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Dynamic interfacial fracture with higher-order vibration modes

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Abstract. The dynamic energy release rate (ERR) of a stationary crack in a double cantilever beam (DCB) under a constant applied opening rate at the free end is derived using beam dynamics, and completely analytical formulae are obtained. It is discovered that the dispersive propagation of flexural waves must be considered otherwise the contributions to the dynamic ERR from the higher-order vibration modes cannot be correctly captured. In addition, it is shown that the intact part of the DCB (ahead of the crack tip) can be replaced with a single vertical spring and a rotation constraint at the crack tip to significantly simplify the analytical modeling. The dynamic ERR is accurately captured in comparison to finite element method (FEM) simulations, including the higher-order vibration modes.

Keywords: Beam dynamics, Energy release rate, Double cantilever beam, Vibration.

1 Introduction

Structures in various engineering applications often experience time-dependent external excitation, where the structure's inertia leads to a significant extra dynamic effect in its response. Among many others, examples include vehicle suspension systems; tower blocks during earthquakes; drilling of plates; and buildings or offshore platforms subject to wind or wave loading. Dynamic effects also play a major role in fracture behavior, particularly along interfaces and joints, which usually represent planes of weakness.

In this work, an analytical theory is developed to predict the vibration-mode partitioned dynamic energy release rate (ERR) of a stationary crack in double cantilever beam (DCB) under a constant applied opening rate at the free end. The DCB is a fundamental structure which is ideal for developing fracture theory, and the findings and methodology directly transfer to more complex structures. Furthermore, the DCB is the

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typical structure used to determine the mode I fracture toughness of materials in experimental testing.

Previous studies which analytically model DCBs including the dynamic effect are represented by Refs. [1,2], in which kinetic energy is also considered in the conservation of energy to derive the dynamic ERR. Static deflection is assumed, and so localized vibration is not considered. Both experiments [3] and numerical simulations [4], however, confirm that the dynamic ERR is in general oscillatory.

The authors [5,6] recently developed an analytical theory for the dynamic ERR of a crack under general applied displacement and achieved excellent agreement with finite element method (FEM) simulations. The theory, however, was for the first vibration mode only, did not consider wave propagation, and was also slightly out-of-phase due to the simplified boundary condition. To the best of the authors' knowledge, the theory of dynamic ERR with consideration for vibration with higher-order modes has not been solved before. This is achieved in this work by taking wave propagation into account. Furthermore, an improved boundary condition in comparison to Ref. [6] is proposed which substantially corrects the phase difference between the FEM results and the analytical theory. The FEM is used to verify the analytical theory, confirming that the phase and amplitude of the dynamic ERR is accurately captured, including higher-order vibration modes. Note that this work is published in full in Ref. [7].

2 Theory

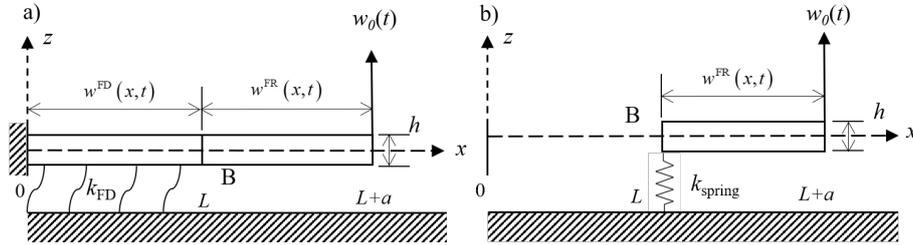


Fig. 1. Schematic of beam partially resting on the elastic foundation.

The dynamic ERR of a symmetric DCB can be investigated by using the figure shown in Fig.1a, which shows a beam of thickness h , partially supported on an elastic foundation of stiffness k , and with time-dependent displacement $w_0(t) = vt$ applied to midplane of its free end. The length of beam supported on the elastic foundation is L , which represents the uncracked region ahead of the crack tip, and a is the length of the crack. The crack tip is labelled as 'B', which is at $x = L$.

