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Wedge elastic waves, with applications to ultrasonic non-destructive testing

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Abstract

Wedge elastic waves are localised vibration modes, both symmetric and anti-symmetric, propagating along tips of elastic wedges. Their existence has been first predicted numerically in 1972 by Lagasse, and then they have been investigated both theoretically and experimentally in a large number of works. The essentially one-dimensional nature of these waves makes them ideally suitable for ultrasonic non-destructive inspection of edges in wedge-like and plate-like structures of different forms and shapes, such as turbine blades, cutting tools, etc. In the present paper, propagation of wedge elastic waves in wedges of different types is discussed, with the emphasis on the theory of anti-symmetric mode propagation in slender wedges developed by the present author using the geometrical acoustics approach. This theory is powerful enough to be applicable to wedges of arbitrary shapes, curved wedges and truncated wedges. In all these cases, with the exception of ideal linear elastic wedges, wedge elastic waves are dispersive. Theory of scattering of wedge elastic waves on small defects located on sharp wedge edges is considered as well using perturbation theory approach. Finally, some theoretical and experimental results are discussed regarding different aspects of propagation and scattering of wedge elastic waves relevant to NDT applications.

1. Introduction

Wedge elastic waves (also known as 'wedge acoustic waves') are localised elastic waves propagating along the tips of elastic wedges (see Figure 1) and of different wedge-like structures. The existence of such waves in an ideal wedge of linear profile formed by two intersecting planes has been first predicted in 1972 by Lagasse ⁽¹⁾ by means of numerical calculations. Similar numerical predictions have been made independently by Maradudin and co-workers, also in 1972, albeit a few weeks later. These waves represent a fundamental type of elastic waves in solids, in addition to longitudinal and shear bulk waves and to different types of surface waves, including Rayleigh waves. They contribute to thermal properties of finite solids studied in statistical physics and to modes of vibration of complex wedge-like structures in different engineering applications. They also can be attractive for non-destructive testing of different wedge-like structures, which will be the main focus of this paper.

According to the more detailed numerical investigations of localised elastic waves in ideal wedges ⁽³⁾ carried out for a material with Poisson's ratio $\sigma = 0.25$, there is a number of antisymmetric modes (for wedges with wedge angles θ between 0 and 100 degrees) and only one symmetric mode (for wedges with wedge angles θ between 125 and 180 degrees).

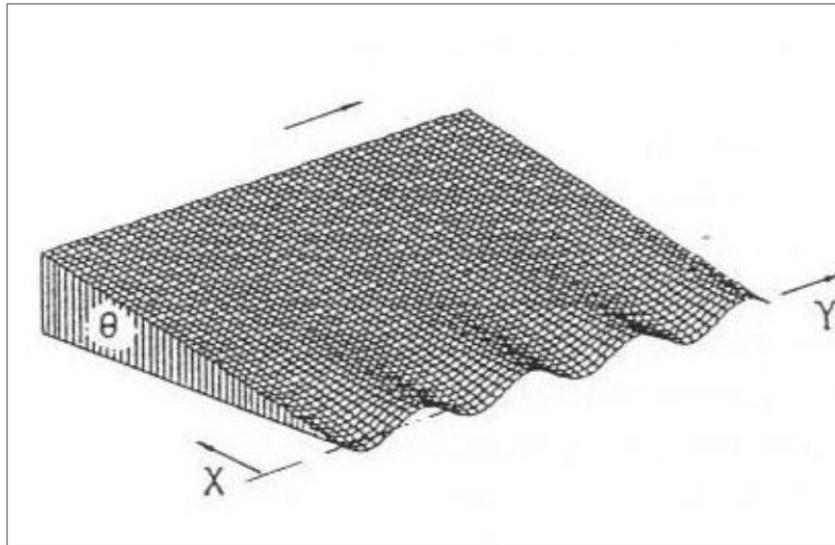


Figure 1. Antisymmetric wedge elastic waves propagating along the tip of an ideal wedge characterised by the wedge angle θ

The above-mentioned symmetric wedge mode is in fact a modified Rayleigh wave propagating with the velocity that is slightly lower than the velocity of Rayleigh waves on a flat surface. This mode decays exponentially on both sides from the tip, and it is relatively weakly localised.

