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# DAMPING OF RESONANT VIBRATIONS UTILISING THE ACOUSTIC BLACK HOLE EFFECT

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## Abstract

*One of the possible ways of damping resonant flexural vibrations of engineering structures or their components, e.g. finite plates or bars, is to reduce reflections of flexural waves from the free edges. A new emerging method of reducing edge reflections, which is being developed by the present author, uses specifically designed attachable plates or bars of variable thickness (non-linear wedges). Such plates or bars utilise gradual change in their thickness from the value corresponding to the thickness of the basic plate or bar, which are to be damped, to almost zero. It is proposed to use specific power-law shapes of these attachable wedges so that they would ideally provide zero reflection of flexural waves even for negligibly small material attenuation, thus materialising the so-called 'acoustic black hole effect'. To make up for real manufactured wedges, that always have edge truncations, and to increase the efficiency of damping it is suggested also to deposit absorbing thin layers on wedge surfaces. According to the theoretical calculations, the deposition of thin absorbing layers on surfaces of wedges utilising the acoustic black hole effect can dramatically reduce the reflection coefficients, thus resulting in very efficient damping systems for flexural vibrations. The theory of the phenomenon based on geometrical-acoustics approach is discussed for different wedge profiles. Theoretical results are complemented by the preliminary experimental measurements carried out on a steel wedge of quadratic shape.*

## 1 Introduction

One of the well-known ways of damping resonant flexural vibrations of engineering structures or their components, such as finite plates or bars, is based on reducing reflections of flexural waves from the free edges of structures. This can be achieved, for example, by introducing graded impedance interfaces along with placing damping material at the edges [1]. The main difficulty in this approach is to create suitable graded impedance interfaces. For example, the authors of the paper [1] have attached the graded impedance interface to the end of the basic steel plate. The interface was constructed as the combination of the three plates made of different materials, all of them having the same thickness as the basic steel plate. Using such a system the authors demonstrated experimentally that as much as 60-80 % of the energy could be damped for frequencies in the range of 2-10 kHz. This was much superior to the commonly used method of reducing edge reflections by embedding edges of plates in sand, which results only in at most 30 % of the energy to be damped at frequencies above 2 kHz [2]. In spite of the impressive results reported in [1], the technological difficulties of building and attaching several additional plates of different materials to the basic plate are restrictive on applying this method for practical purposes.

The alternative way of creating matching impedance interfaces has been proposed by the present author [3] (see also [4-7]). Instead of using combinations of finite plates made of different materials, it was proposed to utilise gradual change in thickness of a plate or a bar from the value corresponding to the thickness of the basic plate to almost zero. In other words, it was proposed to

use elastic wedges as gradual impedance interfaces for flexural waves in plates and bars. The above-mentioned gradual change in thickness of a plate or a bar has to be made according to the special laws that ideally provide zero reflection even for negligibly small material attenuation – the so-called ‘acoustic black hole effect’. To make up for real wedges and to make the damping systems under consideration more efficient it was proposed also to deposit thin absorbing layers on wedge surfaces near the edges.

A possible practical realisation of a non-symmetric wedge-like damper covered by an absorbing film on one side is illustrated in Figure 1. The thick end of a damper is glued to the edge of a basic plate reflections from which are to be suppressed. To avoid reflections from the boundary between the plate and the wedge that may be caused by a sudden change in geometry, the shape of the wedge is smoothed so that it gradually transforms into the plate with the thickness equal to the thickness of the basic plate.

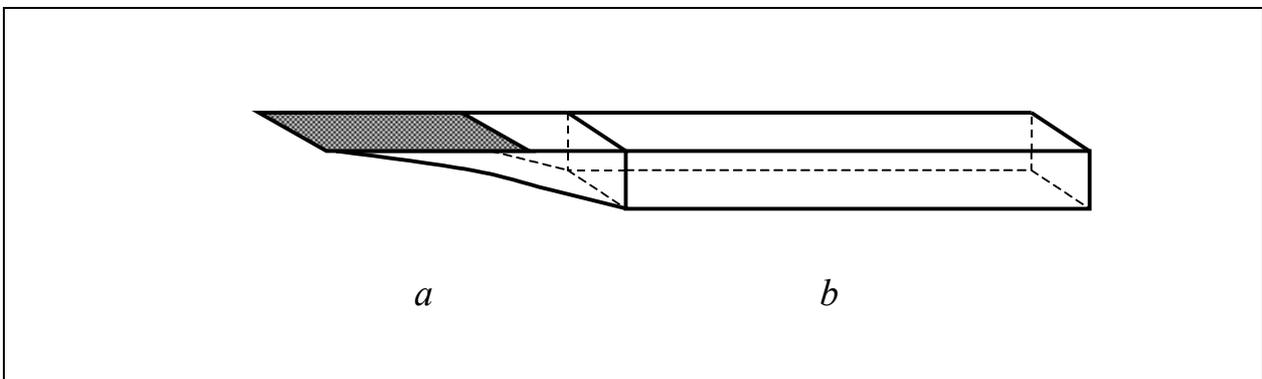


Figure 1. Application of a non-symmetric quadratic wedge-like damper covered by an absorbing film on one side (*a*) for suppression of flexural wave reflections from one of the edges of an elastic plate (*b*) [6].