The foundation stiffness is assumed to be large and so the deflection of the supported section of beam, $w^{\text{FD}}(x,t)$, can be treated as small and static. Fig.1a can thus be approximately represented by Fig.1b, where the beam section ahead of the crack tip is replaced by a vertical spring at the crack tip to produce approximately the same mechanical effect. The crack tip spring stiffness k_{spring} can be written as $k_{\text{spring}} = k_0 EI / a^3$

where k_0 is a dimensionless number that needs to be determined empirically and I is the second moment of area. For a stiff interface with $L \gg h$ and $a \gg h$, a good approximation for k_0 may be $k_0 = \gamma La^3 / (5I)$ where $\gamma = k_{FD} / E$.

The deflection of $w^{FR}(x, t)$ can be written in the form $w^{FR}(x, t) = w_{fv}^{FR}(x, t) + F(x)vt$, where $w_{fv}^{FR}(x, t)$ and $F(x)$ are the vibration component and shifting function respectively. By solving the equation of motion, $Elw^{FR(4)}(x, t) + \rho Av\ddot{w}^{FR}(x, t) = 0$, the deflection of the beam is derived as

$$y(x, t) = -va^2 \sqrt{\frac{\rho A}{EI}} \sum_{i=1}^{\infty} \frac{\Lambda_i}{\lambda_i^3 \xi_i} \phi_i(x) \sin \omega_i t + \frac{(6a^3 + 3ak_0x^2 - k_0x^3)}{2a^3(k_0 + 3)} vt \quad (1)$$

where λ_i is the wavenumber, which is derived from frequency equation, $\phi_i(x)$ is the shape function, $\Lambda_i = -(\sec \lambda_i + \operatorname{sech} \lambda_i)$, and $\xi_i = 0.5(\sec^2 \lambda_i - \operatorname{sech}^2 \lambda_i) + 3\lambda_i^2 k_0^{-1}$.

By considering the conservation of energy for an elastic structure with a crack, together with the deflection in Eq. (1), the time-accumulated energy dissipated from the whole system in opening the crack is

$$\Gamma = - \left\{ \begin{array}{l} -\frac{1}{2} \frac{(3k_0^2 + 9k_0)}{(k_0 + 3)^2} Elv^2 t^2 a^{-3} - \frac{1}{2} \rho Av^2 a \sum_{i=1}^{\infty} \left(\frac{\Lambda_i^2}{\lambda_i^2 \xi_i} \right) \\ + \rho Av^2 a \sum_{i=1}^{\infty} \frac{\Lambda_i^2}{\lambda_i^2 \xi_i} \cos \omega_i t - \frac{3}{280} \rho Av^2 a \frac{(11k_0^2 + 105k_0 + 420)}{(k_0 + 3)^2} \end{array} \right\} \quad (2)$$

Now by considering the energy flux through a small region surrounding the crack tip [8] and accounting for the dispersiveness of flexural waves, the dynamic energy release rate is

$$G = \frac{v^2}{b} \left[\frac{9}{2} \frac{(k_0^2 + 3k_0)}{(k_0 + 3)^2} \frac{El t^2}{a^4} - \frac{3\rho A(11k_0^2 + 105k_0 + 420)}{280(k_0 + 3)^2} + (-1)^j \frac{\sqrt{\rho AEI} t}{a^2} \sum_{i=1}^{\infty} \frac{\lambda_i \Lambda_i^2}{\lambda_i \xi_i} \sin \omega_i t \right] \quad (3)$$

The three terms are, respectively, due to the strain energy of static motion, the kinetic energy of static motion, and due to vibration. Note that the dynamic ERR is proportional to square of the applied opening velocity.

3 Numerical verification

A 2D FEM model of a full DCB was built using plane stress elements (CPS4R) in Abaqus/Explicit. The length of the uncracked region was $L = 20$ mm, the length of the cracked region was $a = 60$ mm, and the thickness of each arm was $h = 2$ mm. The plane stress thickness is a nominal 1 mm. A constant rate opening displacement of 1 m s^{-1} was applied to the DCB tips in opposite directions. The material had a Young's modulus of 10 GPa, a Poisson's ratio of 0.3, and a density of 1 kg m^{-3} . The uncracked region was formed by sharing nodes between the upper and lower arms, and the virtual

crack closure technique was used to determine the ERR. A uniform mesh was used with square elements of size 0.1 mm. All the viscoelastic parameters were set to zero in case of any damping.