In contrast to the symmetric mode, the antisymmetric wedge modes are strongly localised, and their velocities can differ very significantly from the velocity of Rayleigh waves, reducing almost to zero for wedges with very small wedge angles θ . For that reason, antisymmetric modes of wedge elastic waves are the most interesting for practical applications.

In this paper, an overview of theoretical and experimental results on wedge elastic waves that are relevant to their potential non-destructive testing applications will be presented. First, the geometrical acoustics theory of wedge elastic waves propagating in slender wedges will be briefly discussed, following the results obtained by the present author. This theory will be extended to describe localised wave propagation in wedges with more complex geometry. The examples to be considered include truncated wedges and wedges of quadratic profile. After that, theoretical and experimental results on scattering of wedge elastic waves on local defects will be considered. Finally, some possible applications of wedge elastic waves to ultrasonic non-destructive testing of wedge like structures will be briefly discussed.

2. Geometrical acoustics theory of wedge elastic waves

2.1 Waves in ideal wedges of linear profile

Geometrical acoustics approximation is used widely in different problems of classical acoustics for description of sound propagation in ocean and atmosphere ⁽⁴⁾. It can be applied also to Lamb waves propagating in plates of variable thickness ⁽⁵⁾. The most important modes of Lamb waves are lowest order antisymmetric and symmetric modes, or flexural and compression waves respectively. In what follows, a particular type of plates of variable thickness will be discussed initially – plates with linearly variable thickness $h(x) \approx \theta x$ forming slender elastic wedges, i.e. wedges with small values of the wedge angle θ (Figure 2).

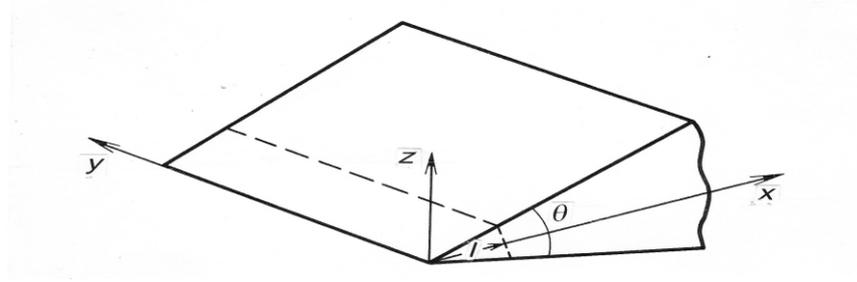


Figure 2. Geometry of a slender solid wedge

To develop a geometrical acoustics theory of wedge elastic waves, one should consider propagation of flexural waves at an arbitrary angle in slender wedges in the geometrical acoustics approximation. To consider this, one can start from the equation for flexural waves in a plate of variable thickness $h(x) \approx \theta x$ (this equation is not shown here for brevity) and seek the solution of this equation in the following form ^(5, 6):

$$w = A(x)e^{i \int [k_a^2(x) - \gamma^2]^{1/2} dx} \quad (1)$$

Here $k_a(x)$ is the local wavenumber of the propagating flexural wave, γ is the projection of the flexural wave vector on the axis y (this projection is considered to be constant), and $A(x)$ represents the wave amplitude that is slowly varying in x -direction. It can be shown that equation (1) satisfies the above-mentioned plate equation for flexural waves in a slender wedge asymptotically at relatively high frequencies ^(5, 6) (see also the monograph ⁽⁷⁾) if the function $A(x)$ has the form

$$A(x) = \frac{G}{x [k_a^2(x) - \gamma^2]^{1/4}}, \quad (2)$$

where G is an arbitrary constant. It can be seen from (2) that the amplitude $A(x)$ increases for small values of x , when the flexural wave approaches the wedge tip, in

agreement with energy conservation law ⁽⁶⁾. Relatively high frequencies mean in practice that these frequencies are well within the range of used frequencies.

