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## Experimental evidence of the acoustic black hole effect for flexural waves in tapered plates

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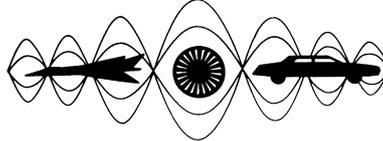
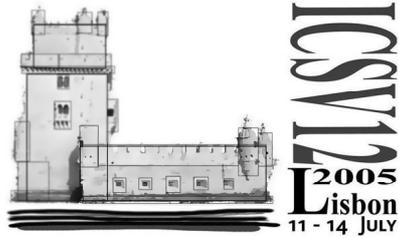
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**Twelfth International Congress  
on Sound and Vibration**

## **EXPERIMENTAL EVIDENCE OF THE ACOUSTIC BLACK HOLE EFFECT FOR FLEXURAL WAVES IN TAPERED PLATES**

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### **Abstract**

A new efficient method of reducing edge reflections of flexural waves in plates or bars based on the 'acoustic black hole effect' has been recently proposed and described theoretically by one of the present authors [1] (see also [2-4]). The method utilises a gradual change in thickness of a plate or bar, partly covered by thin damping layers, from the value corresponding to the thickness of the basic plate or bar (which is to be damped) to almost zero. The present paper describes the results of the experimental investigation of the damping system consisting of a steel plate (wedge) of quadratic shape covered on one side by a strip of absorbing layer. The results of the measurements of point mobility in such a system show that for the wedge covered by an absorbing layer there is a significant reduction of resonant peaks, in comparison with the uncovered wedge or with the covered plate of constant thickness. Thus, the measurements confirm the existence of the acoustic black hole effect for flexural waves and demonstrate the possibility of its use in practice.

### **INTRODUCTION**

This paper describes the results of the experimental investigation of the damping system consisting of a steel wedge of quadratic shape covered on one side by a strip

of absorbing layer located at the sharp edge. Such a system materialises the new efficient method of reducing edge reflections of flexural waves in plates or bars recently proposed and described theoretically by one of the present authors [1] (see also [2-4]) and based on the 'acoustic black hole effect'. It is well-known that damping of resonant flexural vibrations of some engineering structures or their components, such as finite plates or bars, can be achieved by reducing reflections of flexural waves from free edges of the structures. This can be realised, for example, by introducing graded impedance interfaces, such as combinations of finite plates of the same thickness but made of different materials, along with placing damping material at the edges [5]. The main difficulty in implementing this approach is to create suitable impedance interfaces. In contrast to [5], the new method proposed in [1-4] uses gradual change in plate or bar thickness, i.e. it employs elastic wedges of non-linear shape as gradual impedance interfaces. The above-mentioned gradual change in thickness of a plate or a bar has to be made according to the special laws that ideally provide zero reflections even for negligibly small material attenuation – the so-called 'acoustic black hole effect'. To make up for real manufactured wedges and to improve the damping one should cover wedge surfaces near edges by thin absorbing layers (films), e.g. by polymeric films. According to the theoretical calculations [1-4], wedges of power-law profile covered by thin absorbing layers can be very efficient damping systems, with flexural wave reflection coefficients as low as 1-3 %.

The aim of this paper is to present the results of the experimental investigation of the damping system consisting of a steel wedge of quadratic shape covered on one side by a strip of absorbing layer located at the sharp edge. As will be demonstrated, the results of these experiments show that in the wedge covered by an absorbing layer a significant reduction of resonant peaks can be observed, in comparison with the uncovered wedge or with the free and covered plates of constant thickness.

## THEORETICAL BACKGROUND

The physics of the 'acoustic black hole effect' for flexural waves can be understood using the theory of flexural wave propagation near edges of elastic plates of variable thickness gradually decreasing to zero, i.e. near edges of thin elastic wedges of arbitrary shape [6-11]. The phenomenon in question can take place in the special case of wedges having cross sections described by a power law relationship between the local thickness  $h$  and the distance from the edge  $x$ :  $h(x) = \varepsilon x^m$ , where  $m$  is a positive rational number and  $\varepsilon$  is a constant [9-11]. In particular, for  $m \geq 2$  - in free wedges [9-11], and for  $m \geq 5/3$  – in immersed wedges [9,10], the flexural waves incident at an arbitrary angle upon a sharp edge can become trapped near the very edge and therefore never reflect back. Such wedges thus materialise acoustic 'black holes' for flexural waves. In the case of localised flexural waves propagating along edges of such wedges (these waves are also known as *wedge acoustic waves*) the phenomenon of acoustic 'black holes' implies that wedge acoustic wave velocities in such structures become equal to zero [9,10].