To appreciate the ‘acoustic black hole effect’ for flexural waves one has to recall that when flexural waves propagate towards edges of elastic plates of variable thickness gradually decreasing to zero, i.e. towards edges of thin elastic wedges of arbitrary shape, they slow down and their amplitudes grow. After reflection from the edge, with the module of the reflection coefficient normally being equal to unity, the whole process repeats in the opposite direction [8-12]. Especially interesting phenomena may occur in the special case of wedge edges having cross sections described by a power law relationship between the local thickness  $h$  and the distance from the edge  $x$ :  $h(x) = \epsilon x^m$ , where  $m$  is a positive rational number and  $\epsilon$  is a constant [11-13]. In particular, for  $m \geq 2$  - in free wedges [11-13], and for  $m \geq 5/3$  – in immersed wedges [12], the flexural waves incident at an arbitrary angle upon a sharp edge can become trapped near the very edge and therefore never reflect back. Thus, such structures materialise acoustic ‘black holes’. In the case of localised flexural waves propagating along edges of such wedges (these waves are also known as *wedge acoustic waves*) the phenomenon of acoustic ‘black holes’ implies that wedge acoustic wave velocities in such structures become equal to zero [11,12].

The unusual effect of power-low profile on flexural wave propagation in elastic wedges has attracted some initial attention in respect of their possible applications as vibration absorbers.

Mironov [13] was the first to have pointed out that a flexural wave does not reflect from the edge of a quadratically-shaped wedge in vacuum ( $m = 2$ ), so that even a negligibly small material attenuation may cause all the wave energy to be absorbed near the edge. Unfortunately, because of the deviations of manufactured wedges from the ideal power-law shapes, largely due to ever-present truncations of the wedge edges, real reflection coefficients in such homogeneous wedges are as high as 50-70 % [13]. Therefore, in practice such wedges can not be used as vibration absorbers.

The situation, however, can be radically improved by modifying the wedge surfaces. In particular, it can be shown that deposition of thin damping layers on the surfaces of wedges having power-law profile can drastically reduce the reflection coefficients [3-7]. Thus, the combination of wedges with power-law profiles and of thin damping layers can result in very efficient damping systems for flexural vibrations. In what follows we briefly describe the basic theory behind the phenomenon and complement the discussion with the results of some numerical calculations and preliminary experiments.

## 2 Theoretical background

### 2.1 Flexural waves in free wedges of power-law profile

To understand the basic physics of the phenomenon one can consider the above-mentioned simplest case of a plane flexural wave propagating in the normal direction towards the edge of a free wedge described by a power-law relationship  $h(x) = \varepsilon x^m$ , where  $m$  is a positive rational number and  $\varepsilon$  is a constant. Since flexural wave propagation in such wedges can be described in the geometrical acoustics approximation (see [8-10] for more detail), the integrated wave phase  $\Phi$  resulting from the wave propagation from an arbitrary point  $x$  on the wedge medium plane to the wedge tip ( $x = 0$ ) can be written in the form:

$$\Phi = \int_0^x k(x) dx . \quad (1)$$

Here  $k(x)$  is a local wavenumber of a flexural wave for a wedge in contact with vacuum:  $k(x) = 12^{1/4} k_p^{1/2} (\varepsilon x^m)^{-1/2}$ , where  $k_p = \omega/c_p$  is the wavenumber of a symmetrical plate wave,  $c_p = 2c_l(1 - c_l^2/c_t^2)^{1/2}$  is its phase velocity, and  $c_l$  and  $c_t$  are longitudinal and shear wave velocities in a wedge material, and  $\omega = 2\pi f$  is circular frequency. One can easily check that the integral in equation (1) diverges for  $m \geq 2$ . This means that the phase  $\Phi$  becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above mentioned ideal wedges represent acoustic ‘black holes’ for incident flexural waves.

Real fabricated wedges, however, always have truncated edges. And this adversely affects their performance as ‘black holes’. If for ideal wedges of power-law shape (with  $m \geq 2$ ) it follows from equation (1) that even an infinitely small material attenuation, described by the imaginary part of  $k(x)$ , would be sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower integration limit in equation (1) must be changed from 0 to a certain value  $x_0$  describing the length of truncation, thus resulting in the amplitude of the total reflection coefficient  $R_0$  being expressed in the form [13]

$$R_0 = \exp(-2 \int_{x_0}^x \text{Im } k(x) dx). \quad (2)$$

According to equation (2), for typical values of attenuation in the wedge materials, even very small truncations  $x_0$  result in  $R_0$  becoming as large as 50-70 %.