The velocities c of the localised wedge modes propagating in y direction of a wedge can be determined in the geometrical acoustics approximation as solutions of the following Bohr - Sommerfeld type equation ^(5, 6):

$$\int_0^{x_t} [k_a^2(x) - \gamma^2]^{1/2} dx = \pi n, \quad (3)$$

where $\gamma = \omega/c$ is considered as yet unknown wavenumber of a wedge mode, $k_a(x)$ is the already mentioned local wavenumber of a flexural wave in a plate of variable thickness, $n = 1, 2, 3, \dots$ is the mode number, and x_t is the so called ray turning point determined from the equation $k_a^2(x) - \gamma^2 = 0$. For example, in the case of a wedge in vacuum $k_a(x) = 12^{1/4} k_p^{1/2} (\theta x)^{-1/2}$, $x_t = 2\sqrt{3} k_p / \theta \beta^2$, and $k_p = \omega/c_p$, where ω is circular frequency, $c_p = 2c_t(1 - c_t^2/c_l^2)^{1/2}$ is the so called plate wave velocity, c_l and c_t are propagation velocities of longitudinal and shear acoustic waves in plate material. Then, taking analytically the integral in (3) and solving the resulting algebraic equation yields the extremely simple analytical expression for wedge wave velocities ⁽⁵⁻⁷⁾:

$$c = \frac{c_p n \theta}{\sqrt{3}}. \quad (4)$$

The expression (4) agrees well with the other (numerically based) theories in the case of slender wedges ⁽¹⁻³⁾ and with the available experimental results.

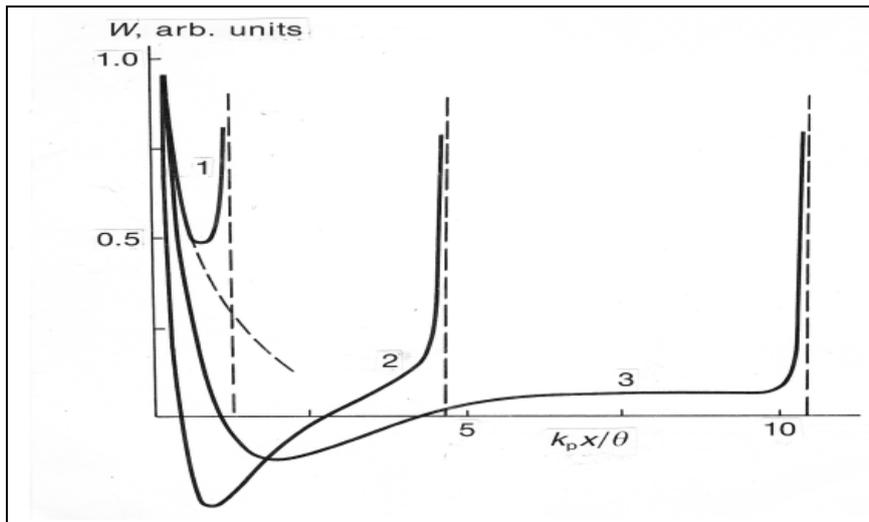


Figure 3. First three modes of wedge elastic waves calculated using geometrical acoustics approximation ⁽⁵⁾

Note that the number of propagating modes of wedge elastic waves depends on the wedge angle θ : the smaller the wedge angle θ , the larger the number of propagating modes n . This number can be roughly estimated from the condition $c < c_R$, where c is defined by (4) and c_R is Rayleigh wave velocity.

The analytical expressions for amplitude distributions of wedge modes are rather cumbersome⁽⁵⁾ and are not displayed here. Figure 3 illustrates the first three modes calculated in geometrical acoustics approximation⁽⁵⁾. The structure of the wedge modes shown in Figure 3 agrees well with the results of numerical calculations, with the exception of the clearly seen singularities in the areas marked by dashed vertical lines. Beyond these lines, which show the locations of caustics (ray congestions), the modes do not penetrate into the depth of a wedge. The above singularities manifest the well-known limitation of all geometrical acoustics (optics) theories: they become invalid near caustics^(4,7).

The above-mentioned geometrical acoustics theory of wedge elastic waves can be generalised to consider localised modes in truncated wedges⁽⁶⁾, quadratically-shaped elastic wedges⁽⁸⁾, wedges immersed in liquids⁽⁹⁾, cylindrical and conical wedge-like structures (curved wedges)^(10,11), wedges of general power-law shape⁽¹²⁾, wedges made of anisotropic materials⁽¹³⁾, and nonlinear wedges⁽¹⁴⁻¹⁶⁾.