The unusual effect of power-law profile on flexural wave propagation in wedges has been first described by Mironov [11]. He pointed out that a flexural wave does not reflect from the edge of a quadratically shaped wedge in vacuum ( $m = 2$ ), so that even a negligibly small material attenuation can cause all the wave energy to be absorbed near the edge. Unfortunately, because of the deviations of real manufactured wedges from the ideal power-law shapes, largely due to ever-present truncations of the wedge edges, the reflection coefficients in such homogeneous wedges are as high as 50-70 % [11], so that they can not be used as practical vibration dampers. To improve the situation, one can consider covering the wedge surfaces by thin damping layers (films) of thickness  $\delta$ , e.g. by polymeric films. Note in this connection that the idea of applying absorbing layers for damping flexural vibrations of plates has been used successfully since the 50-ies (see, e.g. [12]). The new aspect of this idea, which is used in the proposed approach, is to apply such absorbing layers in combination with the specific power-law geometry of a plate of variable thickness (a wedge) to achieve maximum damping.

Two types of wedge geometry can be considered: a symmetric wedge and a non-symmetric wedge bounded by a plain surface at one of the sides. For each of these cases either two or only one of the sides can be covered by absorbing layers. Note that non-symmetric wedges are easier to manufacture. They also have the advantage in depositing absorbing layers: the latter can be deposited on a flat surface, which is much easier. From the point of view of theoretical description, there is no difference between symmetrical and non-symmetrical wedges as long as geometrical acoustics approximation is concerned and the wedge local thickness  $h(x) = \epsilon x^m$  is much less than the flexural wavelength.

To analyse the effect of thin absorbing films on flexural wave propagation in a wedge in the framework of geometrical acoustics approximation one should consider first the effect of such films on flexural wave propagation in plates of constant thickness. The simplest way to approach this problem is to use the already known solutions for plates covered by damping layers (see e.g. [12]). Not specifying physical mechanism of the material damping in the film material, we assume for simplicity that it is linearly dependent on frequency, with non-dimensional constant  $\nu$  being the energy loss factor, or simply the loss factor. Using this approach (see [3,4] for more detail), one can derive the corresponding analytical expressions for the reflection coefficients of flexural waves from the edges of truncated wedges covered by absorbing layers. In particular, for a quadratic wedge covered by an absorbing layer of arbitrary thickness on one side only one can derive the following expression for the corresponding reflection coefficient  $R_0$  [4]:

$$R_0 = \exp \left\{ - \int_{x_0}^x \frac{12^{1/4} k_p^{1/2} (1 + \tilde{\rho} \alpha_2(x))^{1/4} [\eta + \nu \beta_2 \alpha_2(x) (3 + 6\alpha_2(x) + 4\alpha_2(x)^2)]}{2h(x)^{1/2} [1 + \beta_2 \alpha_2(x) (3 + 6\alpha_2(x) + 4\alpha_2(x)^2)]^{5/4}} dx \right\}. \quad (1)$$

Here  $\alpha_2(x) = \delta/h(x) = \delta/\epsilon x^2$ ,  $\beta_2 = E_2/E_1$ ,  $\tilde{\rho} = \rho_l / \rho_w$ , where  $\rho_w$  and  $\rho_l$  are the mass densities of the wedge material and of the absorbing layer respectively,  $k_p = \omega/c_p$  is

the wavenumber of a symmetrical plate wave,  $c_p = 2c_t(1 - c_t^2/c_l^2)^{1/2}$  is its phase velocity, and  $c_l$  and  $c_t$  are longitudinal and shear wave velocities in a wedge material, and  $\omega = 2\pi f$  is circular frequency.. In deriving (1) the following conditions have been used:  $\beta_2 = E_2/E_1 \ll 1$ ,  $\eta \ll 1$  and  $\nu \ll 1$ . These conditions are valid in the majority of practical situations. In the general case of layers of arbitrary thickness the integration in (1) should be carried out numerically, with the exception of thin absorbing layers, where it can be performed analytically.

