coherence and the dependence on turbulence intensity. Further testing of the model should be carried out to confirm that the decay of lateral coherence does not depend on the mean wind speed.

Acknowledgements

This study is funded by National Wind Power Ltd. UK. The authors wish to thank the Rutherford Appleton Laboratory (RAL), UK and in particular Dr. Simon Watson, for supplying data from their test site. We would like to thank the referees for their most valuable comments.

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COHERENCE FOR LARGE SEPARATIONS


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I wish to thank the following persons and organisations:

- David Infield for providing me with the opportunity to carry out and supervising my research at the Centre for Renewable Energy Systems Technology (CREST).

- All staff at National Windpower (NWP), especially John Warren, Paul Hannah and David Magnussen for their support and encouragement. This work would not have been possible without the kind support of NWP. NWP did not only provide a research grant but also valuable data collected at a number of wind farms in the United Kingdom. The thesis would not have been possible without the experimental evidence drawn from this data.

- Rutherford Appleton Laboratory (RAL), UK, and in particular Simon Watson for providing access to data from the RAL test site and from Susetter Hill, Shetlands, UK.

- The directors and staff of Garrad Hassan and Partners Ltd. for their understanding and support during the writing up period.

- Erwin Bossanyi, Fritz Santjer, Herrmann van Radecke, Klaus Kaiser for valuable information and comments.

- Ralph and Elke for Käsespätzle, Tiramisu and Orion.

Thank you!