2.2 Waves in truncated wedges

This important case of real wedges, that are always not ideally sharp but truncated to a certain degree, can be easily analysed using the geometrical acoustics approach to the theory of wedge elastic waves described in the previous section. Namely, the velocities c of wedge elastic waves in such structures still can be obtained as a solution of the Bohr - Sommerfeld type equation similar to (3). The only difference from (3) is that the integration over x is now taken not from zero, but from l indicating the height of truncation (see Figure 2). Performing the integration^(5,7), one can obtain the following algebraic equation in respect of the wedge mode velocities c :

$$\frac{c}{c_0} \pi n = \pi n + \left[\frac{2\sqrt{3}k_p l}{\theta} \left(1 - \frac{c_0^2}{c^2} \frac{\sqrt{3}k_p l}{2n^2\theta} \right) \right]^{1/2} + \frac{c}{c_0} n \cos^{-1} \left(1 - \frac{c_0^2}{c^2} \frac{\sqrt{3}k_p l}{n^2\theta} \right), \quad (5)$$

where $c_0 = c_p n \theta / \sqrt{3}$ are the velocities of wedge modes for an ideal wedge, according to (4). For ideal wedges (without truncations, i.e. for $k_p l = 0$), equation (5) reduces to equation (4), as expected. In the general case of truncated wedges ($k_p l \neq 0$), the numerical solutions of the equation (5) for the velocities of the first three modes are shown in Figure 4 by solid curves. For comparison, the corresponding more precise solutions to the thin plate equation for truncated slender wedges earlier obtained not by means of geometrical acoustics, but using special functions⁽¹⁷⁾ are also shown in Figure 4 by dashed curves. It can be seen from Figure 4 that, according to the solutions obtained using both approaches, all modes of wedge elastic waves in truncated wedges are dispersive, i.e. their velocities depend on frequency. This is what one would expect because, in contrast to the case of ideal wedges, truncated wedges have a characteristic

dimension - the length of truncation l . One can also see that, for the wedge mode with $n = 1$, the geometrical acoustics solution differs significantly from the more precise solution, which is in agreement with the applicability condition for this case ⁽⁶⁾. However, for modes with $n = 2$ and $n = 3$, the agreement is quite good, especially for large values of the non-dimensional parameter $\sqrt{3k_p l/\theta}$. Obviously, the presence of velocity dispersion in the case of truncated wedges could be used for non-destructive inspection of solid wedges, e.g. for control of sharpness of cutting tools.

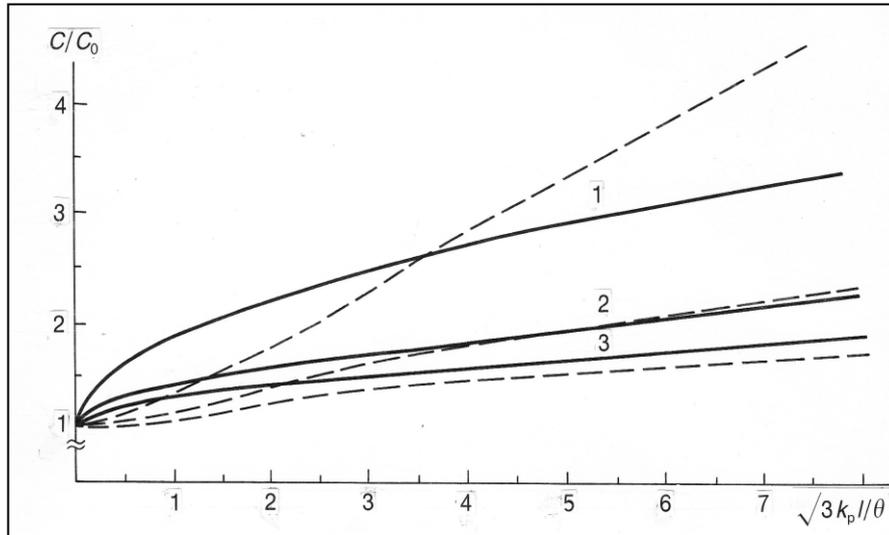


Figure 4. Relative velocities of the first three antisymmetric modes of a truncated slender wedge as functions of the non-dimensional parameter $\sqrt{3k_p l/\theta}$ ⁽⁵⁾