## 2.2 Effect of thin absorbing layers

To improve the situation for real wedges (with truncations), one can consider covering the wedge surfaces by thin damping layers (films) of thickness  $\delta$ , e.g. by polymeric films. Note in this connection that the idea of applying absorbing layers for damping flexural vibrations of plates has been used successfully since the 50-ies (see, e.g. [14-18]). The new aspect of this idea, which is used in the proposed approach, is to apply such absorbing layers in combination with the specific power-law geometry of a plate of variable thickness (a wedge) to achieve maximum damping.

Two types of wedge geometry can be considered: a symmetric wedge and a non-symmetric wedge bounded by a plain surface at one of the sides, the latter being shown in Figure 1. For each of these cases either two or only one of the sides can be covered by absorbing layers. Note that non-symmetric wedges are easier to manufacture. They also have the advantage in depositing absorbing layers: the latter can be deposited on a flat surface, which is much easier. From the point of view of theoretical description, there is no difference between symmetrical and non-symmetrical wedges as long as geometrical acoustics approximation is concerned and the wedge local thickness  $h(x) = \epsilon x^m$  is much less than the flexural wavelength.

In what follows we consider only one possible mechanism of film-induced attenuation – the one associated with in-plane deformations of the film under the impact of flexural waves. Such deformations occur on wedge surfaces as a result of the well known relationship between flexural displacements  $u_z$  and longitudinal displacements  $u_x$  in a plate:  $u_x = -z(\partial^2 u_z / \partial x^2)$ . Not specifying physical mechanism of the material damping in the film material, we assume for simplicity that it is linearly dependent on frequency, with non-dimensional constant  $\nu$  being the energy loss factor, or simply the loss factor.

To analyse the effect of thin absorbing films on flexural wave propagation in a wedge in the framework of geometrical acoustics approximation one should consider first the effect of such films on flexural wave propagation in plates of constant thickness. The latter problem can be approached in different ways. The simplest way is to use the already known solutions for plates covered by damping layers of arbitrary thickness obtained by different authors with regard to the description of damped vibrations in such sandwich plates [14-18]. Using this approach (see [5,6] for more detail), one can derive the corresponding analytical expressions for the reflection coefficients of flexural waves from the edges of truncated wedges covered by thin absorbing layers.

## 2.3 Quadratic wedges

Let us first consider a wedge of quadratic shape, i.e.  $h(x) = \epsilon x^2$ , covered by damping layers on both surfaces. For such a wedge one can derive the following analytical expression for the resulting reflection coefficient  $R_0$  [4-6]:

$$R_0 = \exp(-2\mu_1 - 2\mu_2), \quad (3)$$

where

$$\mu_1 = \frac{12^{1/4} k_p^{1/2} \eta}{4\epsilon^{1/2}} \ln\left(\frac{x}{x_0}\right), \quad (4)$$

$$\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{1/2} \nu \delta}{4\epsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^2} \left(1 - \frac{x_0^2}{x^2}\right). \quad (5)$$

Here  $\nu$  is the loss factor of the material of the absorbing layer and  $\delta$  is its thickness,  $\eta$  is the loss factor of the wedge material,  $x_0$  is the wedge truncation length,  $E_1$  and  $E_2$  are respectively the Young's moduli of the plate and of the absorbing layer. As expected, in the absence of the damping layer ( $\delta = 0$  or  $\nu = 0$ , and hence  $\mu_2 = 0$ ), the equations (3)-(5) reduce to the results obtained in [13] (where a typographical misprint has been noticed). If the damping film is present ( $\delta \neq 0$  and  $\nu \neq 0$ ), it brings the additional reduction of the reflection coefficient that depends on the film loss factor  $\nu$  and on the other geometrical and physical parameters of the wedge and film.

In the case of a wedge of quadratic shape covered by damping layers on one surface only, equations (3) and (4) remain unchanged, whereas equation (5) should be replaced by

$$\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{1/2} \nu \delta}{8\epsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^2} \left(1 - \frac{x_0^2}{x^2}\right). \quad (6)$$

In deriving equations (3)-(6) the effect of thin absorbing layers on flexural wave velocity has been neglected. Note that geometrical acoustics approximation for the above-mentioned quadratic wedges ( $m = 2$ ) is valid for all  $x$  provided that the following applicability condition is satisfied:

$$\frac{\omega}{c_t \epsilon} \gg 1, \quad (7)$$

where  $c_t$  is shear wave velocity in the wedge material. For the majority of practical situations this condition can be easily satisfied even at very low frequencies.