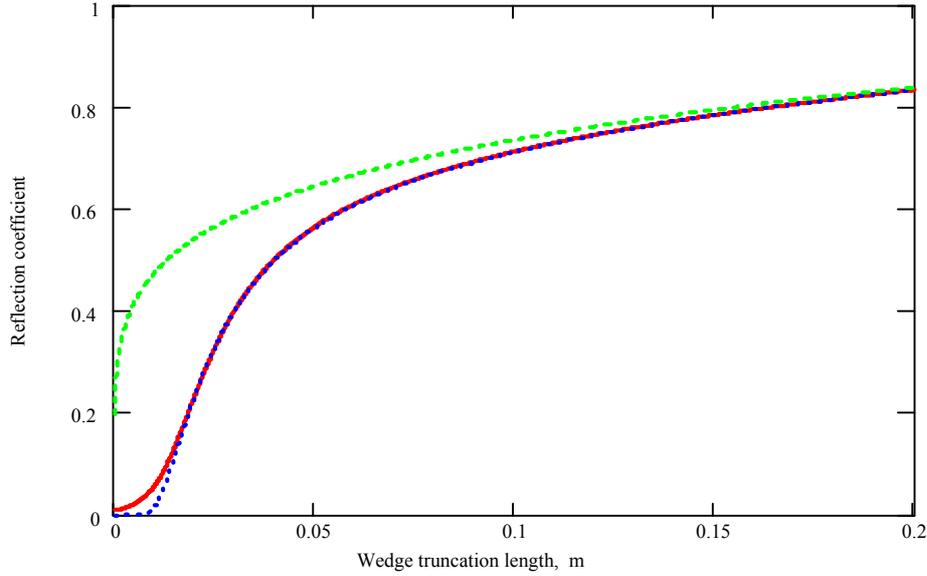


Figure 1. Reflection coefficient  $R_0$  for the non-symmetric wedge covered by the absorbing film on one surface as a function of the wedge truncation length  $x_0$ : solid curve corresponds to the calculations according to equation (1), dotted curve corresponds to the simplified analytical expression, and dashed curve shows the reflection coefficient for the uncovered wedge [4].

Figure 1 shows typical results of calculations of the reflection coefficient  $R_0$  for a quadratic wedge [4]. The reflection coefficient has been calculated at frequency  $f = 10 \text{ kHz}$  as a function of the wedge truncation length  $x_0$  for the non-symmetric wedge covered by the absorbing film on one surface only. The parameters of the wedge and film are:  $\varepsilon = 0.05$ ,  $\delta = 10 \mu\text{m}$ ,  $\nu = 0.2$ ,  $\eta = 0.01$ ,  $x_0 = 2 \text{ cm}$  and  $E_2/E_1 = 0.3$ . For comparison, the behaviour of the reflection coefficient for an uncovered wedge is shown in Figure 1 as well. It is clearly seen that the curves calculated according to the simplified equations and to the more precise formula (1) almost coincide with each other everywhere except very small values of  $x_0$ , where the approximation of thin film becomes invalid. One can see that the values of the reflection coefficient are remarkably low for small values of  $x_0$ . Thus, the

combination of wedges with power-law profiles and of thin damping layers can result in very efficient damping systems for flexural vibrations. Note that almost all absorption of the incident wave energy takes place in the vicinity of the sharp edge of a wedge.

## EXPERIMENTAL INVESTIGATION

In this section we present some of the results of the experimental investigation of the damping system consisting of a steel wedge of quadratic shape covered on one side by an adhesive strips of absorbing layer of various thickness  $\delta$  that was located at the sharp edge of the wedge (see Figure 2). The wedge dimensions were: 280 mm (length) and 200 mm (width). Its thickness at the thick end was 4.5 mm, and the value of the quadratic wedge parameter  $\varepsilon$  was  $5 \times 10^{-5} \text{ mm}^{-1}$ . Measurements of point mobility have been carried out in a wide frequency range (100-6500 Hz) for a free wedge, for a wedge covered by an adhesive strip of absorbing layer, and for a free and covered plates of constant thickness  $h = 4.5 \text{ mm}$  having the same length and width as the quadratic wedge.