2.3 Waves in wedges of quadratic profile

An important advantage of geometrical acoustics approach is its ability of analysing localised modes in solid structures of more complex geometry. One of such structures is a wedge of quadratic profile (see Figure 5) whose local thickness $h(x)$ is described by the function $h(x) = \varepsilon x^2$, where ε is a dimensional parameter. Using this function in the expression for $k_a(x)$ and substituting the result into the Bohr-Sommerfeld type equation (3), in which the integration should be taken from the value of the truncation length x_0 (instead of zero) to the turning point x_t , one can derive the algebraic equation (which is not shown here for brevity) that defines the velocities c of localised modes propagating in a quadratic wedge ⁽⁸⁾. This equation is rather complicated, and in general case it can be solved only numerically.

Typical results of such numerical solutions for relative velocities of localised modes c/c_p as functions of the non-dimensional parameter $G = (2\sqrt{3k_p/\varepsilon})^{1/2}$, where $k_p = \omega/c_p$, are shown in Figure 6. It is assumed that $G \gg 1$. Calculations have been carried out for the value of the non-dimensional parameter $\varepsilon x_0/2\sqrt{3}$ (involving the truncation length x_0) equal to 0.005. It can be seen from Figure 6 that wedge elastic waves in quadratic wedges are dispersive, which reflects the fact that quadratic wedges are characterised by the two dimensional parameters: ε and x_0 . The mode velocities

initially decrease with frequency, which is proportional to G^2 , until they reach their minimum values, before starting to increase. Note that the phase velocities of all these modes are proportional to the dimensionless parameter ϵx_0 describing the wedge sharpness. Obviously, by reducing the value of the parameter ϵx_0 , e.g. by reducing the length of truncation x_0 , the velocities of antisymmetric wedge modes can be made arbitrary small. In particular, for $x_0 = 0$ corresponding to the case of ideal quadratic wedge (without truncation), the velocities of all modes are zero. This behaviour can be explained by the fact that the reflection coefficients of flexural waves from the tips of ideal quadratic wedges and wedges of higher power-law profiles are equal to zero, which prevents such modes from propagation ⁽¹²⁾.