## 2.4 Wedges of power-law profiles with $m = 3$ and $m = 4$

In this section we consider the effects of power-law profiles with higher values of  $m$  on the reflection coefficients of flexural waves from the wedge edges [5]. For a wedge described by the function  $h(x) = \epsilon x^m$  with  $m = 3$  one can obtain:

$$R_0 = \exp(-2\mu^{(3)}_1 - 2\mu^{(3)}_2), \quad (8)$$

where

$$\mu^{(3)}_1 = \frac{12^{1/4} \eta k_p^{1/2}}{2\epsilon^{1/2}} \frac{1}{x_0^{1/2}} \left[1 - \left(\frac{x_0}{x}\right)^{1/2}\right], \quad (9)$$

and

$$\mu^{(3)}_2 = \frac{3 \cdot 12^{1/4} \delta \nu k_p^{1/2}}{7 \epsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^{7/2}} \left[ 1 - \left( \frac{x_0}{x} \right)^{7/2} \right]. \quad (10)$$

Similarly, for a wedge described by the function  $h(x) = \epsilon x^m$  with  $m = 4$ , one can get:

$$R_0 = \exp(-2\mu^{(4)}_1 - 2\mu^{(4)}_2), \quad (11)$$

where

$$\mu^{(4)}_1 = \frac{12^{1/4} \eta k_p^{1/2}}{4 \epsilon^{1/2}} \frac{1}{x_0} \left[ 1 - \left( \frac{x_0}{x} \right) \right], \quad (12)$$

and

$$\mu^{(4)}_2 = \frac{3 \cdot 12^{1/4} \delta \nu k_p^{1/2}}{10 \epsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^5} \left[ 1 - \left( \frac{x_0}{x} \right)^5 \right]. \quad (13)$$

The conditions of geometrical acoustics approximation for power-law-profiled wedges with  $m = 3$  and  $4$  are discussed in [5].

## 2.5 Wedges of sinusoidal profile

Another type of function for a wedge profile that can also result in acoustic ‘black hole’ effects is the function  $h(x) = \epsilon \sin^m(x)$  for  $m \geq 2$ . The choice of such a function is explained by the fact that for small values of  $x$  it can be approximated as a power-law profile  $\epsilon x^m$ , which has been analysed in the previous sections. For clarity, let us consider the function  $h(x) = \epsilon \sin^m(x)$  for  $m = 2$  and  $4$ . For  $m = 2$  one can obtain [5]:

$$R_0 = \exp(-2\mu^{(s2)}_1 - 2\mu^{(s2)}_2), \quad (14)$$

where

$$\mu^{(s2)}_1 = \frac{12^{1/4} \eta k_p^{1/2}}{4 \epsilon^{1/2}} \left[ \ln \frac{(\csc x - \cot x)}{(\csc x_0 - \cot x_0)} \right], \quad (15)$$

and

$$\mu^{(s2)}_2 = \frac{3 \cdot 12^{1/4} \delta \nu k_p^{1/2}}{4 \epsilon^{3/2}} \frac{E_2}{E_1} \left( \ln \frac{(\csc x - \cot x)}{(\csc x_0 - \cot x_0)} + \frac{\cos x_0}{\sin^2 x_0} - \frac{\cos x}{\sin^2 x} \right). \quad (16)$$

Similarly, for  $h(x) = \epsilon \sin^m(x)$  with  $m = 4$ , one can get:

$$R_0 = \exp(-2\mu^{(s4)}_1 - 2\mu^{(s4)}_2), \quad (17)$$

where

$$\mu^{(s4)}_1 = \frac{12^{1/4} \eta k_p^{1/2}}{4\varepsilon^{1/2}} \left( \frac{\cos x_0}{\sin x_0} - \frac{\cos x}{\sin x} \right). \quad (18)$$

and

$$\mu^{(s4)}_2 = \frac{12^{1/4} \delta v k_p^{1/2}}{10\varepsilon^{3/2}} \frac{E_2}{E_1} \left( 3 \left( \frac{\cos x_0}{\sin^5 x_0} - \frac{\cos x}{\sin^5 x} \right) + 4 \left( \frac{\cos x_0}{\sin^3 x_0} - \frac{\cos x}{\sin^3 x} \right) + 8 \left( \frac{\cos x_0}{\sin x_0} - \frac{\cos x}{\sin x} \right) \right). \quad (19)$$

The discussion of the conditions of geometrical acoustics approximation for sinusoidal wedges with  $m = 2$  and  $4$  can be found in [5]. Note that in the case of sinusoidal profile the applicability of geometrical acoustics approximation is significantly improved in comparison with power-law-profiled wedges with  $m = 3$  and  $m = 4$ , for which the geometrical acoustics approximation becomes invalid far away from the wedge edges.

### 3 Wedges covered by absorbing layers of arbitrary thickness

The expressions for the reflection coefficient of flexural waves derived above are valid if the thickness of layers is much smaller than the local thickness of the main wedge. Therefore, although being very useful and simple, these expressions can only be applicable for wedges covered by very thin absorbing layers (thin films). But even so, their applicability fails near wedge edges, where even very thin films may become comparable or even thicker than wedge local thickness [6]. To extend the analysis to smaller values of wedge truncation and/or to thicker films one has to consider flexural wave propagation in wedges covered by damping layers of arbitrary thickness. As the wave phase behaviour near wedge edges is important for materialising the acoustic ‘black hole’ effect, the influence of the film thickness on flexural wave velocity dispersion in this case should be taken into account as well.