Experiments were performed within the Noise and Vibration Laboratory of the Aeronautical and Automotive Engineering Department at Loughborough University. All signal processing was performed using a HP 3566 FFT analyser. Other equipment included a Ling Dynamic Systems 200 series electromagnetic shaker; a Bruel & Kjaer Type 8200 force transducer; a Bruel & Kjaer Type 4374 accelerometer; and ENDEVCO Model 27218 charge amplifier.



Figure 2. Experimental set-up.

A shaker was providing the excitation input to the wedge (see Figure 2). The shaker was attached to the bottom surface of the wedge using a steel ‘stinger’. A Bruel & Kjaer Type 8200 force transducer was then attached to the end of the ‘stinger’ using a studded fastener. The force transducer was attached to the plate surface via adhesive to minimise mass loading and improve result validity. Support of the wedge and plate examined in the study was provided by foam, as illustrated in Figure 2. In the case of the rectangular plate, the entire surface of the plate was supported by a foam block to ensure a uniform reaction. The support of the quadratic wedge applied the same approach but necessitated the partial removal of foam support in the proximity of the thin edge to enable the free vibration of this section of the wedge. A small aperture was made in the foam to allow access of the force transducer and ‘stinger’ assembly. A random signal was generated using the HP analyser. Frequency response analysis was then performed on the force transducer and accelerometer measurements.

The results of measurements of point mobility of the free plate are shown in Figure 3. Note that the attachment of adhesive strips of absorbing layers of various thickness at any of the plate edges did not cause noticeable changes in the frequency response functions, which therefore are not shown here.

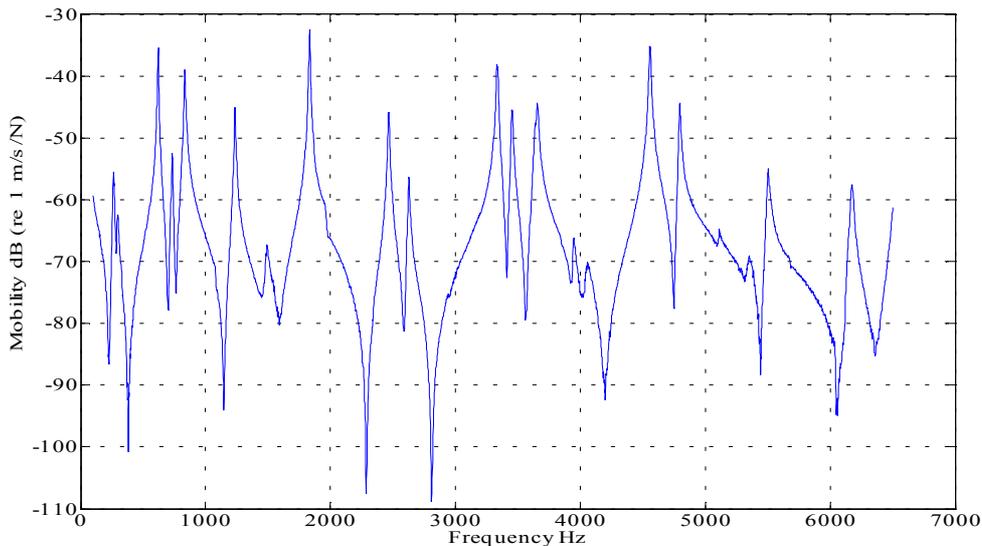


Figure 3. Point mobility of the free plate; the position of the shaker was at 100 mm from one of the shorter edges and at 100 mm from both longer edges.

The results of typical measurements of point mobilities of the free quadratic wedge and of the same wedge covered by a strip of thin absorbing layer are shown in Figure 4. As one can see from the above figures, in the wedge covered by an absorbing layer there is a significant reduction of resonant peaks, in comparison with the uncovered wedge. This can be attributed to the significant reduction of the reflection coefficient

of flexural waves from the sharp edge of the wedge, in agreement with the theory briefly discussed in the previous section.