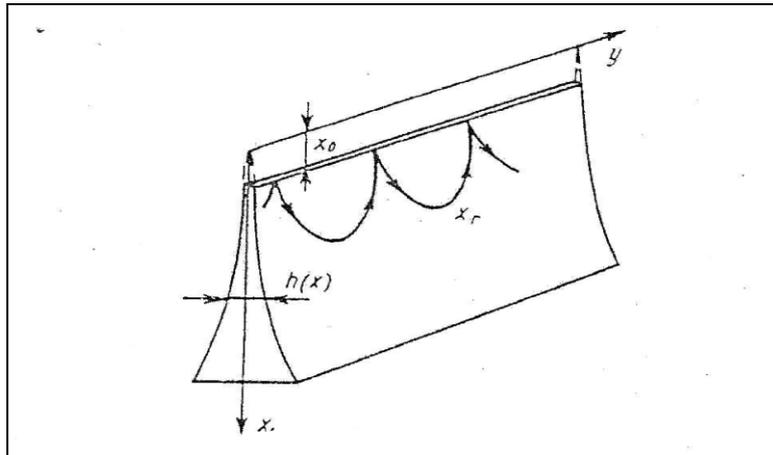


Figure 5. Elastic wedge of quadratic profile: $h(x) = \epsilon x^2$

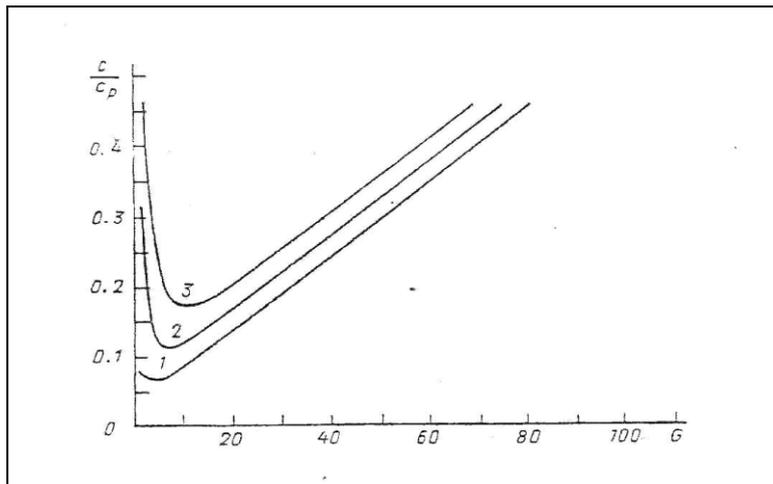


Figure 6. Dispersion curves of the three lowest order modes of a quadratic wedge as functions of the non-dimensional parameter $G = (2\sqrt{3}\omega/\epsilon c_p)^{1/2}$ ⁽⁸⁾

3. Scattering of wedge elastic waves

Studying of scattering of wedge elastic waves on different defects of wedge tips or on abrupt edges of wedge-like structures is paramount for using such waves in non-destructive testing. There have been very few publications in this area so far.

3.1 Theoretical studies

Theoretical analysis of scattering of antisymmetric wedge waves has been first performed analytically for the case of a shallow notch on the tip of a slender wedges of linear profile⁽¹⁸⁾. The geometry of the problem is shown in Figure 7. The shallowness of the notch, in comparison with the wavelength of propagating wedge waves, means that the first (Born) approximation of the perturbation approach can be used in developing the theory of scattering. For slender wedges, the total elastic field can be found as a sum of the incident wedge elastic wave of a given mode and of the scattered field represented as the sum of all existing wedge modes propagating in forward and in backward directions (the details are not presented here for shortness). Scattering into the backward-travelling mode of the same order as the incident wave represents the reflection of the wedge mode from the notch.

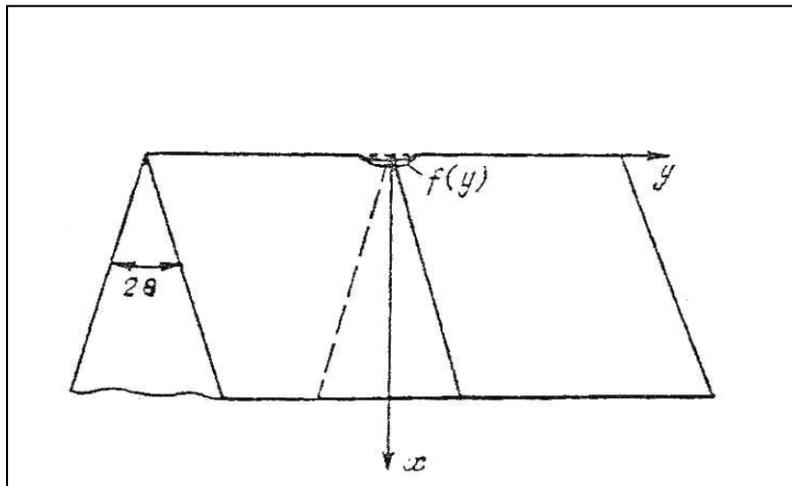


Figure 7. Geometry of a slender wedge with a notch on its tip

Note that analytical expressions for different modes of wedge waves of the unperturbed wedge (without a notch) that are present in the perturbation solution for the scattered field should be accurate enough in the vicinity of the wedge tip. Obviously, this is not the case for wedge modes defined in the geometrical acoustics approximation, which is invalid near wedge tips. Therefore, for consideration of the scattering problem⁽¹⁸⁾, the known solution for an ideal slender wedge in terms of special functions⁽¹⁷⁾ has been used.

From the practical point of view, the fact that scattering of a wedge wave on a notch takes place into all wedge modes that can propagate in a wedge with a given wedge

angle is a factor that makes it difficult to interpret the scattered signal in non-destructive testing applications. Indeed, all modes propagate at different velocities. Therefore, the scattered signal contains several pulses, which would require application of special mode filtering to select a pulse representing a desired wedge mode. There is no such complication if a wedge angle is large enough so that only one antisymmetric mode can propagate in the wedge, like in the case of rectangular wedge to be discussed in the next sub-section. However, for large wedge angles, analytical solution of the scattering problem is not possible and numerical calculations should be used instead.