The analytical results for the dynamic ERR with successively more vibration modes with $\gamma = 0.9$ are compared against the FEM results in Fig. 2. The analytical results are in good agreement with the FEM, except for small phase shift, which is reasonable considering the approximate boundary condition applied at the crack tip. Furthermore, this phase shift is substantially smaller than that seen in Ref. [6] which uses a fixed boundary condition at the crack tip. Note that using only the first three vibration modes is adequate to capture the oscillation of the dynamic ERR and adding more vibration modes makes an insignificant difference.

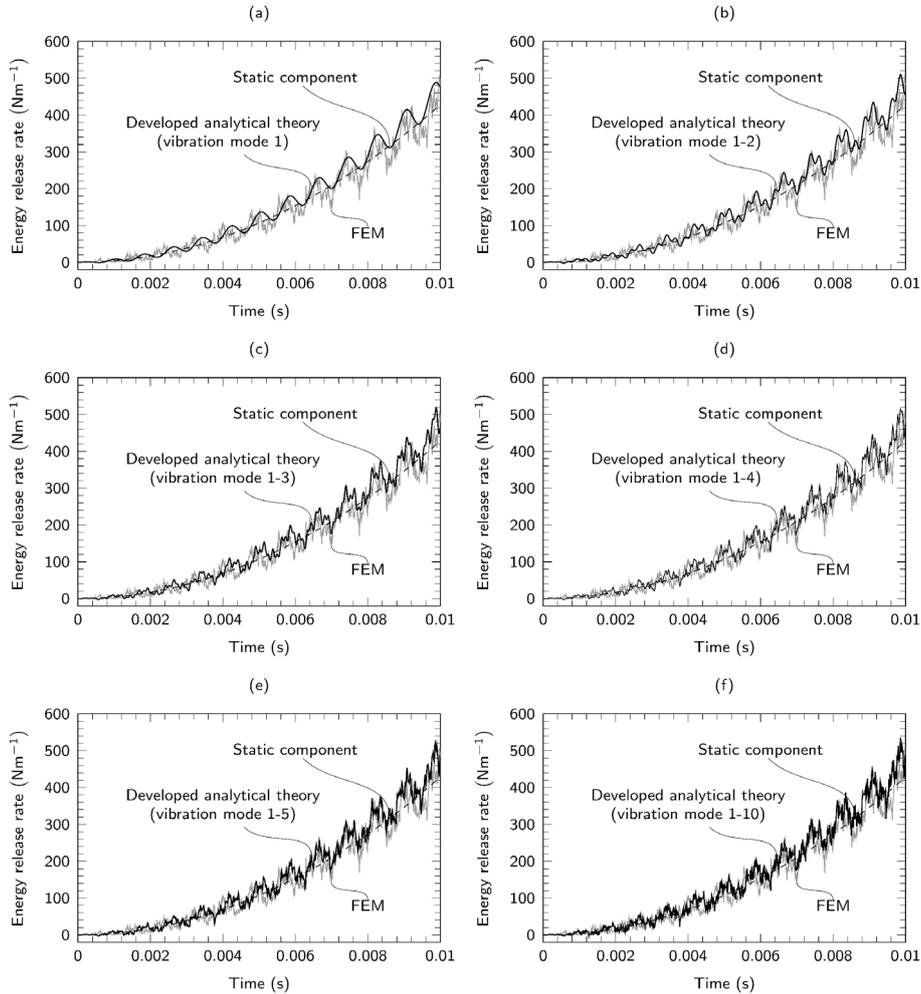


Fig. 2. Analytical results for dynamic ERR compared against FEM result.

4 Conclusion

An analytical theory based on beam dynamics, energy conservation, and wave propagation has been derived that determines the dynamic ERR of a DCB under a constant applied opening rate at the free end. It has been shown that the dispersive propagation of flexural waves must be considered in the calculation of dynamic ERR, otherwise contributions from the higher-order vibration modes cannot be correctly captured. In addition, it has been shown that the intact part of the DCB (ahead of the crack tip) can be replaced with a vertical spring and a rotation constraint at the crack tip to significantly simplify the analytical modeling. The dynamic ERR is accurately captured in comparison to finite element method (FEM) simulations, including the higher-order vibration modes. This novel analytical theory can be applied to calculate dynamic fracture toughness in experimental tests with DCBs, and to predict dynamic fracture behavior in engineering structures.

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