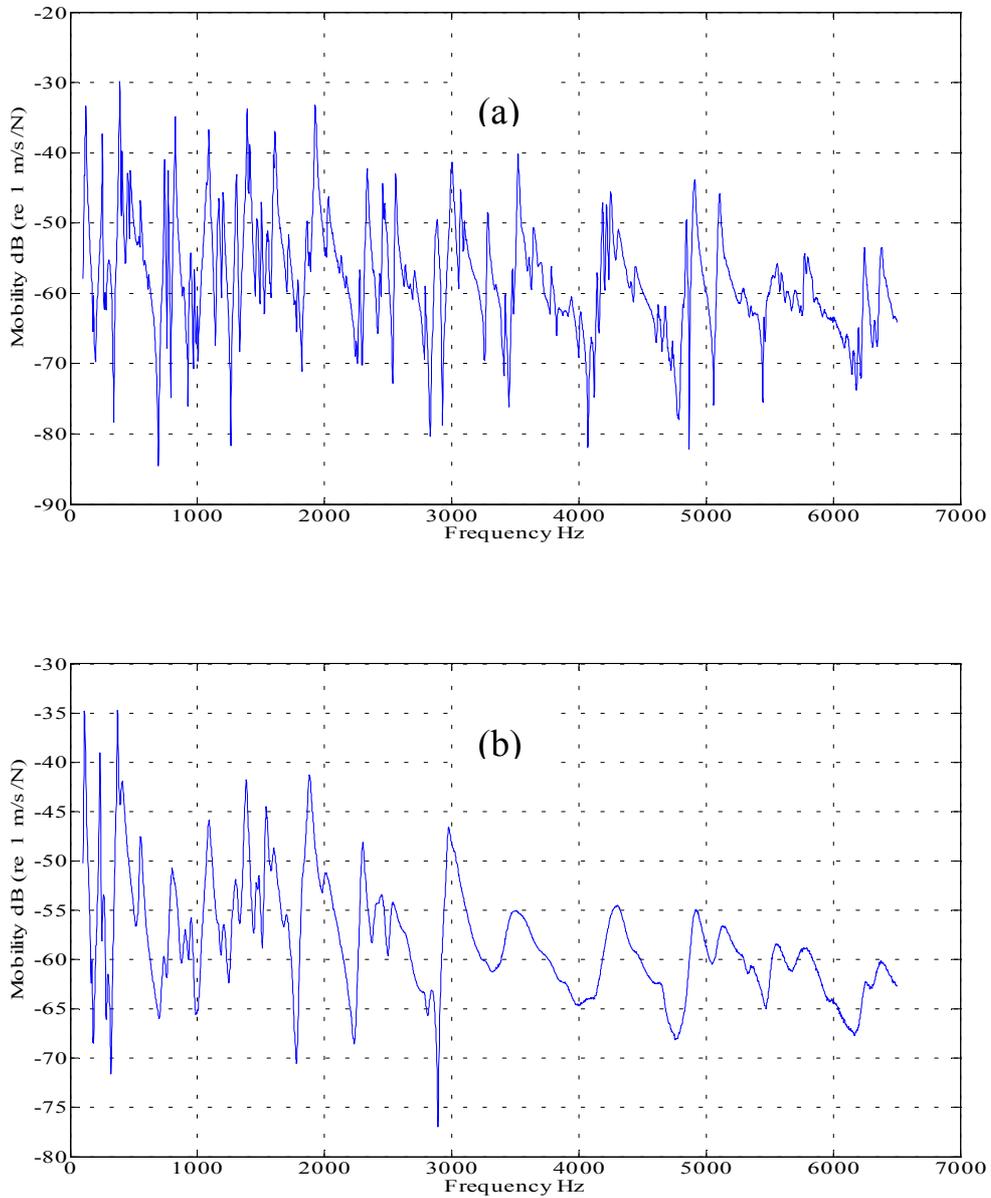


Figure 4. Point mobilities of the free quadratic wedge (a) and of the same wedge covered at the sharp edge by a thin adhesive strip ( $\delta = 0.2$  mm) of absorbing layer (b); the position of the shaker was at 100 mm from the thick end of the wedge and at 100 mm from both side ends.

## CONCLUSION

The reported results of the experimental measurements of point mobilities of the free quadratic wedge and of the same wedge covered by adhesive strips of absorbing layers demonstrate a significant suppression of resonant peaks. This can be attributed to the significant reduction of the reflection coefficient of flexural waves from the sharp edge of the wedge, in agreement with the above-mentioned theory utilising the acoustic black hole effect.

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