3.2 Experimental studies

The first experimental study of wedge wave scattering⁽¹⁹⁾ has been carried out using an Aluminium prism of height 160 mm having a base representing a rectangular triangle with legs of 60 mm (see Figure 8). Pulses of a single antisymmetric wedge mode (for a 90⁰-wedge) of duration 4 μ s and with central frequency of 2.1 MHz were generated by means of a horizontally polarised plate made of piezoelectric ceramics via a pad made of acrylic plastic (1). By rotating the transducer (1) by 90⁰ it was also possible to generate a symmetric wedge mode, which in this case was a leaky wave.

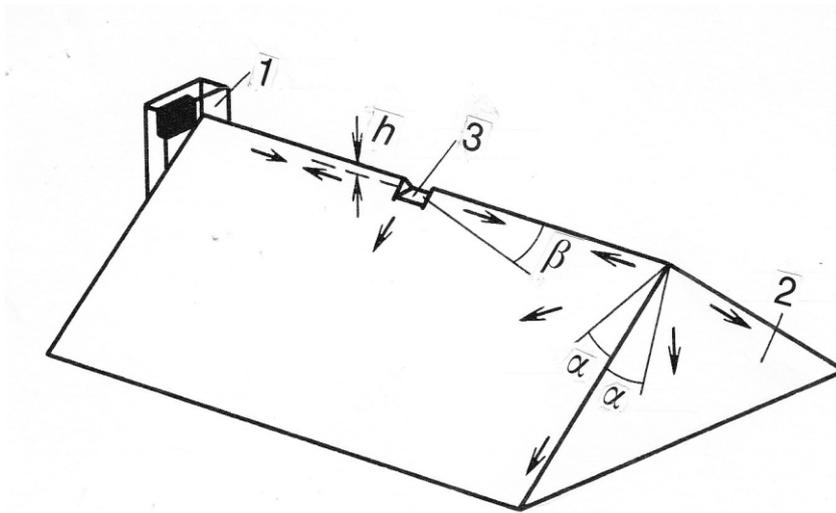


Figure 8. Scattering of an antisymmetric wedge wave by irregularities of a rectangular wedge made of Aluminium⁽¹⁹⁾

Generated and scattered antisymmetric wedge modes were detected by a common surface wave wedge transducer. The same transducer was also used as a receiver of Rayleigh waves. The velocities of the antisymmetric and symmetric wedge modes have been measured with a precision up to 1%. The measured velocity of the antisymmetric mode was 2800 m/s, and of the symmetric mode - 2920 m/s. The measured Rayleigh wave velocity was 2880 m/s, which means that the velocity of the symmetric wedge mode was larger than the velocity of Rayleigh waves, thus confirming that this mode for a 90⁰-wedge is indeed a leaky mode propagating with a significant attenuation. The measured value of this attenuation was 1.5 dB/cm, whereas the antisymmetric mode did not practically vary with distance.

Measurements of scattering have been carried out for antisymmetric wedge waves only⁽¹⁹⁾ (see also the monograph⁽⁷⁾). Two basic types of wedge irregularities have been considered: a rectangular face of the wedge (2) and a relatively shallow notch on the wedge tip (3). Note that in both cases the scattering takes place into other wedge elastic waves (reflected and transmitted), into Rayleigh waves propagating along side surfaces, and into longitudinal and shear bulk waves. In particular, for the scattering on a rectangular face of the wedge (2), the measurements have shown that the absolute values of the reflection coefficient $|R|$ into the wedge mode propagating in the opposite direction and of the transmission coefficients $|T|$ into the two wedge modes propagating along branching edges were $|R| = 0.5 \pm 0.05$ and $|T| = 0.4 \pm 0.06$ respectively. Measurements of angular dependence of scattering into Rayleigh waves on all three intersecting surfaces have been carried out as well⁽¹⁸⁾ (the details are not discussed here for shortness).

Measurements of scattering of an antisymmetric wedge mode by a notch made on the wedge tip have been carried out for a rectangular notch of width $d = 0.5$ mm. The depth h varied from 0.02 to 2 mm with step 0.1-0.15 mm. For each value of h , the reflection and transmission coefficients into wedge modes, $|R|$ and $|T|$ respectively, have been measured, as well as angular dependence of scattering into Rayleigh waves on side surfaces. The values of $|R|$ and $|T|$ as functions of h/λ , where λ is the wavelength of the wedge wave ($\lambda = 1.32$ mm at 2.1 MHz) are shown in Figure 9.