Using the more general approach to the analysis of the effect of damping layers on complex flexural rigidity of a sandwich plate [15] and applying it to a quadratic wedge covered by an absorbing layer on one side only, one can derive the following expression for the corresponding reflection coefficient  $R_0$  [6]:

$$R_0 = \exp \left\{ - \int_{x_0}^x \frac{12^{1/4} k_p^{1/2} (1 + \tilde{\rho} \alpha_2(x))^{1/4} [\eta + v \beta_2 \alpha_2(x) (3 + 6\alpha_2(x) + 4\alpha_2(x)^2)]}{2h(x)^{1/2} [1 + \beta_2 \alpha_2(x) (3 + 6\alpha_2(x) + 4\alpha_2(x)^2)]^{5/4}} dx \right\}. \quad (20)$$

Here  $\alpha_2(x) = \delta/h(x) = \delta/\varepsilon x^2$ ,  $\beta_2 = E_2/E_1$ ,  $\tilde{\rho} = \rho_l / \rho_w$ , where  $\rho_w$  and  $\rho_l$  are the mass densities of the wedge material and of the absorbing layer respectively. Other notations in equation (20) are the same as in the previous sections. In deriving (20) we have used the conditions  $\beta_2 = E_2/E_1 \ll 1$ ,  $\eta \ll 1$  and  $v \ll 1$ , which are valid in the majority of practical situations. Unfortunately, even under such simplifying conditions, equation (20) can not be simplified any further, and the integration in it should be carried out numerically.

## 4 Some numerical examples

Let us first perform calculations according to the simplest formulae (3)-(6) and let us choose for illustration purposes the following values of the film parameters:  $\nu = 0.25$  (i.e., consider the film as being highly absorbing),  $E_2/E_1 = 0.3$  and  $\delta = 15 \mu\text{m}$ . Let the parameters of the quadratically shaped wedge be:  $\varepsilon = 0.05 \text{ m}^{-1}$ ,  $\eta = 0.01$ ,  $x_0 = 2 \text{ cm}$ ,  $x = 50 \text{ cm}$  and  $c_p = 3000 \text{ m/s}$ . Then, e.g. for a wedge covered by absorbing films on both sides and at the frequency  $f = 10 \text{ kHz}$  it follows from equations (3)-(5) that in the presence of the damping film  $R_0 = 0.022$  (i.e. 2.2 %), whereas in the absence of the damping film  $R_0 = 0.542$  (i.e. 54.2 %). Thus, in the presence of the damping film the value of the reflection coefficient is much smaller than for a wedge with the same value of truncation, but without a film. Obviously, it is both the specific geometrical properties of a quadratically-shaped wedge in respect of wave propagation and the effect of thin damping layers that result in such a significant reduction of the reflection coefficient. Note that almost all absorption of the incident wave energy takes place in the vicinity of the sharp edge of a wedge.

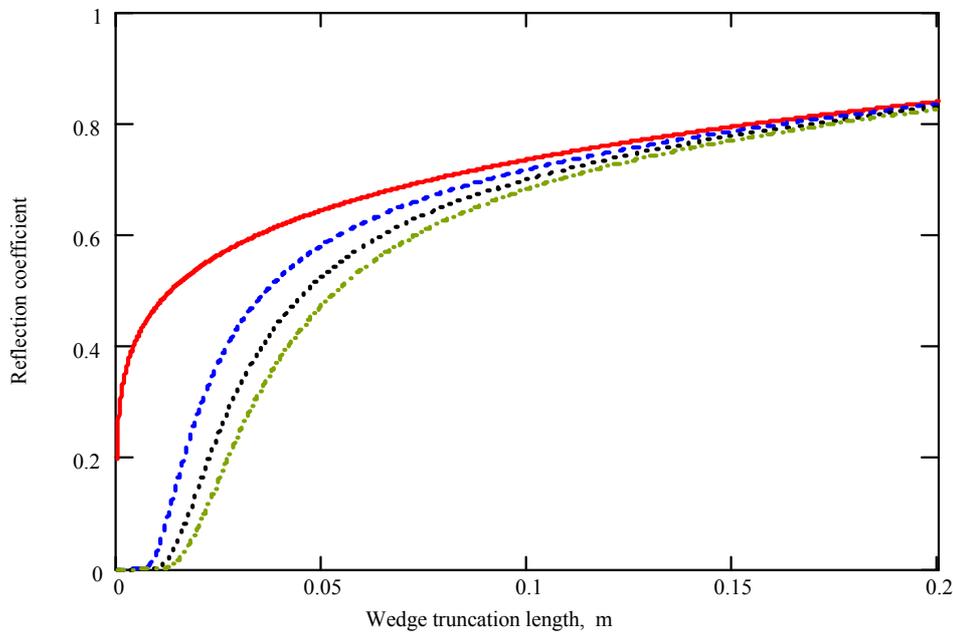


Figure 2. Effect of wedge truncation length  $x_0$  (in  $m$ ) on the reflection coefficient  $R_0$ : solid curve corresponds to an uncovered wedge, dashed, dotted and dash-dotted curves correspond to a wedge covered by absorbing films with the values of relative film stiffness  $E_2/E_1$  equal to 0.1, 0.2 and 0.3 respectively; the film material loss factor  $\nu$  is 0.15, and the film thickness  $\delta$  is  $15 \mu\text{m}$ . [6]

The effect of wedge truncation length  $x_0$  on the reflection coefficient  $R_0$  in the above example is shown in Figure 2. For comparison, the curve corresponding to the wedge not covered by absorbing films is shown on the same Figure as well. One can see that the behaviour of the reflection coefficient  $R_0$  as a function of  $x_0$  is strongly influenced by the parameter  $\beta_2 = E_2/E_1$  describing relative stiffness of the absorbing film. The larger the film stiffness the higher values of truncation  $x_0$  can be allowed to keep the reflection coefficient low.