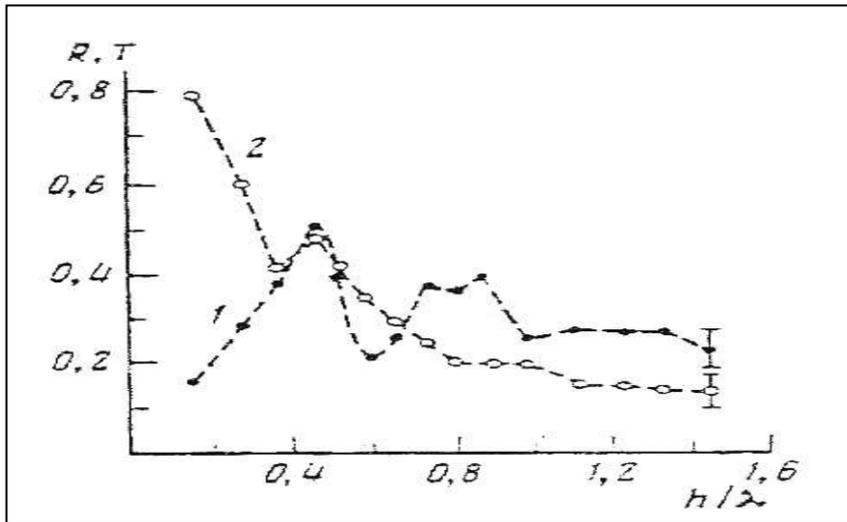


Figure 9. Measured absolute values of the reflection coefficient (curve 1) and transmission coefficient (curve 2) of the lowest order antisymmetric mode on a notch made on the tip of a rectangular Aluminium wedge⁽¹⁹⁾

In the later work⁽²⁰⁾, experimental investigations of scattering of antisymmetric wedge modes on several types of irregularities in wedges made of different materials have been carried out. Some of the wedge angles were relatively small, so that more than one wedge mode could propagate along the wedge tips. In particular, four wedges have been made of Duraluminium, with the wedge angles of 35° , 45° , 50° and 65° . The numbers of

propagating antisymmetric wedge modes in these wedges were 3, 2, 2 and 1 respectively. Also, wedges have been made of Plexiglas and of Polyvinyl (two wedges for each of these materials, with the wedge angles of 35° and 65°). The numbers of propagating antisymmetric wedge modes in these wedges were 2 and 1 respectively. Measurements of forward scattering, among other investigations, have shown that a process of mode conversion occurs in this case, which agrees with the above-mentioned theoretical prediction for the scattering on a shallow notch ⁽¹⁸⁾.

4. On some possible applications to non-destructive testing

As in the cases of ultrasonic non-destructive testing involving bulk, surface or plate waves, it is envisaged that scattering of wedge elastic waves on various defects of the tips of different wedge-like structures would be the main physical phenomenon to be used in non-destructive testing employing wedge waves. The most important wedge-like structures that could be tested in this way are turbine blades and propellers. The advantage of wedge elastic waves over traditional types of elastic waves in testing such structures is in their localisation at the tips, where damage may occur and where traditional types of waves are not very suitable. The factor that may restrict using wedge waves for non-destructive testing is the complexity of their scattering on defects, in particular their scattering into several wedge modes that can exist in wedges with a relatively small wedge angles. Further research into scattering of wedge waves is needed to explore their full potential for non-destructive testing applications.

Another possible application of wedge waves for non-destructive testing could be their use for inspection of sharpness of blades and cutting tools via measuring velocity dispersion of propagating waves. As was mentioned above, for ideally sharp wedge, there is no velocity dispersion. However, for any deviation from an ideal shape, be it a truncation, a more complex shape or curvature, there will be a change in velocity with frequency that can be used for non-destructive evaluation.

5. Conclusions

In the present paper, it has been demonstrated that processes of propagation and scattering of wedge elastic waves have some unique features that could be used for non-destructive testing of some special wedge-like structures, such as turbine blades and propellers. Using wedge elastic waves for testing edges of such structures could be more efficient and less time consuming than employing more traditional types of bulk and surface waves. Practical confirmations of these expectations are yet to come.

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