Numerical calculations have been carried out also for wedges with power-law profiles corresponding to  $m = 3$  and  $m = 4$  and for wedges with sinusoidal profiles corresponding to  $m = 2$  and  $m = 4$  [5]. The results of these calculations (that are not shown here) demonstrate that there is a remarkable improvement in the reduction of the reflection coefficient for larger values of  $m$ .

Let us now turn to the calculations of the reflection coefficient  $R_\theta$  according to the more precise formula (20), where the integration should now be performed numerically. Figure 3 shows the reflection coefficient  $R_\theta$  at frequency  $f = 10 \text{ kHz}$  as a function of the wedge truncation length  $x_0$  for the non-symmetric wedge covered by the absorbing film on one surface only. The parameters of the wedge and film are:  $\varepsilon = 0.05$ ,  $\delta = 10 \mu\text{m}$ ,  $\nu = 0.2$ ,  $\eta = 0.01$ ,  $x_0 = 2 \text{ cm}$  and  $E_2/E_1 = 0.3$ . A dotted curve corresponds to the reflection coefficient calculated according to the simple analytical expressions (3), (4) and (6). A solid curve corresponds to the reflection coefficient calculated according to the more precise formula (20). For comparison, the behaviour of the reflection coefficient for an uncovered wedge is shown in Figure 3 as well.

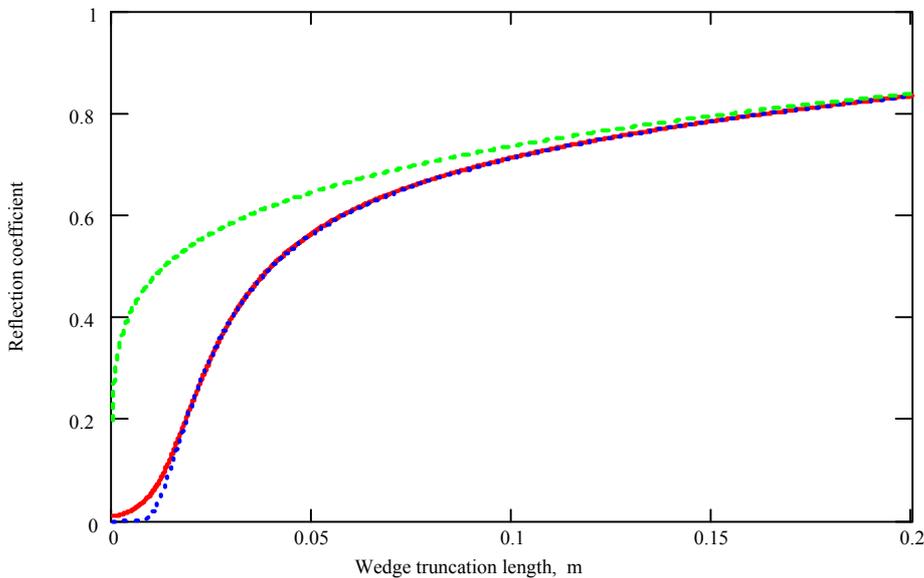


Figure 3. Reflection coefficient  $R_\theta$  for the non-symmetric wedge covered by the absorbing film on one surface only as a function of the wedge truncation length  $x_0$ : solid curve corresponds to the calculations according to equation (20), dotted curve corresponds to the calculations according to the simplified equations (3), (4) and (6), dashed curve shows the reflection coefficient for the uncovered wedge [6].

It is clearly seen that the curves calculated according to the simplified equations and to the more precise formula (20) almost coincide with each other everywhere except very small values of  $x_0$ , where the approximation of thin film becomes invalid. As expected, the values of  $R_\theta$  calculated for very small  $x_0$  according to equation (20) are always larger than zero since even for the ideal case of the absence of wedge truncation ( $x_0 = 0$ ) the presence of the film implies that as  $x \rightarrow 0$  the wedge covered by a film reduces gradually to a film of constant thickness only, and the reflection coefficient is determined entirely by an absorbing film. Nevertheless, the curve

calculated according to the simplified analytical formulae (equations (3), (4) and (6)) represents a very good approximation of the reflection coefficient for realistic values of wedge truncation  $x_0$ .

## 5 Results of the preliminary experiments

The present section describes the results of the preliminary experimental investigation of the damping system comprising a steel wedge of quadratic shape covered on one side by a strip of absorbing adhesive layer ( $\delta = 0.2 \text{ mm}$ ) that was located at the sharp edge of the wedge [7].

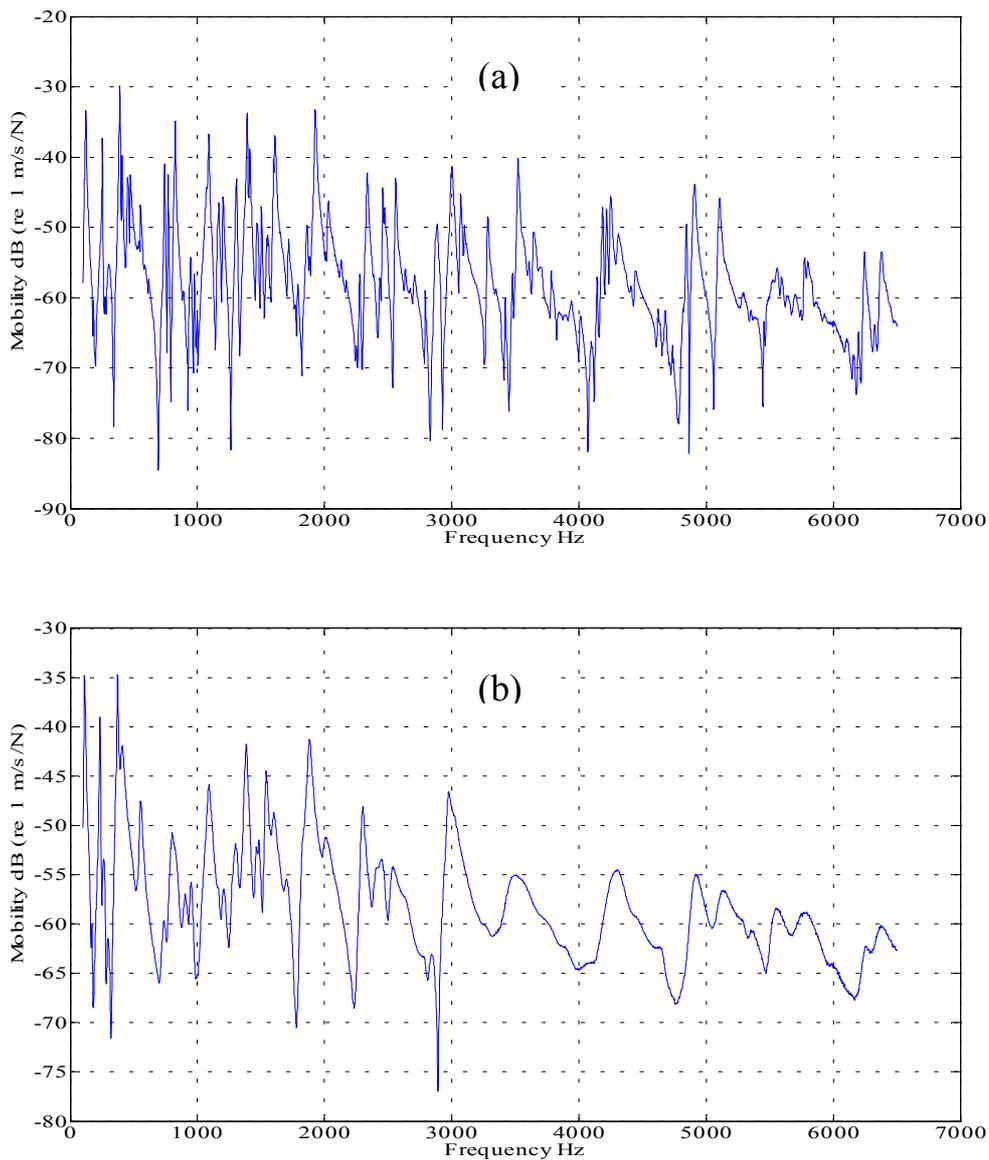


Figure 4. Point mobilities of the free quadratic wedge (a) and of the same wedge covered at the edge by a strip of absorbing adhesive layer (b); the position of the shaker was at 100 mm from the thick end of the wedge and at 100 mm from either side [7].

The wedge dimensions were: 280 mm (length) and 200 mm (width); its thickness at the thick end was 4.5 mm, and the value of the quadratic wedge parameter  $\epsilon$  was  $5 \times 10^{-5} \text{ mm}^{-1}$ . The point mobility measurements have been carried out in a wide frequency range for a free wedge, for a wedge covered by a strip of absorbing thin layer, and for a plate of constant thickness  $h = 4.5 \text{ mm}$  having the same length and width as the quadratic wedge. The results of typical measurements of point mobilities of the free quadratic wedge and of the same wedge covered by a strip of absorbing adhesive layer are shown in Figure 4.

As one can see from Figure 4, in the wedge covered by an absorbing layer there is a significant reduction of resonant peaks, in comparison with the uncovered wedge. This can be attributed to the significant reduction of the reflection coefficient of flexural waves from the sharp edge of the wedge, in agreement with the theory discussed in the previous sections.

## 6 Conclusions

Some theoretical results have been described on the possible practical utilisation of the acoustic ‘black hole’ effect for flexural waves propagating in elastic wedges of power-law profile and in sinusoidal wedges covered by thin absorbing layers for damping flexural vibrations. It has been demonstrated that the presence of thin absorbing layers on the surfaces of such wedges can result in very low reflection coefficients of flexural waves from their edges even in the presence of edge truncations. As a result of this, such wedges can be used as efficient attachable devices providing very low reflection coefficients from edges of structural elements to be damped, e.g. finite plates or bars. This in turn would result in very efficient damping of basic structures’ resonant vibrations.

The additional advantage of using the proposed wedge-like vibration dampers over traditional types of vibration dampers lays in the fact that wedge-like dampers are compact and relatively easy to manufacture. They can be produced as separate parts to be attached to the structure, or they can be integrated into such a structure on a design stage as its inseparable components.

The reported results of the preliminary experimental measurements of point mobilities of a free quadratic wedge and of the same wedge covered by a strip of absorbing adhesive layer generally confirm the above-mentioned theoretical conclusions.

In spite of the encouraging theoretical and experimental results described in this paper, further theoretical and experimental investigations are needed to further validate the principle and to explore the most efficient ways of developing the proposed wedge-like dampers of flexural vibrations.

## 7 References

- [1] C. Vemula and A.N. Norris, Attenuation of waves in plates and bars using a graded impedance interface at edges. *Journal of Sound and Vibration* **196**, 107-127 (1996).
- [2] L. Cremer, M. Heckl, and E.E. Ungar, *Structure-Borne Sound*. Berlin, Springer Verlag 1980.

- [3] V.V. Krylov, Laminated plates of variable thickness as effective absorbers for flexural vibrations. *Proceedings of the 17th International Congress on Acoustics*, Rome, Italy, 2-7 September 2001, Ed. A. Alippi, Vol. 1, pp. 270-271 (2001).
- [4] V.V. Krylov, Acoustic 'black holes' for flexural waves and their potential applications. *Proceedings of the Institute of Acoustics Spring Conference*, Salford, UK, 25-27 March 2002 (on CD).
- [5] V.V. Krylov and F.J.B.S. Tilman, Acoustic 'black holes' for flexural waves as effective vibration dampers. *Journal of Sound and Vibration*, **274**, 605-619 (2004).
- [6] V.V. Krylov, New type of vibration dampers utilising the effect of acoustic 'black holes'. *Acustica-Acta Acustica*, **90**(5), 830-837 (2004).
- [7] V.V. Krylov and R.E.T.B. Winward, Experimental evidence of the acoustic black hole effect for flexural waves in tapered plates, *Proceedings of the 12<sup>th</sup> International Congress on Sound and Vibration*, Lisbon, Portugal, 11-14 July 2005 (to be published).
- [8] V.V. Krylov, Conditions for validity of the geometrical-acoustic approximation in application to waves in an acute-angle solid wedge. *Soviet Phys. -Acoust.* **35**, 176-180 (1989).
- [9] V.V. Krylov, Geometrical-acoustics approach to the description of localized vibrational modes of an elastic solid wedge. *Soviet Phys. - Tech. Phys.* **35**, 137-140 (1990).
- [10] V.V. Krylov, On the velocities of localised vibration modes in immersed solid wedges. *J. Acoust. Soc. Am.* **103**, 767-770 (1998).
- [11] V.V. Krylov, Localised acoustic modes of a quadratically-shaped solid wedge. *Moscow Univ. Phys. Bull.*, **45**, 65-69 (1990)
- [12] V.V. Krylov and A.L. Shuvalov, Propagation of localised flexural vibrations along plate edges described by a power law. *Proceedings of the Institute of Acoustics*, **22**(2), 263-270 (2000).
- [13] M.A. Mironov, Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval. *Sov. Phys. – Acoustics*, **34**, 318-319 (1988).
- [14] H. Oberst, Ueber die Dämpfung der Biegeschwingungen dünner Bleche durch fest haftende Beläge. *Acustica*. **2**, 181-194 (1952).
- [15] D. Ross, E.E. Ungar and E.M. Kerwin, Jr., Damping of plate flexural vibrations by means of viscoelastic laminae. In: J.E. Ruzicka (ed) *Structural Damping*, Pergamon Press, Oxford 1960, p. 49-87.
- [16] A.S. Nikiforov and S.V. Budrin, *Propagation and attenuation of vibrations in ships*, Sudostroenie, Leningrad 1968 (in Russian).
- [17] P.N. Kravchun, *Generation and methods of reduction of noise and vibration*, Moscow University Press, Moscow 1991 (in Russian).
- [18] E.E. Ungar, Vibration isolation and damping. In: M.J. Crocker (ed) *Handbook of Acoustics*, John Wiley and Sons, New York 1998, p. 675-687.